Elastically Damped Transient Response of Axially FG Straight Beams

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Abstract: - The objective of this study is to investigate the influence of damping ratio, material gradients and cross-sectional properties on the transient response of axially functionally graded (FG) straight beams. The mixed finite element formulation based on Timoshenko beam theory including the shear influence and the rotary inertia is used. Time integration analysis of axially FG straight beams is performed using Newmark time integration method.

Key-Words: - transient analysis, functionally graded, damping ratio, Timoshenko beam, finite element

1 Introduction

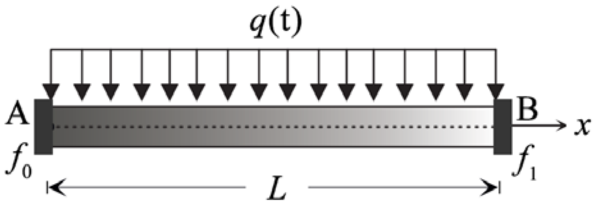
Recent technology leads composite materials to gain great importance in many application fields e.g. civil, mechanical, and aerospace engineering. Functionally graded materials are preferable in some complex conditions where they show some advantages due to their properties such as; low weight to strength ratio, high stiffness, resistance to temperature, avoiding stress concentrations *etc*. Some studies about the free vibration analyses of axially FG beams are cited as [1-7]. The studies concerned with forced vibration analyses of axially FG beams are summarized here. By using the B-spline method, [8] studied dynamics of rotating axially FG tapered beams based on a new dynamic model. Complementary functions method is adopted in [9] in order to investigate the transient analysis of axially FG straight Timoshenko beam with variable cross-section. [10] studied the dynamic response of an axially FG beam reflecting longitudinal-transverse coupling effect under a moving transverse/longitudinal harmonic load.

Inclusion of the damping effects are important due to the internal friction of the structural elements under dynamic loadings. Some studies about damping effect in structures can be cited as follows: [11] examined the conditions under which a damped linear system have classical normal modes. [12] investigated necessary and sufficient conditions under which both discrete and continuous damped linear dynamic systems have classical normal modes. [13] presented a frequency-domain method about estimation the mass, stiffness and damping matrices of the model of a structure. [14] proposed a simple identification method in order to obtain the damping matrix using the generalized proportional damping model. This method requires only natural frequencies and modal damping factors. [15] developed a simple and easy-to-implement algorithm for a generalized proportional viscous damping matrix. [16] evaluated the effects of Rayleigh damping model on the engineering demand parameters of two-steel moment-resisting frame buildings. [17] reported the vibration and damping performances of hybrid carbon fiber composite pyramidal truss sandwich panels with viscoelastic layers embedded in the face sheets. [18] presented the natural frequencies and the modal damping ratios of steel building model for different data analysis options used in Operational Modal Analysis method. [19] investigated a classical damping matrix in terms of a Caughey series. [20] formulated a least squares approach to determine the coefficients in a Caughey series representation of a classical damping matrix. [21] developed methods in order to estimate boundary parameters and tension for elastic beams using measured natural frequencies, mode shapes, and damping coefficients.

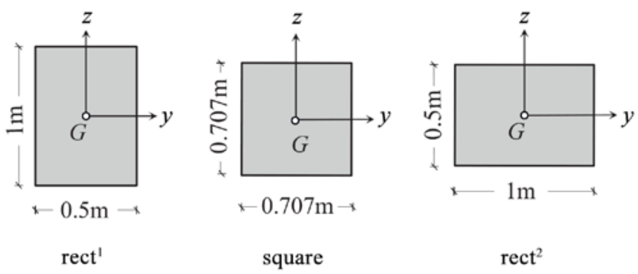
In this study, the transient responses of axially FG straight Timoshenko beams are investigated in time domain using a mixed finite element method (MFEM). The transient analyses are performed using Newmark algorithm, and the damping is considered to be proportional to stiffness. Mixed finite element method and time integration scheme based on Newmark algorithm was also implemented in [22] for nonlinear transient analysis of functional graded (FG) and fiber metal laminated (FML) plates under blast loading. The influence of material gradients, cross-sectional properties and damping ratios on the dynamic behaviour of axially FG straight beams is investigated. Axially FG material distribution is assumed as a power-law relation. The same cross-sectional area are chosen for the square and rectangular cross sections. The boundary condition is imposed to be fixed-fixed. The damping ratio is considered to be proportional to stiffness of the beam.

2 Formulation

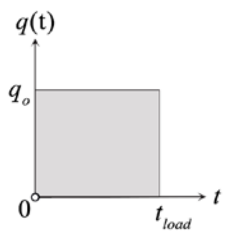
The mixed finite element formulation for elastically damped transient analysis of axially functionally graded straight beam is developed based on [23], [24], and [22]. The field equations, the functional and the mixed FE formulation for the static analyses of an isotropic homogenous spatial beam relying on Timoshenko beam theory exist in [23] and [24]. In this study, the field equations and functional of the isotropic homogenous spatial beam is revised for the transient analysis of an axially functionally graded straight beam including damping effect. In the transient analysis, Newmark time integration method is adopted by considering the velocities and accelerations of the force and moment components in the MFE [22]. The Rayleigh damping ξ [22], whose coefficients are proportional to the stiffness and first two natural frequencies of the beam, is used in this study.



(a) axially FG beam having fixed-fixed ends



(b) cross-sections having same area



(c) rectangular type impulsive load

Fig. 1. Axially FG beam, cross section and impulsive load

2.1 Axially FG straight beam

Axially FG material distribution is assumed by a power-law relation along the axis of straight beam as follows:

$$f(x) = f_0 + (f_1 - f_0)\left(\frac{x}{L}\right)^m \tag{1}$$

where f denotes a material property (*e.g.* modulus of elasticity: E, density: ρ or shear modulus: G), m is the material gradient index, the subscript "0" and "1" denotes the materials at left and right ends of the beam, respectively. x is the axis of beam and L is the length of the beam (Fig. 1.a).

3 Numerical Examples

The objective of the example is to investigate the effect of damping ratios, material index and cross sectional properties (square and rectangular) on force and moment (T_z^A, M_y^A) exerted at point A, and maximum displacement ($u_z^{\max}$) of axially FG straight beam having fixed-fixed boundary condition. All the cross-sections are chosen to have the same constant area. The geometrical parameters of axially FG straight beam: the length of beam is $L = 5\,\mathrm{m}$, the dimensions of the rectangular cross-sections for rect^1 and rect^2, and dimension of square cross-section are given in Fig.1(b). The material properties are, $E_0 = 210\,\mathrm{GPa}$, $\rho_0 = 7500\,\mathrm{kg/m^3}$, $E_1 = 70\,\mathrm{GPa}$, $\rho_1 = 2500\,\mathrm{kg/m^3}$, the Poisson's ratio $\upsilon = 0.3$. The material gradients are $m = 0.5;1;3$. The damping ratios are $\xi = 0, 0.01, 0.02, 0.05$. The intensity and the duration of loading are $q_o = 200\,\mathrm{kN/m}$ and $t_{load} = 0.021\,\mathrm{s}$, respectively. The dynamic response of the beam is determined within $0 \le t \le 0.084\,\mathrm{s}$.

3.1 Convergence analysis

A convergence analysis of the mixed finite element formulation is performed with respect to the time step Δt and the number of elements n_e. The selected parameters are: the material gradient index $m = 3$, the damping ratio $\xi = 0$ and the cross-section is square. First, the dynamic analysis of the beam is carried out using 40 finite elements for the selected time steps $\Delta t = 0.65625\,\mathrm{ms}$, $0.16406\,\mathrm{ms}$, $0.08203\,\mathrm{ms}$, $0.04102\,\mathrm{ms}$. Time history of $u_z^{\max}$ is as shown in Fig. 2. Next, the dynamic analysis is carried out for $\Delta t = 0.04102\,\mathrm{ms}$ using $n_e = 20$, 30 and 40 finite elements. The time histories of $u_z^{\max}$, T_z^A and M_y^A are as shown in Fig. 3. According to the results of convergence analysis, in the following examples, the time increment $\Delta t = 0.04102\,\text{ms}$ and 40 elements are employed.

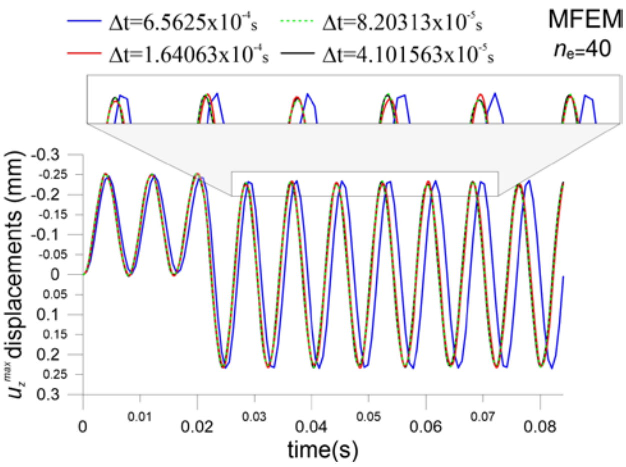
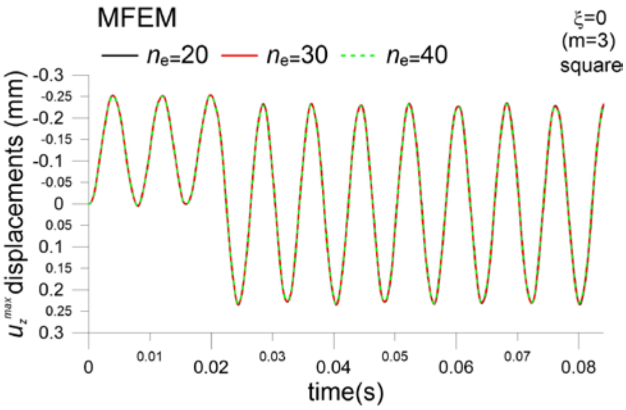
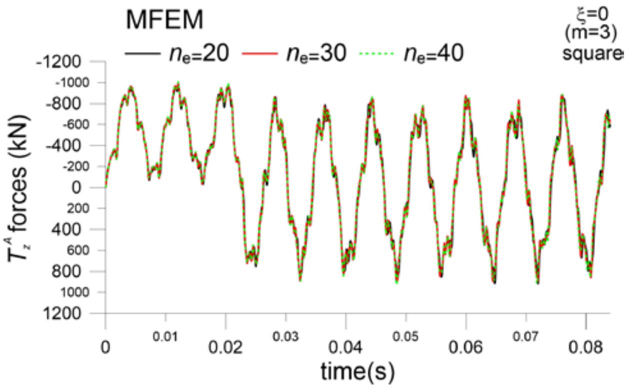


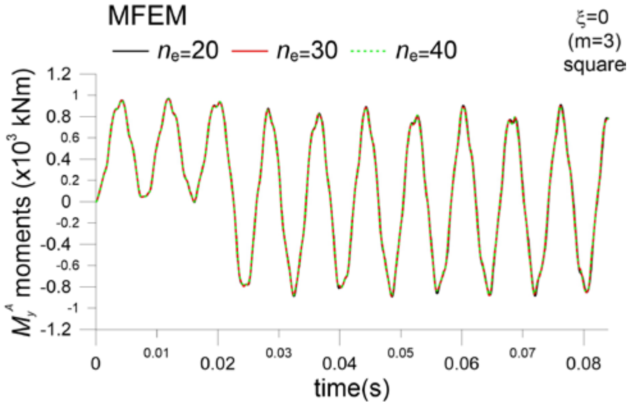
Fig. 2. The convergence analysis of axially FG straight beam according to the time steps, $\Delta t = 0.65625\,\text{ms}$, $0.16406\,\text{ms}$, $0.08203\,\text{ms}$ and $0.04102\,\text{ms}$.



(a) The convergence analysis of $u_z^{\max}$



(b) The convergence analysis of T_z^A at point A

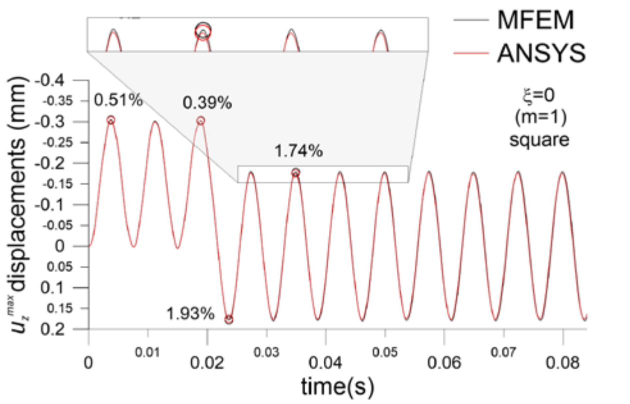


(c) The convergence analysis of M_y^A at point A

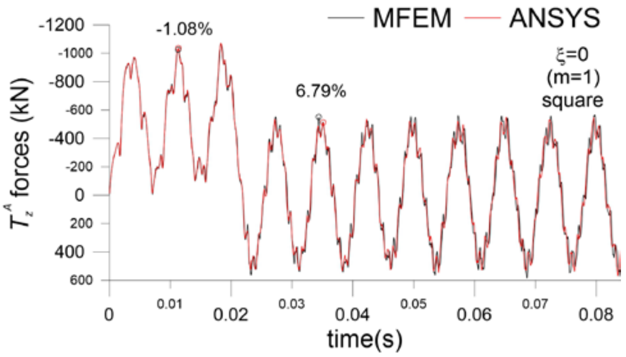
Fig. 3. The convergence analysis of MFEM of axially FG straight beam according the number of finite elements for $\Delta t = 0.04102\,\text{ms}$.

3.2 Comparison with ANSYS

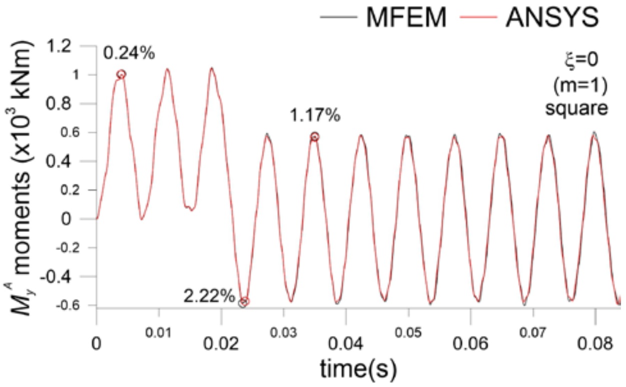
The dynamic analysis of axially FG straight beam with a square cross-section is performed by mixed FEM and ANSYS (version 18.1) for $m = 1$, $\Delta t = 0.04102\,\text{ms}$ and $n_e = 40$. In ANSYS solution, the beam elements (BEAM188) are used and the axially FG material is defined by the average value of material properties between each node of the straight beam (41 nodes). The comparison of MFEM and ANSYS results for $u_z^{\max}$, T_z^A and M_y^A of axially FG straight beam is given in Figs. 4(a-c) and Figs. 4(d-f) for the two different damping ratios $\xi = 0$ and 0.01, respectively. It is observed that, the results of MFEM and ANSYS show a well agreement with each other (see Fig. 4). For the damping ratio $\xi = 0$, the percent differences for some peak points obtained by ANSYS with respect to the MFEM results are given in Fig. 4.



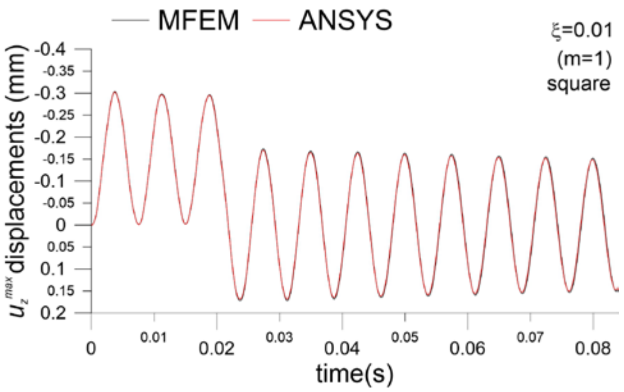
(a) The comparison of $u_z^{\max}$ for $\xi = 0$.



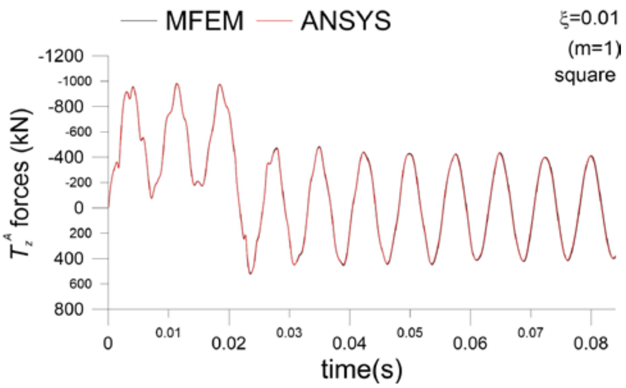
(b) The comparison of T_z^A at point A for $\xi = 0$.



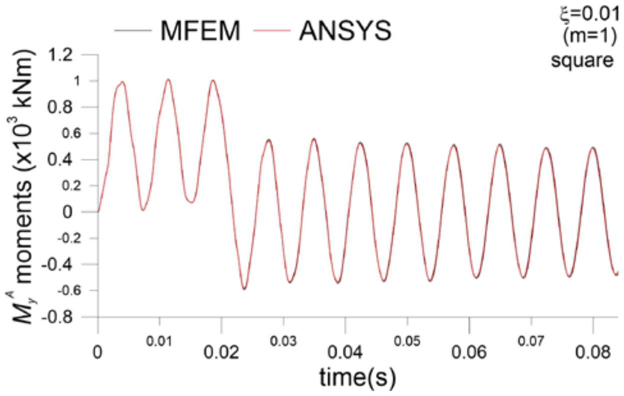
(c) The comparison of M_y^A at point A for $\xi = 0$.



(d) The comparison of $u_z^{\max}$ for $\xi = 0.01$



(e) The comparison of T_z^A at point A for $\xi = 0.01$

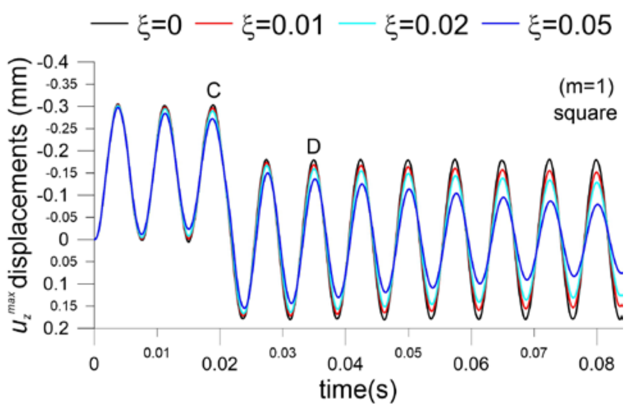


(f) The comparison of M_y^A at point A for $\xi = 0.01$

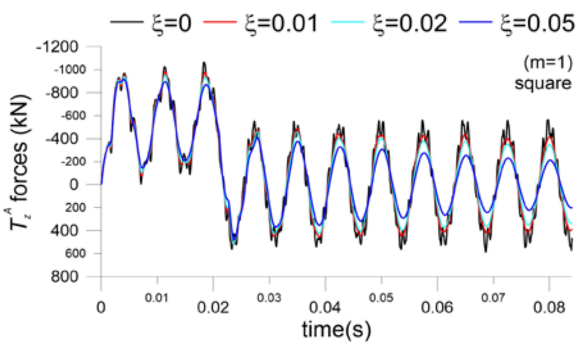
Fig. 4. The comparison of MFEM and ANSYS results for $n_e = 40$ and $\Delta t = 0.04102\,\text{ms}$. (○: result of MFEM, ○ : result of ANSYS)

3.3 The influence of damping ratio, material gradient index and different cross sections

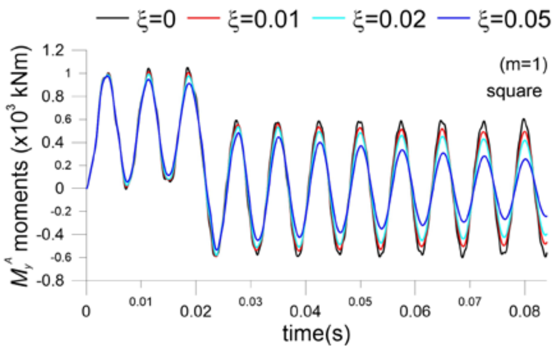
The dynamic response of the axially FG straight beam with square cross-section is investigated for the damping ratios $\xi = 0, 0.01, 0.02, 0.05$ and material gradient $m = 1$. The time histories of $u_z^{\max}$, T_z^A and M_y^A at point A are as shown in Fig. 5 for $\xi = 0, 0.01, 0.02, 0.05$. $u_z^{\max}$ for the damping ratios $\xi = 0.01, 0.02, 0.05$ are normalized with respect to $u_z^{\max}$ for $\xi = 0$ at points C and D in Fig. 5(a). The percent reductions in the amplitude of $\xi = 0.01, 0.02, 0.05$ at point C are 2.27%, 4.38% and 10.14%, respectively. The percent reductions in the amplitude of $\xi = 0.01, 0.02, 0.05$ at point D are 3.58%, 6.47% and 13.11%, respectively. As a consequence, when the damping ratio increases, the amplitude of the times histories decreases.



(a) $u_z^{\max}$



(b) T_z^A at point A

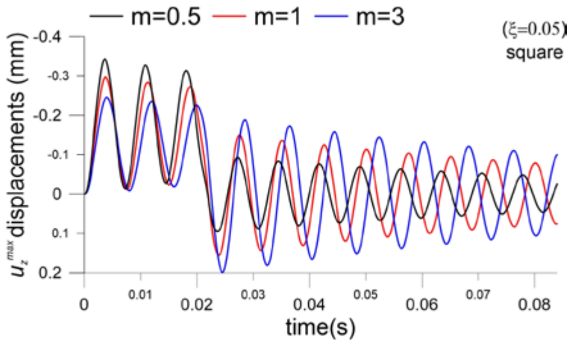


(c) M_y^A at point A

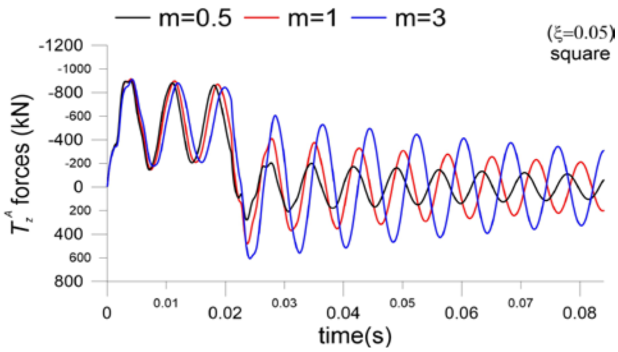
Fig. 5. The time histories of the axially FG straight beam having $m=1$ and square cross section for different damping ratios $\xi=0, 0.01, 0.02, 0.05$.

The influence of material gradient index on the dynamic behavior of the axially FG straight beam having the damping ratio $\xi=0.05$ is investigated for $m=0.5,1,3$ and the results are presented in Fig.6. When the values of first extrema of forced zone $0 \le t \le t_{load}=0.021$s and free vibration zone $0.021 < t \le 0.084$s are considered for u_z^{max}, the absolute maximum amplitude is obtained for axially FG beam having $m=0.5$ and $m=3$ for forced vibration zone, respectively, whereas the maximum and minimum amplitude is obtained for axially FG beam having $m=3$ and $m=0.5$ for free vibration zone. The values of first extrema of the forced vibration and free vibration zone for the u_z^{max} are determined from Fig. 6(a) and the MFEM results of axially FG beam for $m=0.5$ are compared with the results associated with $m=1,3$. For this purpose, the ratio $\beta=\left(u_z^{max}\right)^{0.5}/\left(u_z^{max}\right)^{m}$ where $m=1,3$ is calculated. For the forced vibration zone the ratio $\beta>1$ and it is 1.15 and 1.40 for $m=1$ and 3, respectively. For the free vibration zone the ratio $\beta<1$ and it is 0.61 and 0.47 for $m=1$ and 3, respectively.

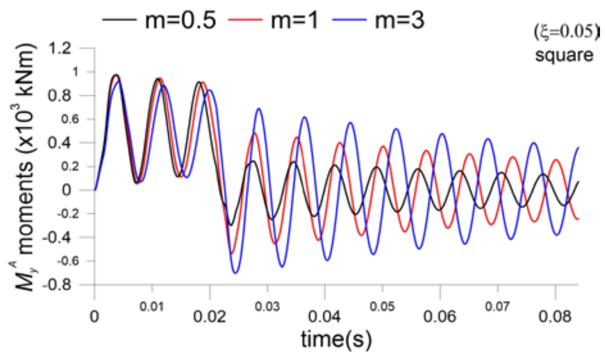
Dynamic response of three constant cross-sections (one square, two rectangles) of the axially FG straight beam having the damping ratio $\xi=0.05$ for $m=1$ is investigated for u_z^{max}, T_z^A and M_y^A at point A. The results are presented in Fig. 7. The values of first extrema of the forced vibration zone for u_z^{max} are determined from Fig. 7(a) and MFEM results for axially FG beam having rect^2 cross-section are compared with the results associated with axially FG beam having square and rect^1 cross-sections. The percent reductions for the axially FG beam having square and rect^1 cross-sections are 44.1% and 66.2%, respectively.



(a) u_z^{max}

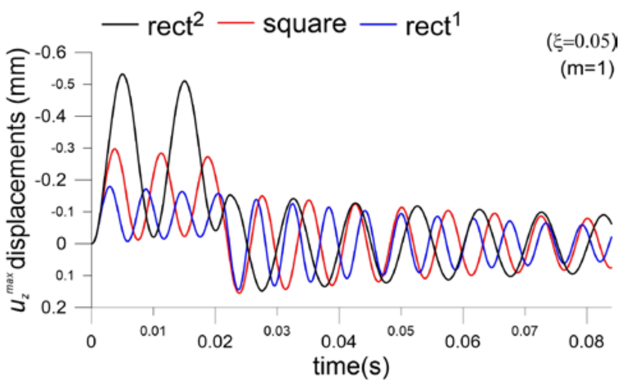


(b) T_z^A at point A

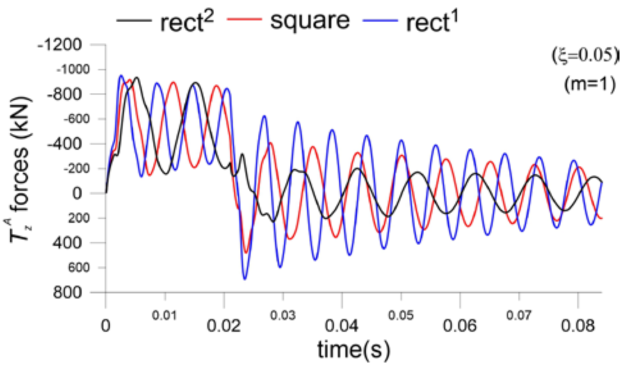


(c) M_y^A at point A

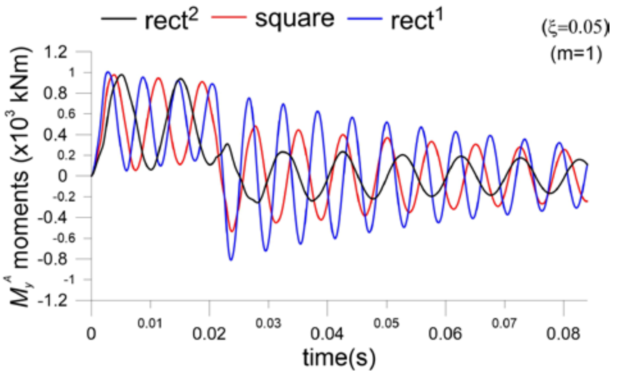
Fig. 6. The time histories of the axially FG straight beam having $\xi=0.05$ and square cross section for different material gradients $m=0.5,1,3$.



(a) $u_z^{\max}$



(b) T_z^A at point A



(c) M_y^A at point A

Fig. 7. The time histories of the axially FG straight beam for $\xi = 0.05$ and $m = 1$ for different cross-sections (see Fig.1b).

4 Conclusion

The damped/undamped forced vibration analysis of axially functionally graded straight beams under rectangular type impulsive load is investigated using the mixed finite element formulation based on the Timoshenko beam theory. The analyses are carried out in time domain using Newmark algorithm. The damping ratio used in this study considers the stiffness proportional coefficient of the Rayleigh damping using first two natural frequencies of the beam. The results of damped/ undamped axially FG straight beams are compared with the commercial finite element program ANSYS. Some parametric studies are performed to observe the effect of the material gradient index, damping ratios and the type of cross section on the dynamic response of the axially FG straight beam. Following remarks can be cited:

- The results of MFEM and ANSYS are in good agreement with each other.
- As the damping ratio increases, the amplitude of the times histories ($u_z^{\max}$, T_z^A and M_y^A at point A) decreases.
- As the material gradient index increases, the amplitude of time history $u_z^{\max}$ decreases for forced vibration zone and increases for free vibration zone.
- An increase in the thickness of the cross-section causes a reduction of the vibration period of time histories ($u_z^{\max}$, T_z^A and M_y^A at point A).

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