Nonlinear model predictive control of a Stewart platform based on improved dynamic model

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Abstract This paper presents a nonlinear model predictive control (NMPC) for six-degree-of-freedom (DOF) Stewart Platform based on parallel mechanisms. Nowadays, NMPC has been used in many applications in industry. First, nonlinear equations related to Stewart platform dynamic are extracted using Lagrange method. The advantages of this dynamic model are improved and are highly accurate because we take into account rational velocity and acceleration of pods around their longitudinal axis and accurately model the friction of joint. In this controller, outputs are anticipated at any time. The main advantage of the proposed controller is that constraints can be applied to inputs and outputs. Another benefit is its high precision. If the weights are added to inputs and outputs, the errors of tracking will reduce in outputs. In current work, three different trajectories were used in simulation to verify the performance of designed and proposed NMPC. Also, we extracted PID controller results to compare and validate the NMPC.

Key-Words: Nonlinear Model Predictive Control, Stewart Platform, parallel mechanisms, Lagrange method

1 Introduction

Model predictive control (MPC), also known as moving horizon control (MHC) or receding horizon control (RHC), is a control technique based on which the applied input is calculated on-line by solving an open-loop optimal control problem over affixed prediction horizon into the future. The first part of the obtained open-loop signal is implemented until measurements become available. Based on the new information, the open-loop optimal control problem is solved again and the whole procedure is repeated. One of the advantages of the MPC controller is its use in MIMO systems (multiple input and output). Most systems are inherently non-linear, but the MPC applications are widely based on linear dynamic model under uncertainty or noise. Therefore, linear models are often not useful in describing the dynamics, so non-linear models must be considered. The inadequacy of linear models is one the motivations for the increasing interest in nonlinear model predictive control [1-3]. In this paper, the 6 DOFs Stewart platform is observed to NMPC. Stewart platform has many advantages such as: good dynamic performance, high accuracy, high rigidity, and high load to weight ratio compared to a serial robot. Due to nonlinear characteristics of dynamic parameters such as Inertial and Coriolis terms, the dynamic models of Stewart platforms are strongly nonlinear. In most research studies to design a controller, using the simplified linear dynamic model creates problems for the system in hard conditions [4-6]. Hence, application of nonlinear strategies is strongly advised. The dynamic modeling of a parallel mechanism in terms of the ability to control its movement, especially when accurate positioning and good dynamic performance of mechanism with the loading on it are intended, has special importance and is considered as the first step in the analysis of control mechanism, which is the reason why it has been the subject of several studies like the present study. Unlike series of open-loop mechanisms, dynamic modeling of parallel mechanisms has inherent computational complexity due to kinematic constraints and closed-loop chain. Several methods, based on the principles of classical mechanics, have been provided by researchers for the dynamic analysis of mechanism. Regarding the dynamic solution of the Stewart platform, some important contributions are summarized as below: According to the fact that dynamic analysis is performed by Newton-Euler method for Stewart platform by applying the simplification assumptions and is presented by researchers in several studies, there have been some technical problems. Therefore, Pedrammehr et al. [7] studied the inverse kinematic and dynamic equations of mechanism with the application of accurate assumptions and also reforming the most bugs available in the previous research. They conducted inverse kinematics and dynamics analysis of the desired platform by accurately calculating the centres of basic mass and taking into account all frictional forces, Coriolis, inertia and external forces using Newton-Euler method and provided accurate and complete results of kinematic and dynamic equations. However, using the simplified friction model in their dynamic analysis is the main drawback of their contribution. Given the location and general motion of moving Stewart platform of 6-DOFs parallel mechanism and using inverse kinematics analysis, Stefan [8] initially determined the position, velocity and acceleration of the mechanism links and then solved the problem of inverse dynamics of the platform using the principle of virtual work to provide kinematic and dynamic solution of the platform. Antonio [9] provided an approach based on a generalized momentum of the actuator for dynamic modeling of Stewart platform which is used to calculate the generalized forces applied on Stewart platform. Also in this study, analytical expressions are brought for terms inertial, Coriolis and central which are effective in calculating the gravitational areas of generalized forces obtained through potential energy of the actuator. Ouarda [10] provided a new method based on recursive Newton-Euler method for direct and inverse dynamic modeling of Stewart platform.

Proportional integral derivative (PID) classic controllers, which are widely used in control of industrial mechanisms, never could guarantee functional control of parallel mechanisms alone [11]. Therefore, controlling of parallel mechanisms as a nonlinear multivariable system [12,13] and providing control strategies and ideas to increase the accuracy of the performance of the parallel mechanisms have always been of particular interest to researchers [14-16]. The idea of adaptive, robust and sliding mode strategies [17-19], and controllers based on intelligent controller model using neural networks are cases many of which have been recently used by researchers in the control of parallel mechanisms [20-22]. Chifu Yanga et al. [23-24] have conducted an extensive body of research on the dynamics and control of Stewart parallel platform including designing of decoupling controller equipped with a mutual interference pre-compensator (CCPC), designing a robust adaptive controller, and providing proportional - derivative control algorithm with gravity compensator. Ashraf Omran et al. [25] provided an algorithm for the optimal control of Stewart platform including two optimized phases. The first phase seeks an optimal polynomial approximation from direct kinematic model of Stewart platform using the function square error of prediction, and the second phase optimally determines the interest of controller depending on the platform working conditions. Hong Bo et al. [26] presented a sequential control algorithm based on sliding mode for a 6-DoF parallel mechanism with a hydraulic actuator. Their strategy includes two internal and external control loops for dynamic separation of the mechanical part from the hydraulic part. Yangjun Pi et al. [27] examined the control problem of the 6-DoF parallel mechanism with the hydraulic actuators in the joint space. In their study, at first a nonlinear disturbance observer for estimation and compensation of external uncertain disturbances is designed and then sequential control algorithm for separation of hydraulic and mechanical dynamic is provided. Yang Bo et al. [28] designed a robust control algorithm for 6-DoF Stewart parallel mechanism using a combination of PID controller with fuzzy logic control rules. The comparison of the PID controller in the joint space and generalized predictive controller (GPC) for 6-DoF Stewart platform was presented by Rosario et al. [29]. Sung-Hua et al. [30] presented the nonlinear observer and a sliding mode controller to control the six states of the 6-DoF moving platform directly. They solved the forward kinematics problem to achieve the output feedback control. Recently, application of new and efficient control strategies for Stewart mechanism is developed and it can be a subject for extensive researches [31-35].

In this paper, the nonlinear model predictive control is designed for Stewart Platform. First, the nonlinear equations related to kinematic and dynamic of Stewart platform are extracted by using homogeneous matrix and Lagrange method, respectively. We extracted a high accurate dynamic model using friction for joints and considering the rotational movement of pods around their longitudinal axis which were neglected for simplification by pervious researchers. Then, NMPCs were designed and simulated for Stewart platform with 6-DOFs. To this end, we applied essential constraints and weights to inputs and outputs to optimize the cost function which leads to error reduction in reference trajectory tracking. To predict, the plant was used for desired cost function over the predication horizon. In current work, three different paths as circle curve, oscillating circle curve and eight curve that were used for dynamic analysis and NMPC simulation in Stewart platform. Likewise, the results of PID controller were extracted to compare and validate the performance and accuracy of the designed NMPC. The results show high accuracy and strong performance of our proposed NMPC. Briefly, the main contribution of the current work is application of a nonlinear control strategy versus using the simplified linear dynamic model.

2 Kinematic and Dynamic Analysis of Stewart Platform

In this paper, the Stewart platform is considered with 6 inputs and 6 degrees of freedom (Fig. 1). In the present mechanism, platforms which are input driven of Stewart platform have been formed from two lower and upper parts. The lower part of the platforms is connected to the bottom plate via a universal joint (U), and the upper part is connected to the upper moving platform by a spherical connection (S). Also, both the upper and lower parts of pod are linked together via a Ball Screw system that converts DC motor rotational motion transmitted into linear motion with the help of coupling to the screw. This part of the platforms can be a reciprocating linear connection (P) due to their linear motion.

Fig 1: Stewart Platform Mechanism

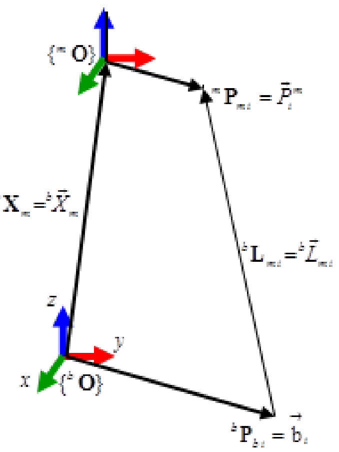

Fig 2: Coordinate axis used in Stewart platform analysis

According to Figure 1, two separate coordinate axes for analyzing Stewart platform will be used as follows:

1- Coordinate axes connected to a lower fixed platform which are attached to the geometric center of the fixed platform and are fixed $\{^{b}O\}$

2- - Coordinate axes connected to upper moving platform which are attached to the geometric center of the upper moving platform and are moving $\{^{m}O\}$

For kinematic analysis of the platform, we use a homogeneous matrix method. Transfer homogeneous matrix method is in fact the accumulated mode of vector chain method, but its advantage is that kinematic model of Stewart platform with infinitely many modules can be easily obtained by the homogeneous transfer method. Of course, the main reason for presenting the method in this paper is to use it in removed algorithm and direct kinematic solving the Stewart platform. According to the definition of homogeneous matrix, the matrix is defined as follows:

$$H_{^{m}O,^{b}O} = \begin{pmatrix} {}^{b}R_{m} & {}^{b}X_{m} \\ 0 & 1 \end{pmatrix} \tag{1}$$

where, matrix R is a rotation matrix which will be defined as follows:

$${}^{b}R_{m} = \begin{pmatrix} C\theta_z C\theta_y & C\theta_x S\theta_y S\theta_x - S\theta_z C\theta_x & C\theta_z S\theta_y C\theta_x + S\theta_z S\theta_x \\ S\theta_z C\theta_y & S\theta_z S\theta_y S\theta_x + C\theta_z C\theta_x & S\theta_z S\theta_y C\theta_x - C\theta_z S\theta_x \\ S\theta_z & C\theta_y S\theta_x & C\theta_y C\theta_x \end{pmatrix} \tag{2}$$

Where $H_{^{m}O,^{b}O}$, is homogeneous transfer matrix from coordinates $\{^{m}O\}$ to $\{^{b}O\}$. With regard to transfer relations of vectors from a one coordinate to the other, vectors of platforms in each module is determined with the help of homogeneous transfer matrix as follows:

$$ {}^{b}\mathbf{L}'_{mi} = \mathbf{H}_{{}^{m}O,\,{}^{b}O}\ {}^{m}\mathbf{P}'_{mi} - {}^{b}\mathbf{P}'_{bi} \tag{3} $$

In equations above, considering that the matrix, $\mathbf{H}_{{}^{m}O,{}^{b}O}$, is square matrix 4×4, thus parameters $\mathbf{L}'$, $\mathbf{P}'$ which are $\mathbf{L}$, $\mathbf{P}$ defined as vector , which is given below, so that the above equations in terms of dimensions are true:

$$ \mathbf{P}' = [\mathbf{P},1]^T = [p_{xi}\ \ p_{yi}\ \ 0\ \ 1]^T \tag{4} $$

$$ \mathbf{L}' = [\mathbf{L},1]^T = [L_{xi}\ \ L_{yi}\ \ L_{zi}\ \ 1]^T \tag{5} $$

Since dynamic modeling of mechanisms is very important because of the importance of their role in simulating behaviour and designing the control of mechanism, the dynamic analysis of Stewart platform with approach to energy and using Lagrange - Euler method was done in this section. For this purpose, kinetic and potential energy of moving platforms and platforms of each module in terms of generalized coordinates and velocities, $\mathbf{q}, \dot{\mathbf{q}}$ are extracted and then Lagrange equations are used to determine the dynamic equations. The use of Lagrange method is better preferred compared to Newton-Euler method, because it determines the terms inertial, Coriolis, centrifugal, gravity and friction in the derived dynamic equations separately, although the volume of calculations will increase. Also in this analysis, joint frictional forces using Coulomb's friction model the dynamic equations have been applied to determine.

Dynamic equations for Stewart mechanism are obtained using the Lagrangian formulation by applying the following Lagrange equation:

$$ \frac{\partial}{\partial t}\left(\frac{\partial K}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial K}{\partial \dot{\mathbf{q}}} + \frac{\partial U}{\partial \mathbf{q}} = \boldsymbol{\tau} \tag{6} $$

In which, K, U are kinetic and potential energy respectively and $\mathbf{q}, \dot{\mathbf{q}}$ are joint space variables. Lagrangian formulation yields the dynamic equations in the standard form:

$$ \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{C}_F + \mathbf{J}^{-T}\ \mathbf{F}_P = \boldsymbol{\tau} \tag{7} $$

Where $\mathbf{M(q)}$ is the positive-definite inertia matrix, $\mathbf{N}(\mathbf{q},\dot{\mathbf{q}})$ the centrifugal and Coriolis term, $\mathbf{G}(\mathbf{q})$ the gravitational term, $\mathbf{J}^{-T}$ is velocity Jacobian matrix, $\mathbf{C}_F$ and $\mathbf{F}_P$ describes coulomb and viscous frictions torques/forces and $\boldsymbol{\tau}$ states the actuated joint forces. $\mathbf{M(q)}$ can be determined from kinetic energy directly and $\mathbf{N}(\mathbf{q},\dot{\mathbf{q}})$ is calculated as:

$$ \mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) = \dot{\mathbf{M}}(\mathbf{q})\ \dot{\mathbf{q}} + \frac{\partial}{\partial \mathbf{q}}\left(\frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\ \dot{\mathbf{q}}\right) \tag{8} $$

Where:

$$ \dot{\mathbf{M}}(\mathbf{q})\ \dot{\mathbf{q}} = \sum_{j=1}^{6}\sum_{n=1}^{6}\left(\dot{q}_j\ \dot{q}_n\ \frac{\partial M_{kj}}{\partial q_n}\right)_{k=1,\dots,6} \tag{9-a} $$

$$ \frac{\partial}{\partial \mathbf{q}}\left(\frac{1}{2}\dot{\mathbf{q}}^T\,\mathbf{M}(\mathbf{q})\ \dot{\mathbf{q}}\right) = \frac{1}{2}\left(\sum_{j=1}^{6}\sum_{n=1}^{6}\dot{q}_j\ \dot{q}_n\ \frac{\partial M_{nj}}{\partial q_k}\right)_{k=1,\dots,6} \tag{9-b} $$

Therefore:

$$ \mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) = \left[\sum_{j=1}^{6}\sum_{n=1}^{6}\dot{q}_j\ \dot{q}_n\ \frac{\partial M_{kj}}{\partial q_n} - \frac{1}{2}\sum_{j=1}^{6}\sum_{n=1}^{6}\dot{q}_j\ \dot{q}_n\ \frac{\partial M_{nj}}{\partial q_k}\right]_{k=1,\dots,6} \tag{10} $$

And:

$$ \mathbf{G}(\mathbf{q}) = \frac{\partial U}{\partial \mathbf{q}} \tag{11} $$

3 Nonlinear Model Predictive Control Problem

Our plant that should be controlled is described by the following discrete-time, nonlinear, state-space model:

$$ \begin{aligned} x_{k+1} &= f(x_k, u_k, v_k, w_k) \\ y_k &= g(x_k) + \xi_k \end{aligned} \tag{12} $$

Where $u_k \in R^{12\times1}$ is the vector of inputs or manipulated variables (MV's), $y_k \in R^{12\times1}$ is the vector of outputs or controlled variables (CV's), $x_k \in R^{12}$ is a the vector of state variables, $v_k \in R^{12}$ is the vector of measured disturbance variables (DV's), $w_k \in R^{12}$ is the vector of unmeasured DV's or noise and $\xi_k \in R^{12}$ is the vector of measurement noise.

The bias term that compares the current predicted output y_k to the current measured output y_k^m is calculated:

$$\hat{y} = y_k^m - y_k \tag{13}$$

We added bias term to model for use in subsequent predictions:

$$y_{k+j} = g(x_{k+j}) + \hat{y}_k \tag{14}$$

The nonlinear model predictive control algorithms are described here by minimizing the following dynamic objective:

$$J = \sum_{j=1}^{N} \| y_{k+j} - y_s \|_{Q_j}^2 + \sum_{j=0}^{M-1} \| \Delta u_{k+j} \|_{S_j}^2 + \sum_{j=0}^{M-1} \| u_{k+j} - u_s \|_{R_j}^2 + \| s \|_T^2 \tag{15}$$

Subject to constraint

$$\begin{aligned} & x_{k+j} = f(x_{k+j-1}, u_{k+j-1}) \\ & y_{k+j} = g(x_{k+j}) + \hat{y}_k \\ & \underline{y}_j - s \le y_{k+j} \le \overline{y}_j + s \\ & \underline{u} \le u_{k+j} \le \overline{u} \\ & \Delta\underline{u} \le \Delta u_{k+j} \le \Delta\overline{u} \\ & s \ge 0 \end{aligned} \tag{16}$$

The desired steady-state (x_s, y_s, u_s) is determined by local steady-state optimization, which may be based on economic objective. The relative importance of the objective function contributions is controlled by setting the time dependent weight matrices Q_j, S_j and R_j; these are chosen to be positive definite. S is used to minimize the size of output constraint violations; T is chosen positive definite.

The basic structure of a NMPC control loop is shown in Figure 3. A state estimate is calculated based on the applied input and the measured outputs. This estimate is added to the NMPC controller which calculates a new state and applied to the plant. The set point or trajectory is added to overall loop.

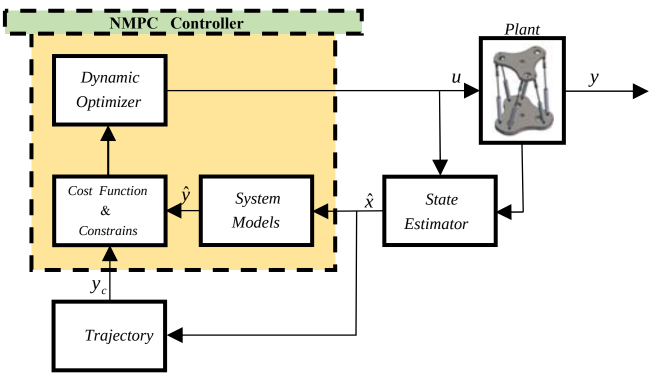

Fig 3: NMPC Controller

Summarizing, a NMPC scheme works as follows:
1. Calculate measurements of state of the plant
2. Calculate an optimal input minimizing the desired cost function over the predication horizon using the plant for prediction.
3. Implement the first part of the optimal input until the next recalculation instant
4. Continue with 2

4 Simulation and Results

The simulation is carried out with a six-degree freedom Stewart platform with geometric specification given in Table 1. Our trajectory is used to test the performance in Table 2.

Table 1: Geometric specifications of Stewart platform

Parameters	Values
Platform Radius	0.5 m
Pod Radius	0.5 m
Mass Platform	1.5 kg
Mass of Upper Pod	0.1 kg
Mass of Lowe Pod	0.1 kg
Platform Inertia	diag[0.08,0.08,0.08]
Upper Pod Inertia	diag[0.00625,0.00625,0]
Lower Pod Inertia	diag[0.00625,0.00625,0]

Table 2: The trajectories used to simulation

Trajectories	x(t)	y(t)	z(t)
I-Circle Curve	*7*sin(t)*	*7*cos(t)*	*0*
II-Circle oscillating Curve	*7*sin(t)*	*10*cos(t)*	*2.5*sin(4t)*
III-Eight Curve	*7*sin(t)*	*7*sin(t)*cos(t)*	*0*
IV Eight oscillating Curve	*7*sin(t)*	*7*sin(t)*cos(t)*	*2.5*sin(4t)*

The forces changing for each pod by using equation (12) to tracking trajectory (*II*) are shown in Figure 4:

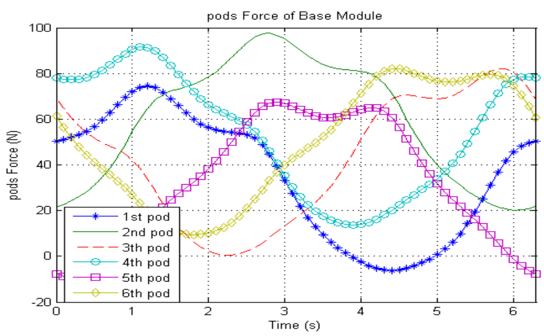

Fig4: Forces for each pods for trajectory II

To validate the performance of NMPC controller, we compared it PID controller. PID gains were selected as $k_p = 200, k_i = 800, k_d = 10$, which guarantees a good tracking performance. In this paper, sampling time for controller is 0.01 and duration of the simulation is 2000 samples.

We assumed that the input u has to satisfy the following constraint:

$$u = \{u \in R^{12} \mid -20 \leq u \leq 20\}$$
(17)

The weighting matrices Q and R are the objective functional and are chosen as:

$$\mathbf{Q} = diag\ [10,\ 10,\ 10,\ 5,\ 5,\ 5] \qquad (18)$$

$$\mathbf{R} = diag\ [0.1,\ 0.1,\ 0.1,\ 0.2,\ 0.2,\ 0.2] \qquad (19)$$

The norm of matrices is second order (q=2).

Figure 5 shows the tracking I trajectory in x, y, z axes:

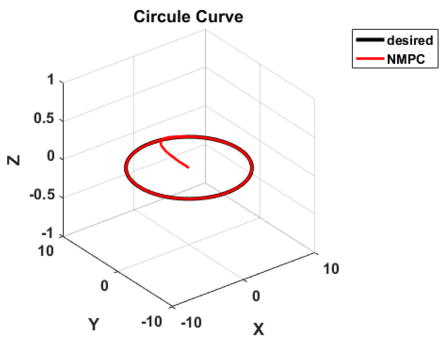

Fig 5: Tracking Circle Curve

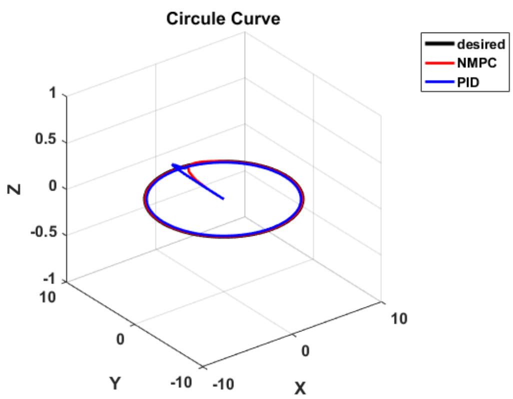

Fig 6: Results of comparison of the NMPC and PID controllers for Circle Curve

Figure 5 shows tracking the circle curve by Stewart platform in x, y, z axes. Platforms cannot track well in first samples but after 0,4 second, platforms can track the desired trajectory as well.

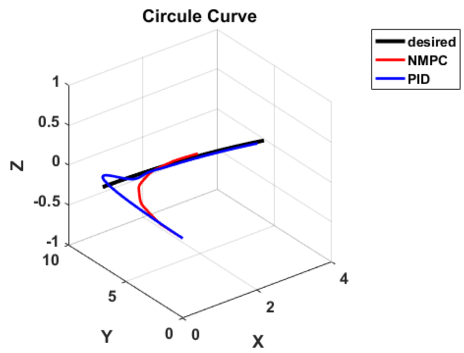

Fig 7: Compare NMPC with PID controller in first 0.5 second

To validate the NMPC output, we can compare its PID controller; the results are presented in figure 6. Figure 7 shows tracking in first 0.4 second. It can show tracking desired trajectory by NMPC controller better than PID controller because unlike PID controller, the dynamic model is nonlinear. Also, PID controller is faster than NMPC controller in tracking the trajectory because of constraints. As well, the accuracy of NMPC controller is higher than PID due to weight and etc.
To validate results and compare them with those obtained from PID controller, two other trajectories simulate for Stewart platform. Figures 8 and 9 are the tracking trajectory II and compare it with PID controller.

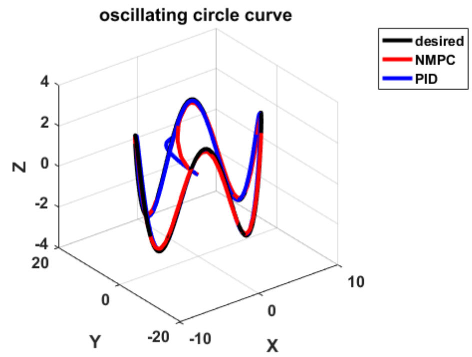

Fig 8: Results of comparison of the NMPC and PID controllers for Circle Oscillating Curve

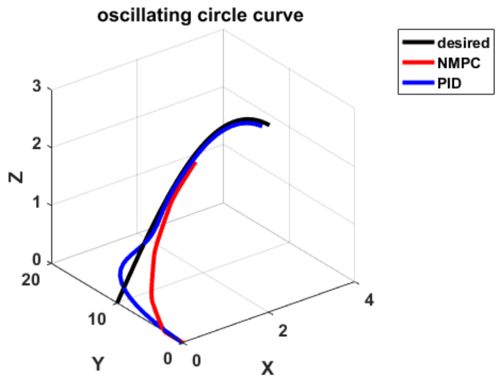

Fig 9: Compare NMPC with PID controller in first 0.4 second

Tracking trajectory III and IV are shown Figures 10 and 11.

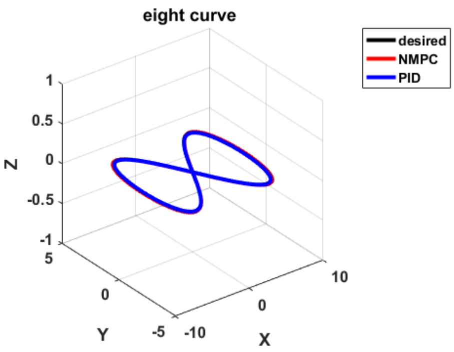

Fig 10: Results of comparison of the NMPC and PID controllers for Eight Curve

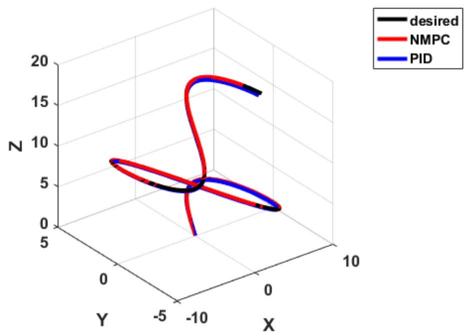

Fig 11: Results of comparison of the NMPC and PID controllers for Eight Oscillating Curve

Generally, NMPC is improved and showed a better performance compared to PID controller. So accuracy in NMPC is better compared with PID controller.

A brief comparison between NMPC and PID controllers is presented in Figures 5 to 11 as follows:

1. NMPC uses nonlinear state space model for predication.
2. NMPC gives us an explicit consideration of state and input contortions.
3. For prediction, the system state must be estimated.
4. For the application of NMPC, a real-time solution of an open-loop optimal control problem is necessary.
5. PID controller is faster than NMPC controller in tracking the trajectory because of constraints.
6. Accuracy of NMPC controller is higher than PID because of weight and etc.
7. The error of NMPC is less than PID controller.

4 Conclusion

The goal of this paper is to design and simulate nonlinear model predictive control (NMPC) for parallel 6-DOFs Stewart mechanism. First, nonlinear Model related to kinematic and dynamic of Stewart mechanism are driven by using homogeneous matrix and Lagrange method, respectively. Because model predictive controllers rely on dynamic models, we try to generate an improved dynamic model of Stewart mechanism by considering joints frictions as coulomb model. Also, the rotational movement of the pods around the axial direction as an extra degree of freedom is taken into account to extract accurate dynamic model in comparison with previous ones. Then, NMPCs were designed and simulated for Stewart platform with 6-DOFs. For this purpose, we applied essential constraints and weights to inputs and outputs for optimizing cost function which leads to error reduction in reference trajectory tracking. For prediction, the plant was used for desired cost function over the predication horizon. The main advantage of the proposed controller is that constraints can be applied to inputs and outputs. Another benefit is the high precision of this method. If weights are added to inputs and outputs, the errors of tracking will reduce in outputs. In the current work, three different paths of circle curve, oscillating circle curve and eight-curve, were used for dynamic analysis and NMPC simulation in Stewart platform. Moreover, we extracted PID controller results to compare and validate the NMPC. Finally, the obtained results showed high accuracy and strong performance of our proposed NMPC.

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