developed the theory of thermomicrostretch elastic solids.

A microelongated elastic solid possesses four degrees of freedom, three for translation and one for microelongation. In microelongation theory, the material particles can perform volumetric microelongation in addition to classical deformation of the medium. The material particles of such medium have a tendency to contract and stretch independently of their translations. Composite materials reinforced with chopped elastic fibers, solid-liquid crystals, porous media with pores filled with nonviscous fluid or gas can be categorized as a microelongated medium.

Kiris and Inan [3] found the Eshelby tensors for a spherical inclusion in a microelongated elastic field and they introduced a special micromorphic model to describe the damaged material that defines the damage as the deformation and the growth of microvoids and microcracks occurred in the material at the microstructural level. Shaw and Mukhopadhyay [4] investigated a functionally graded isotropic unbounded microelongated solid under periodically varying heat sources using Laplace-Fourier transform techniques. Shaw and Mukhopadhyay [5] investigated the influence of moving heat source in a thermoelastic microelongated solid in the context of the generalized theory of heat conduction. Ailawalia, Sachdeva and Pathania [6] studied a two dimensional deformation problem in a thermoelastic microelongated medium with internal heat source.

In this paper, a two dimensional problem of an infinite microelongated thermoelastic circular plate is solved by using the eigenvalue approach following the Laplace and Hankel transforms. Using the numerical inversion technique of integral transforms, the results are obtained in the physical domain, numerically for a particular model. The effect of microelongation on displacements, temperature distribution, normal stress and tangential stress are presented graphically to discuss the results and to make the conclusions.

2 Basic Equations

Following Eringen [7], Lord-Shulman [8] and Green-Lindsay [9], the basic equations and the constitutive relations for a linear microelongated thermoelastic solid are given as

$$(\lambda+\mu)\nabla(\nabla.\vec{u})+\mu\nabla^2\vec{u}+\lambda_0\nabla\psi$$
$$-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T=\rho\frac{\partial^2\vec{u}}{\partial t^2}, \quad (1)$$

$$\alpha_0\nabla^2\psi-\lambda_0(\nabla.\vec{u})-\lambda_1\psi$$
$$+m\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\frac{1}{2}\rho j_0\frac{\partial^2\psi}{\partial t^2}, \quad (2)$$

$$K_1^*\nabla^2T-\mathrm{v}\mathrm{T}_0\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla.\vec{u})$$
$$\mathrm{m}\mathrm{T}_0\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\psi$$
$$=\rho C^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T, \quad (3)$$

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)$$
$$-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)T\delta_{ij}+\lambda_0\delta_{ij}\psi, \quad (4)$$

where $\vec{u}$ is the displacement vector and ψ is the microelongation scalar; $\lambda,\mu,\alpha_0,\lambda_0,\lambda_1,j_0,m$ are the material constants, ρ is the density, K_1^* is the coefficient of thermal conductivity, $\nu=(3\lambda+2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion, T is the change in temperature of the medium at any time, C^* is the specific heat at constant strain; $\tau_0\, and\, \tau_1$ are the thermal relaxation times; t_{ij} is the stress tensor; δ_{ij} is the kroneckor delta and ∇ is the Laplacian operator.
For L-S theory; $\tau_1=0,\ \tau_0>0$, and $\eta_0=1$.
For G-L theory; $\tau_1\geq\tau_0>0$, and $\eta_0=0$.

3 Formulation of the Problem

We consider an infinite homogeneous isotropic microelongated thermoelastic circular plate having thickness $2d$. A transient axisymmetric temperature field and an instantaneous normal ring force are acted upon the plate. Also, the plate is considered as thermally insulated. The cylindrical polar coordinates $(r,\ \theta,\ z)$ having origin in the middle surface of the plate and z-axis along the normal to the plate, i.e., along the thickness of the plate are taken. The problem considered is a two dimensional axisymmetric problem with z-axis as the axis of symmetry. The plane $(r,\ z)$ is taken as the plane of incidence and all the components depend upon r, z and t only. The initial temperature in the thick plate is taken as a constant temperature T_0.

4 Solution of the Problem

For the two dimensional axisymmetric problem, we take

$$\vec{u} = (u_r,\ 0,\ u_z). \tag{5}$$

Equations (1)-(3) with the use of (5) take the form

$$(\lambda+\mu)\frac{\partial}{\partial r}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)+\mu\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}+\frac{\partial^2 u_r}{\partial z^2}-\frac{u_r}{r^2}\right)+\lambda_0\frac{\partial\psi}{\partial r}-\nu\frac{\partial}{\partial r}\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\rho\frac{\partial^2 u_r}{\partial t^2}, \tag{6}$$

$$(\lambda+\mu)\frac{\partial}{\partial z}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)+\mu\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{\partial^2 u_z}{\partial z^2}\right)+\lambda_0\frac{\partial\psi}{\partial z}-\nu\frac{\partial}{\partial z}\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\rho\frac{\partial^2 u_z}{\partial t^2}, \tag{7}$$

$$\alpha_0\left(\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial z^2}\right)-\lambda_0\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)-\lambda_1\psi+m\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\frac{1}{2}\rho j_0\frac{\partial^2\psi}{\partial t^2}, \tag{8}$$

$$K_1^*\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)=\rho C^*\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)+\nu T_0\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)+mT_0\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\psi. \tag{9}$$

The non dimensional quantities are introduced as

$$r'=\frac{\omega^* r}{c_1},\quad z'=\frac{\omega^* z}{c_1},\quad u_r'=\frac{\rho c_1\omega^* u_r}{\nu T_0},\quad u_z'=\frac{\rho c_1\omega^* u_z}{\nu T_0},\quad \psi'=\frac{\rho c_1^2\psi}{\nu T_0},$$

$$T'=\frac{T}{T_0},\quad t'=\omega^* t,\quad \tau_0'=\omega^*\tau_0,\quad \tau_1'=\omega^*\tau_1,\quad t_{ij}'=\frac{t_{ij}}{\nu T_0}, \tag{10}$$

where

$$c_1^2=\frac{\lambda+2\mu}{\rho},\qquad \omega^*=\frac{\rho C^* c_1^2}{K_1^*}.$$

Equations (6)-(9), with the aid of dimensionless quantities (10) and after suppressing the primes, yield

$$(1-\delta^2)\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}+\frac{\partial^2 u_z}{\partial r\partial z}-\frac{u_r}{r^2}\right)+\delta^2\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}+\frac{\partial^2 u_r}{\partial z^2}-\frac{u_r}{r^2}\right)+p_0\frac{\partial\psi}{\partial r}-\frac{\partial}{\partial r}\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\frac{\partial^2 u_r}{\partial t^2}, \tag{11}$$

$$(1-\delta^2)\left(\frac{\partial^2 u_r}{\partial r\partial z}+\frac{1}{r}\frac{\partial u_r}{\partial z}+\frac{\partial^2 u_z}{\partial z^2}\right)+\delta^2\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{\partial^2 u_z}{\partial z^2}\right)+p_0\frac{\partial\psi}{\partial z}-\frac{\partial}{\partial z}\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\frac{\partial^2 u_z}{\partial t^2}, \tag{12}$$

$$\left(\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial z^2}\right)-p_0\delta_1^*\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)-p_1\delta_1^*\psi+\bar{\nu}\delta_1^*\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\delta_2^*\frac{\partial^2\psi}{\partial t^2}, \tag{13}$$

$$\nabla^2 T=\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)+\epsilon\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)+\bar{\nu}\epsilon\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\psi. \tag{14}$$

where

$$c_2^2=\frac{\mu}{\rho},\qquad \delta^2=\frac{c_2^2}{c_1^2},\qquad p_0=\frac{\lambda_0}{\rho c_1^2},$$

$$p_1=\frac{\lambda_1}{\rho c_1^2},\qquad \delta_1^*=\frac{\rho c_1^4}{\alpha_0\omega^{*2}},\qquad \delta_2^*=\frac{\rho j_0 c_1^2}{2\alpha_0},$$

$$\bar{\nu}=\frac{m}{\nu},\qquad \epsilon=\frac{\nu^2 T_0}{\rho K_1^*\omega^*}.$$

Now, we define the Laplace transform with respect to time variable 't' with transformed variable 's' as

$$\bar{f}(r,z,s)=L\{f(r,z,t)\}=\int_0^\infty f(r,z,t)e^{-st}dt, \qquad (15)$$

and the Hankel transform of order n with respect to the variable 'r' with transformed variable 'ξ' as

$$\widetilde{f_n}(\xi,z,s)=H_n\{\bar{f}(r,z,s)\}=\int_0^\infty r\bar{f}(r,z,s)J_n(\xi r)dr. \quad (16)$$

Applying the Laplace and then the Hankel transforms defined by (15) and (16) on the set of equations (11)-(14), we obtain

$$\widetilde{u_r''}=a_{11}\tilde{u}_r+a_{13}\tilde{\psi}+a_{14}\tilde{T}+b_{12}\widetilde{u_z'}, \qquad (17)$$

$$\widetilde{u_z''}=a_{22}\tilde{u}_z+b_{21}\widetilde{u_r'}+b_{23}\widetilde{\psi'}+b_{24}\tilde{T}', \qquad (18)$$

$$\widetilde{\psi''}=a_{31}\tilde{u}_r+a_{33}\tilde{\psi}+a_{34}\tilde{T}+b_{32}\widetilde{u_z'}, \qquad (19)$$

$$\widetilde{T''}=a_{41}\tilde{u}_r+a_{43}\tilde{\psi}+a_{44}\tilde{T}+b_{42}\widetilde{u_z'}, \qquad (20)$$

where

$$a_{11}=\left(\frac{\xi^2+s^2}{\delta^2}\right),\qquad a_{13}=\frac{p_0\xi}{\delta^2},$$

$$a_{14}=-\frac{\xi}{\delta^2}(1+\tau_1 s),\qquad a_{22}=(\xi^2\delta^2+s^2),$$

$$a_{31}=p_0\delta_1^*\xi,\qquad a_{33}=(\xi^2+\delta_2^*s^2+p_1\delta_1^*),$$

$$a_{34}=-\bar{\nu}\delta_1^*(1+\tau_1 s),\qquad a_{41}=\xi\epsilon(s+\eta_0\tau_0 s^2),$$

$$a_{43}=\bar{\nu}\epsilon(s+\eta_0\tau_0 s^2),$$

$$a_{44}=(\xi^2+Q^*(s+\tau_0 s^2)),$$

$$b_{12}=\frac{\xi(1-\delta^2)}{\delta^2},\qquad b_{21}=-\xi(1-\delta^2),$$

$$b_{23}=-p_0,\qquad b_{24}=(1+\tau_1 s),$$

$$b_{32}=p_0\delta_1^*,\qquad b_{42}=\epsilon(s+\eta_0\tau_0 s^2).$$

The system of equations (17)-(20) may be written as

$$\frac{d}{dz}W(\xi,z,s)=A(\xi,s)W(\xi,z,s), \qquad (21)$$

where

$$W=\begin{bmatrix}U\\DU\end{bmatrix},\qquad A=\begin{bmatrix}O & I\\A_2 & A_1\end{bmatrix},$$

$$U=\begin{bmatrix}\tilde{u}_r\\\tilde{u}_z\\\tilde{\psi}\\\tilde{T}\end{bmatrix},\qquad D=\frac{d}{dz},$$

$$O=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},\qquad I=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},$$

$$A_2=\begin{bmatrix}a_{11}&0&a_{13}&a_{14}\\0&a_{22}&0&0\\a_{31}&0&a_{33}&a_{34}\\a_{41}&0&a_{43}&a_{44}\end{bmatrix}$$

$$A_1=\begin{bmatrix}0&b_{12}&0&0\\b_{21}&0&b_{23}&b_{24}\\0&b_{32}&0&0\\0&b_{42}&0&0\end{bmatrix}.$$

The solution of the equation (21) is assumed as

$$W(\xi,z,s)=X(\xi,s)e^{qz}, \qquad (22)$$

so that, we have

$$A(\xi,s)W(\xi,z,s)=qW(\xi,z,s),$$

which leads to the eigenvalue problem, and the characteristic equation of the problem is obtained as

$$|A-qI|=0,$$

which on simplification, by using row and column operations, yields

$$q^8-\lambda_1 q^6+\lambda_2 q^4-\lambda_3 q^2+\lambda_4=0, \quad (23)$$

where

$$\lambda_1 = a_{11} + a_{22} + a_{33} + a_{44} + a_{55} + b_{12}b_{21} + b_{23}b_{32} + b_{24}b_{42},$$

$$\lambda_2 = a_{11}a_{22} - a_{13}a_{31} + a_{11}a_{33} + a_{22}a_{33} - a_{14}a_{41} - a_{34}a_{43} + a_{11}a_{44} + a_{22}a_{44} + a_{33}a_{44} + (a_{33} + a_{44})b_{12}b_{21} + (a_{11} + a_{44})b_{23}b_{32} + (a_{11} + a_{33})b_{24}b_{42} - (a_{13}b_{21} + a_{43}b_{24})b_{32} - (a_{14}b_{21} + a_{34}b_{23})b_{42} - (a_{31}b_{23} + a_{41}b_{24})b_{12},$$

$$\lambda_3 = a_{13}a_{34}a_{41} + a_{14}a_{31}a_{43} + (a_{11}a_{33} - a_{13}a_{31})a_{22} - (a_{22} + a_{33})a_{14}a_{41} - (a_{11} + a_{22})a_{34}a_{43} + (a_{11} + a_{33})a_{22}a_{44} + (a_{11}a_{33} - a_{13}a_{31})a_{44} + (a_{33}a_{44} - a_{34}a_{43})b_{12}b_{21} + (a_{34}a_{41} - a_{31}a_{44})b_{12}b_{23} + (a_{31}a_{43} - a_{33}a_{41})b_{12}b_{24} + (a_{14}a_{43} - a_{13}a_{44})b_{21}b_{32} + (a_{11}a_{44} - a_{14}a_{41})b_{23}b_{32} + (a_{13}a_{41} - a_{11}a_{43})b_{24}b_{32} + (a_{13}a_{34} - a_{14}a_{33})b_{21}b_{42} + (a_{14}a_{31} - a_{11}a_{34})b_{23}b_{42} + (a_{11}a_{33} - a_{13}a_{31})b_{24}b_{42},$$

$$\lambda_4 = (a_{13}a_{34} - a_{14}a_{33})a_{22}a_{41} + (a_{14}a_{31} - a_{11}a_{34})a_{22}a_{43} + (a_{11}a_{33} - a_{13}a_{31})a_{22}a_{44}.$$

The eigenvalues are the roots of the equation (23) and the roots are say q_i^2 $(i = 1, 2, 3, 4)$. The eigenvector $X(\xi, s)$ corresponding to the eigenvalue q can be obtained by solving the system of equations

$$[A - qI]X(\xi, s) = 0.$$

Thus, the eigenvectors $X_i(\xi, s)$ corresponding to different eigenvalues $\pm q_i$ $(1, 2, 3, 4)$ are obtained as

$$X_i(\xi, s) = \begin{bmatrix} X_{i1}(\xi, s) \\ X_{i2}(\xi, s) \end{bmatrix},$$

where

for $q = q_i$ $(i = 1, 2, 3, 4)$, we have

$$X_{i1}(\xi, s) = \begin{bmatrix} a_i q_i \\ b_i \\ d_i \\ e_i \end{bmatrix}, \quad X_{i2}(\xi, s) = \begin{bmatrix} a_i q_i^2 \\ b_i q_i \\ d_i q_i \\ e_i q_i \end{bmatrix},$$

for $q = -q_i$ $(i = 1, 2, 3, 4)$, we have

$$X_{j1}(\xi, s) = \begin{bmatrix} -a_i q_i \\ b_i \\ d_i \\ e_i \end{bmatrix}, \quad X_{j2}(\xi, s) = \begin{bmatrix} a_i q_i^2 \\ -b_i q_i \\ -d_i q_i \\ -e_i q_i \end{bmatrix},$$

$$j = i + 4,$$

in which

$$a_i = \frac{\xi}{\delta^2}[r_1^2\{p_0^2\delta_1^* - (1 - \delta^2)r_2\} + \epsilon r_1 r_4\{r_2 - (1 - \delta^2)\bar{v}^2\delta_1^* + 2\bar{v}p_0\delta_1^*\}],$$

$$b_i = \frac{1}{\delta^2}[r_1^2(r_2 r_3 - p_0^2\delta_1^*\xi^2) + \epsilon r_1 r_4(\xi^2 r_2 + \bar{v}^2\delta_1^* r_3 - 2\delta_1^*\bar{v}p_0\xi^2)],$$

$$d_i = \delta_1^*[(p_0 r_1 + \epsilon\bar{v}r_4)(\xi a_i + b_i)q_i] / (-r_1 r_2 - \epsilon\bar{v}^2\delta_1^* r_4),$$

$$e_i = [\epsilon(s + \eta_0\tau_0 s^2)\{(r_1 r_2 + \epsilon\bar{v}^2\delta_1^* r_4) - \bar{v}\delta_1^*(p_0 r_1 + \epsilon\bar{v}r_4)\}](\xi a_i + b_i) / \{r_1(-r_1 r_2 - \epsilon\bar{v}^2\delta_1^* r_4)\},$$

$$r_1 = \left(\xi^2 + (s + \tau_0 s^2) - q_i^2\right),$$

$$r_2 = \left(\xi^2 + \delta_2^* s^2 + p_1\delta_1^* - q_i^2\right),$$

$$r_3 = \left(\xi^2 + s^2 - q_i^2\delta^2\right),$$

$$r_4 = (1 + \tau_1 s)(s + \eta_0\tau_0 s^2), \quad (i = 1, 2, 3, 4, 5).$$

Thus, the solution of (21) is obtained as

$$W(\xi, z, s) = \sum_{i=1}^{4} N_i X_i(\xi, s)\cosh(q_i z), \quad (24)$$

where N_1, N_2, N_3 and N_4 are arbitrary constants. Now, using (5) in equations (4), the stress components for the two dimensional problem are obtained as

$$t_{zz} = (\lambda + 2\mu)\frac{\partial u_z}{\partial z} + \lambda\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right)$$
$$-\nu\left(1 + \tau_1 \frac{\partial}{\partial t}\right)T + \lambda_0 \psi, \qquad (25)$$

$$t_{zr} = \mu\frac{\partial u_r}{\partial z} + \mu\frac{\partial u_z}{\partial r}. \qquad (26)$$

Using the non dimensional quantities defined by (10) and applying the Laplace and then the Hankel transform defined by (15) and (16) on the equations (25)-(26), and then using (24), we get

$$\widetilde{t_{zz}} = L_1 N_1 \cosh(q_1 z) + L_2 N_2 \cosh(q_2 z)$$
$$+L_3 N_3 \cosh(q_3 z) + L_4 N_4 \cosh(q_4 z), \quad (27)$$

$$\widetilde{t_{zr}} = M_1 N_1 \cosh(q_1 z) + M_2 N_2 \cosh(q_2 z)$$
$$+M_3 N_3 \cosh(q_3 z) + M_4 N_4 \cosh(q_4 z), \quad (28)$$

where

$$L_i = \left[\frac{\lambda \xi a_i q_i}{\rho c_1^2} + \frac{\lambda_0}{\rho c_1^2} d_i - (1 + \tau_1 s) e_i + \left(\frac{\lambda + 2\mu}{\rho c_1^2}\right) b_i q_i\right],$$

$$M_i = \left[-\frac{\mu \xi b_i}{\rho c_1^2} + \frac{\mu a_i q_i^2}{\rho c_1^2}\right], \qquad (i = 1, 2, 3, 4).$$

5 Boundary Conditions

The circular plate occupies the region defined by $0 \le r < \infty$ and $-d \le z \le d$. The plate is acted upon by an instantaneous normal ring force and a transient axisymmetric temperature field dependent on the radial and axial directions of the cylindrical coordinate system. Also, the plate is thermally insulated. Therefore, the nondimensional boundary conditions at the surface $z = \pm d$ of the plate are taken as

$$\frac{dT}{dz} = \pm g_0 F(r, z), \qquad (29)$$

$$t_{zz} = \delta(t)\delta(a - r), \qquad (30)$$

$$t_{zr} = 0, \qquad (31)$$

$$\frac{d\psi}{dz} = 0, \qquad (32)$$

where $F(r, z) = z^2 e^{-\omega r}$, $\omega > 0$, $F(r, z)$ is a function that increases in the axial direction symmetrically and falls off exponentially as one moves away from the centre of the plate along the radial direction. The constant temperature applied on the boundary is g_0. $\delta(-)$ is the Dirac delta function.

Applying the Laplace and the Hankel transforms defined by (15) and (16) on the boundary conditions (29)-(32), we obtain

$$\frac{d\tilde{T}}{dz} = \pm g_0 \frac{z^2 \omega}{(z^2 + \omega^2)^{3/2}}, \qquad (33)$$

$$\widetilde{t_{zz}} = a J_0(\xi a), \qquad (34)$$

$$\widetilde{t_{zr}} = 0, \qquad (35)$$

$$\frac{d\tilde{\psi}}{dz} = 0. \qquad (36)$$

Making use of (24) and (27)-(28) in the transformed form of boundary conditions (33)-(36), we obtain

$$S_1 N_1 + S_2 N_2 + S_3 N_3 + S_4 N_4 = Q, \qquad (37)$$
$$T_1 N_1 + T_2 N_2 + T_3 N_3 + T_4 N_4 = R, \qquad (38)$$
$$U_1 N_1 + U_2 N_2 + U_3 N_3 + U_4 N_4 = 0, \qquad (39)$$
$$V_1 N_1 + V_2 N_2 + V_3 N_3 + V_4 N_4 = 0, \qquad (40)$$

where

$$S_i = e_i q_i \cosh(q_i d), \qquad T_i = L_i \cosh(q_i d),$$
$$U_i = M_i \cosh(q_i d), \qquad V_i = d_i q_i \cosh(q_i d),$$

$$Q = \pm g_0 \frac{z^2 \omega}{(z^2 + \omega^2)^{3/2}}, R = a J_0(\xi a),$$
$$(i = 1, 2, 3, 4).$$

After solving the system of equations $(37) - (40)$, we obtain the values of N_i $(i = 1, 2, 3, 4)$ as

$$N_i = \frac{\Delta_i}{\Delta}, \qquad (41)$$

where

$$\Delta = \begin{vmatrix} S_1 & S_2 & S_3 & S_4 \\ T_1 & T_2 & T_3 & T_4 \\ U_1 & U_2 & U_3 & U_4 \\ V_1 & V_2 & V_3 & V_4 \end{vmatrix},$$

and $\Delta_i (i = 1, 2, 3, 4)$ are obtained from Δ by replacing i^{th} column of Δ with $|Q, R, 0, 0|^{tr}$, ($'tr'$ stands for transpose).

Using the values of N_i $(i = 1, 2, 3, 4)$ from (41) in the equations (24) and $(27) - (28)$, we obtain the expressions of displacements, microelongation, temperature distribution and stresses in the transformed domain as

$$(\tilde{u}_r,\ \tilde{u}_z,\ \tilde{\psi},\tilde{T}) = \frac{1}{\Delta}\sum_{i=1}^{4}(a_i q_i,\ b_i,\ d_i,\ e_i)\Delta_i \cosh(q_i z)\,, \quad (42)$$

$$(\widetilde{t_{zz}},\ \widetilde{t_{zr}}) = \frac{1}{\Delta}\sum_{i=1}^{4}(L_i,\ M_i)\Delta_i \cosh(q_i z)\,. \quad (43)$$

The above expressions $(42)-(43)$ provide the solution of the problem in the transformed form of components of displacement, microelongation, temperature distribution and stresses.

6 Particular Case

In case of absence of microelongation, that means the circular plate is of thermoelastic medium, then the boundary conditions for the problem become,

$$\frac{dT}{dz} = \pm g_0 F(r,z),$$

$$t_{zz} = \delta(t)\delta(a-r),$$

$$t_{zr} = 0.$$

Accordingly, the expressions for displacements, temperature distribution and stresses are obtained from (42) and (43) as

$$(\tilde{u}_r,\tilde{u}_z,\tilde{T}) = \frac{1}{\Delta}\sum_{i=1}^{3}(a_i q_i,\ b_i,\ e_i)\Delta_i \cosh(q_i z)\,,$$

$$(\widetilde{t_{zz}},\ \widetilde{t_{zr}}) = \frac{1}{\Delta}\sum_{i=1}^{3}(L_i,\ M_i)\Delta_i \cosh(q_i z)\,,$$

where

$$\Delta_1 = (\mathrm{RS}_3 - \mathrm{QT}_3)\mathrm{U}_2 + (\mathrm{QT}_2 - \mathrm{RS}_2)\mathrm{U}_3,$$

$$\Delta_2 = (\mathrm{QT}_3 - \mathrm{RS}_3)\mathrm{U}_1 + (\mathrm{RS}_1 - \mathrm{QT}_1)\mathrm{U}_3,$$

$$\Delta_3 = (\mathrm{RS}_2 - \mathrm{QT}_2)\mathrm{U}_1 + (\mathrm{QT}_1 - \mathrm{RS}_1)\mathrm{U}_2,$$

$$\Delta = (\mathrm{S}_2\mathrm{T}_3 - \mathrm{S}_3\mathrm{T}_2)\mathrm{U}_1 + (\mathrm{S}_3\mathrm{T}_1 - \mathrm{S}_1\mathrm{T}_3)\mathrm{U}_2 + (\mathrm{S}_1\mathrm{T}_2 - \mathrm{S}_2\mathrm{T}_1)\mathrm{U}_3,$$

and

$$S_i = e_i q_i \cosh(q_i d), \qquad T_i = L_i \cosh(q_i d),$$

$$V_i = d_i q_i \cosh(q_i d),$$

$$L_i = \left[\frac{\lambda\xi a_i q_i}{\rho c_1^2} - (1+\tau_1 s)e_i + \left(\frac{\lambda+2\mu}{\rho c_1^2}\right)b_i q_i\right],$$

$$M_i = \frac{\mu}{\rho c_1^2}\left(a_i q_i^2 - \xi b_i\right), \qquad (i=1,2,3).$$

7 Inversion of Transforms

The transformed displacement, microelongation, temperature distribution and stresses are the functions of the form $\tilde{f}(\xi,z,s)$. Therefore, to get the solution in physical domain, we have to obtain the function, $f(r,z,t)$. So, we first invert the Hankel transform by using

$$\bar{f}(r,z,s) = \int_0^\infty \xi\tilde{f}(\xi,z,s)J_n(\xi r)d\xi\,. \quad (44)$$

Press et. al. [10] described the method for evaluating the integral by using the Romberg's integration with adaptive step size. This method uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

The expression (44) provides the Laplace transform $\bar{f}(\xi,z,s)$ of the function, $f(r,z,t)$. Now the function $\bar{f}(r,z,s)$ can be considered as the Laplace transform $\bar{g}(s)$ of some function $g(t)$, for the fixed values of r and z. The Laplace transform $\bar{g}(s)$ can be inverted by using the inversion technique given by Honig and Hirdes [11] by taking the inverse Laplace transform as

$$g(t) = \frac{1}{2\iota\pi}\int_{c-\iota\infty}^{c+\iota\infty}\bar{g}(s)e^{st}ds, \quad (45)$$

where c is an arbitrary constant and is greater than all the real parts of the singularities of $\bar{g}(s)$.

8 Numerical Results and Discussion

To illustrate the problem considered and solved above theoretically in the transformed domain, here we take the numerical parameters for a model of microelongated thermoelastic medium to get the solution of the problem in physical domain, numerically by using the inversion technique described above. The physical parameters for the model considered are given as under:-

Following Eringen [12], the values of micropolar constants are taken as

$\lambda = 9.4\times10^{10}Nm^{-2},\ \mu = 4.0\times10^{10}Nm^{-2},$

$\rho = 1.74\times10^{3}Kgm^{-3},$

and the Microstretch parameters are given by

$j_0 = 0.19 \times 10^{19} Nm^{-2}$,

$\alpha_0 = 0.779 \times 10^{-9} N$,

$\lambda_0 = 0.5 \times 10^{10} Nm^{-2}, \quad \lambda_1 = 6.5 \times 10 Nm^{-2}$.

The values of thermal parameters are given by Dhaliwal and Singh [13] as

$K_1^* = 1.7 \times 10^6 Jm^{-1}s^{-1}K^{-1}$,

$C^* = 1.04 \times 10^3 JKg^{-1}K^{-1}$,

$\alpha_t = 2.33 \times 10^{-5} K^{-1}$,

$\tau_0 = 6.131 \times 10^{-13} sec$,

$\tau_1 = 8.765 \times 10^{-13} sec$,

$m = 1.13849 \times 10^{10} N/m^2$,

$T_0 = 0.298 \times 10^3 K$.

Taking the above parameters into consideration and using a computer program for the numerical inversion of the integral transforms in MATLAB, we draw the variations of displacements, normal stress, tangential stress and temperature distribution with radial distance 'r', for the middle surface of the plate and for $t = 0.01$, and are shown in figures (1)-(5), respectively for the cases of microelongated thermoelastic medium (MTM) and thermoelastic medium (TM). To notice the variations for the two cases in the same figure, the figures are shown by multiplying the field components by some constant factors as per requirements in different figures for the different cases and are mentioned accordingly for each figure, as the magnitude values for some field components are very large/small in comparison to others. In all these figures, the cases of MTM and TM correspond to the solid line (——) and dashed line (- - - - -), respectively.

Fig. 1 depicts the variation of radial displacement u_r with radial distance r. The variations are drawn after multiplying the values for the case MTM by 10^4 and for the case TM by 10^{-2} to notice the variations. It is noticed that in both the cases, i.e., MTM and TM, the magnitude values of radial displacement are large initially, i.e., near the origin, which goes on decreasing with the increase in the value of r and ultimately tends to zero with further increase in the value of r, following the oscillatory pattern.

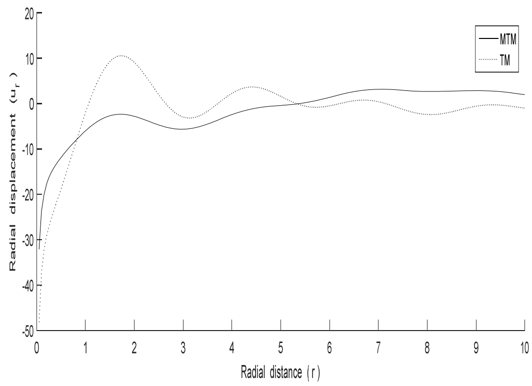

Fig. 1 Variations of radial displacement u_r

Also, the radial displacement vanishes for large values of r, i.e., at a far away distance from the point of application of the source for both the cases. Also, the curves for the cases MTM and TM show the impact of the microelongation, and it is seen that this impact is mainly on the magnitude values of radial displacement and the magnitude values are large if the microelongation factor is not taken into consideration.

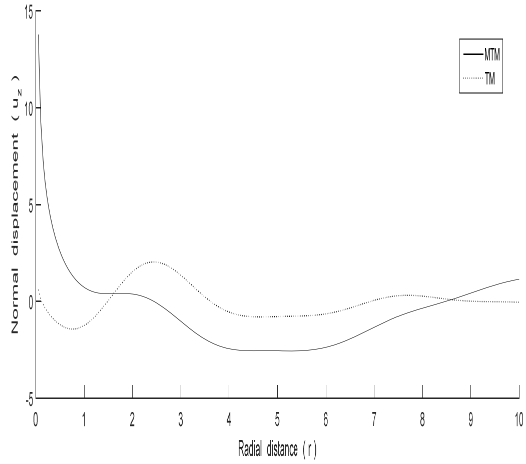

Fig. 2 Variations of normal displacement u_z

Fig. 2 describes the variation of normal displacement u_z with radial distance r. The variations are drawn after multiplying the values for the case MTM by 10^3 and for the case TM by 10^{-3} to notice the variations. It is noticed that in both the cases, i.e., MTM and TM, the magnitude values of normal displacement tend to zero values with increase in the value of r, following the oscillatory pattern. Also, the curves for the cases MTM and TM show that the magnitude values of normal displacement are large if the microelongation factor is not taken into consideration.

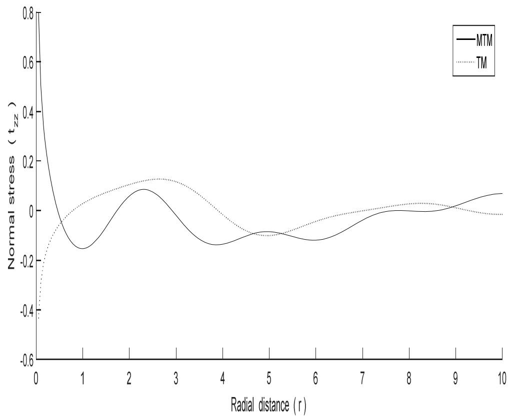

Fig. 3 Variations of normal stress t_{zz}

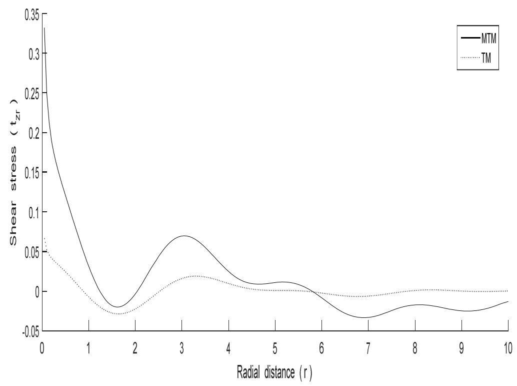

Fig. 4 Variations of shear stress t_{zr}

Fig. 3 exhibits the variation of normal stress t_{zz} with radial distance r. The variations are drawn after multiplying the values for the case TM by 10^{-6} to notice the variations. It is noticed that the magnitude values of normal stress tend to zero values with increase in the value of r, following the oscillatory pattern for both the cases, i.e., MTM and TM. Also, the curves for the cases MTM and TM show that the magnitude values of normal stress are large for the case of TM as compared to MTM, i.e., the microelongation factor decreases the magnitude values of normal stress. Similar trends are observed for the variation of shear stress t_{zr} with radial distance r from fig. 4. However, the variations are drawn after multiplying the values for the case TM by 10^{-7} to notice the variations.

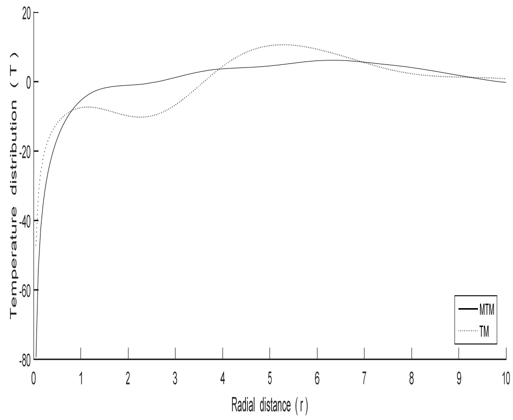

Fig. 5 Variations of temperature distribution T

Fig. 5 gives the variation of temperature distribution T with radial distance r for the cases MTM and TM. The variations are drawn after multiplying the values for the case MTM by 10^4 and for the case TM again by10^4 to notice the variations. It is noticed that the magnitude values of temperature distribution tend to zero values with increase in the value of r, following the oscillatory pattern for both the cases, i.e., MTM and TM. Also, the curves for the cases MTM and TM show that the magnitude values of temperature distribution are comparable, i.e., the microelongation factor has not much impact on the temperature distribution.

9 Conclusion

An axisymmetric problem of an infinite circular plate of microelongated thermoelastic medium acted upon by thermomechanical sources is solved and the impact of microelongation is analysed graphically. It is seen that in all the cases the values for various components are large initially and the variation curves follow the oscillatory pattern to tend to zero value, which show the characteristic of the source applied as the impact of the source become negligible, if we move away from the point of application of the source. Further, there is a large variation in the magnitude values of the various components due to microelongation factor except in case of temperature distribution. This analysis shows the properties of the medium considered. The problem discussed will be useful for the researchers in the field of continuum mechanics and related fields for further studies and research.

References:-

[1] Eringen, A. C., Micropolar elastic solids with stretch (Prof. Dr. Mustafa Inan Anisma,) Ari Kitapevi Matbaasi, Istanbul, Turkey, 1971.

[2] Eringen, A. C., Theory of thermomicrostretch elastic solids, *International Journal of Engineering Science*, 28, 1990, 1291–1301.

[3] Kiris , A., Inan, E., Eshelby tensors for a spherical inclusion in microelongated elastic fields, International Journal of Engineering Science, 43, 2005, 49-58.

[4] Shaw, S., Mukhopadhyay, B., Periodically varying heat source response in a functionally graded microelongated medium, *Applied Mathematics and Computation*, 218, 2012, 6304-6313.

[5] Shaw, S., Mukhopadhyay, B., Moving heat source response in a thermoelastic microelongated solid, *Journal of Engineering Physics and Thermophysics*, 86, 2013, 716-722.

[6] Ailawalia, P., Sachdeva, S. K., Pathania, D. S., Plane strain deformation in a thermoelastic microelongated solid with internal heat source, *International Journal of Applied Mechanics and Engineering*, 20, 2015, 717-731.

[7] Eringen, A. C., Microcontinuum field theories, I, Foundations and Solids, Springer-Verlag, New York, 1999.

[8] Lord, H. W., Shulman, Y., A generalized dynamical theory of thermoelasticity, *Journal of Mechanics and Physics Solids*, 15, 1967, 299-309.

[9] Green, A. E., Lindsay, K. A., Thermoelasticity, *Journal of Elasticity*, 2, 1972, 1-7.

[10] Press, W. H., Teukolsky, S. A., Vetterling W. T. and Flannery, B. P. ; Numerical recipes in C, *Cambridge University Press, Cambridge*, 1986.

[11] Honig, G., Hirdes, U., A method for the numerical inversion of the Laplace transform, *Journal of Computational and Applied and Mathematics*, 10, 1984, 113-132.

[12] Eringen, A. C., Plane waves in non local micropolar elasticity, *International Journal of Enineering Science*, 22, 1984, 1113-1121.

[13] Dhaliwal, R. S., Singh, A., Dynamical coupled thermoelasticity, Hindustan Publication Corporation, New Delhi, 1980.