Resonant Orbits for the Space Debris

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Abstract: The present paper has the goal of studying the changes of the orbital parameters considering a fragment that makes a close approach with the Saturn.Successive swing-by maneuvers with the planet was performed to determine the trajectory. It is also assumed the presence of only two massive bodies (Sun and Saturn) that are in circular and planar orbits. Those derivations are based in the "patched-conics" approximation, which means that a series of keplerian orbits are assumed for the debris. It is then searched for geometries of the swing-by maneuvers that cause a series of passages by the planet. Those orbits have to be resonant with the motion of Saturn. After deriving the equations they are verified using the Tisserand's Criterion, which is a rule that must be followed by the keplerian elements before and after a swing-by. It is necessary to verify if the orbits are physically possible, having in mind that the periapsis of the orbits around the Sun needs to be above its surface, as well as the closest approach with Saturn needs to be above the surface of the planet.

Key–Words: Astrodynamics, Maneuvers, Space debris, Swing-Bys.

1 Introduction

Since the launch of the first satellite in 1957, there is a large expansion in findings related to space, due to the great advancements space technology obtained in the last decades. From time to time, satellites are sent into space, discovering new horizons and increasingly monitoring the Solar System. The artificial satellites launched are used for several purposes, including the observations of the Saturn, climate monitoring, studies of the atmosphere and the Saturn's gravitational field, etc. Space exploration requires that the orbits are accurately determined, where limits are established for each type of mission. However, since 1961, more than 190 man-made objects are orbiting the Earth, which causes serious problems due to the large number of spacecrafts and particles orbiting the Earth. It increases the risks of collisions. These space debris result from natural explosions of some bodies as meteorites, asteroids, etc; or the fragmentation of satellite launchers and non-operational satellites. Another major source of such debris is the disassemble of space platforms that remained in orbit, after becoming inactive, as well as tools that can be lost during space repairs required by some satellites.

In astrodynamics, depending on the mission, the trajectory of the space debris can be controlled by orbital maneuvers. The literature shows several studies to minimize the space debris in orbit considering the gravity force. This study will be used Swing-By maneuvers. It is a maneuver used to change the energy of a spacecraft by making a passage close to a massive body. The description of this type of maneuver can be seen in several publications, like [1]-[7]. Applications of this maneuver are also widely available, like in studies of transfer orbits to/from the Lagrangian points [8]-[11]; in the description of real missions that used this concept, like in [12]-[18]. Also variations of this problem is studied, like using the combination of impulsive maneuvers with the close approach [19], the presence of an atmosphere of the planet during the passage [20], elliptical orbits for the main bodies [21], the simultaneous passage of a group of particles instead of a single one [22]- [24], the combination with multiobjective optimization [25] or in the scattering of comets by a planet [26].

On the other hand, some researchers are engaged in searching solutions to understand the behavior of space debris in the Solar System: [27],[28] , [29], [30] and [31] are some examples of this type of work. However, there is an increasing effort to formulate analytic and computational models or techniques to provide more accurate estimates of those space particles: [35]; [32]; [36].

In the present research the goal is to find series of close passages by Saturn to allow a spacae debris to study the space in that region of the Solar System by changing its orbit without allowing an escape to occur. The dynamics is assumed to be given by the patched-conics. The orbital characteristics of the orbits involved are studied (velocity, energy, orbital elements and angular momentum) after each passage to show the evolution of the trajectories. Fig.1 shows a sketch of the maneuver.

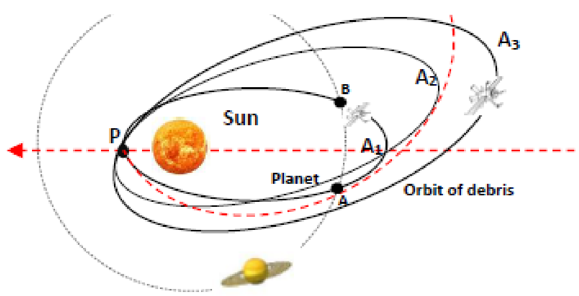

Figure 1: Overview of the orbital characteristics for some maneuvers.

2 Definition of the problem and mathematical model

In the problem the formulation hypothesis, will be considered a system formed by 3 bodies: The Sun (m_1), the Saturn (m_2) and a space debris, with an infinitesimal mass (m_3), with an initial orbit around the Sun and it makes a close approach with the Saturn. It is assumed that the problem is composed of three phases, where each of these phases is modeled by the classic Two-Body Problem, this formulation is called Patched Conics. The procedure can be split into three parts:

1. First, the gravitational effect of m_2 is despised and it is considered as a Keplerian orbit movement of m_3 around m_1;

2. Secondly, it is assumed that m_1 is too far and the system m_2m_3 makes a new two-body problem, it is assumed that m_3 invades the influence sphere m_2 (place where m_2 has a gravitational force of greater intensity acting in m_3);

3. Third step of the patched conics maneuver, m_2 is neglected one more time and the system m_1m_3 makes another two-body problem. Finally, under these conditions the space debris enters a new orbit Keplerian around m1 and the maneuver is complete.

This encounter changes the orbit m_3 of with respect to m1 and, by the hypothesis assumed for the problem, it is considered that the orbits of m_1 and m_2 do not change. The standard maneuver can be identified by the following three parameters: r_{ap}, the distance between the each space debris and the celestial body during the closest approach; v_{inf}^{-} and v_{inf}^{+}, the velocities of the space debris regarding the Saturn, before and after the passage, respectively, in the inertial frame; v_2, the Saturn velocity concerning the Sun; δ, half of the angle of the curvature due to the close approach; and Ψ, the angle of the approach, that is, the angle between the line connecting the primaries and the periapsis of the close approach trajectory. The velocity and orbital elements of the m_2 are changed by the close approach with the saturn [33, 34]. The orbital elements of space debris concerning the Sun before maneuver are: semi-major axis (a), eccentricity (e), energy (E) and angular momentum (C). They are obtained from Equations 1 shown below when ($-$) and ($+$) used for before and after the maneuver, respectively.

$$a_{-} = \frac{r_{a_{-}} + r_{p_{-}}}{2}; e_{-} = 1 - \frac{r_{p_{-}}}{a_{-}}; \tag{1}$$

$$E_{-} = -\frac{\mu_s}{2a_{-}}; C_{-} = \sqrt{\mu_s a_{-}(1 - e_{-}^2)} \tag{2}$$

where r_p is the periapsis of the orbit of the space debris around the Sun, r_a is the apoapsis of that orbit and μ_s is the gravitational parameter of the Sun. It is possible to obtain the variations of velocity, energy and angular momentum for each fragment from the Equations 2, 3 and 4 ([5]).

$$\Delta v {=} \vec{v}_0 - \vec{v} {=} 2|\vec{v}_\infty| \sin\delta \tag{3}$$

$$\Delta E = E_{+} - E_{-} = -\vec{v}_2 \cdot \vec{v}_\infty \sin\delta \sin\psi \tag{4}$$

$$\Delta C {=} \frac{\Delta E}{\omega} \tag{5}$$

where ω is the angular velocity of the motion of the primaries; δ is half of the angle of deflection due to the close approach; and E_{-}, E_{+} are the energy before and after the maneuver of each fragment, respectively. Finally, having determined the variations of energy and angular momentum due to the maneuver, it is possible to obtain the semi-major axis and the eccentricity of the orbit for debris after the close approach, by the using Eq. 6 shown next.

$$a_{+} = -\frac{\mu_s}{2E_{+}}; e_{+} = \sqrt{1 - \frac{C_{+}^2}{\mu_s a_{+}}} \tag{6}$$

2.0.1 Tisserands Criterion

The Tisserands criterion is an important method that can be used in the study of gravity-assisted maneuvers. It was presented by the French astronomer Francois Felix Tisserand and can be obtained from the Jacobian constant by making the approximation of neglecting the mass of the secondary primary. It is an equation, developed in dimensionless coordinates, based on the circular-restricted three-body problem model. This method can be used to find sequences of orbits similarly to what is done here, but it does not give the parameters required to obtain the sequence, as done by the present methodology. But, besides those limitations, it is an important form to verify the results found in the present research, because it can validate the sequence of orbits. In order to verity the orbits found here, the Tisserands criterion is used. It means that the orbits before and after the passage have to follow the equation [37]:

$$\begin{aligned} &\frac{1}{a_-} + 2\sqrt{a_-(1-e_-^2)}\cos(i_-) \approx \\ &\approx \frac{1}{a_+} + 2\sqrt{a_+(1-e_+^2)}\cos(i_+) \end{aligned} \tag{7}$$

When the orbital elements a_-, e_- and i_- are the ones before the passage and the orbital elements and a_+, e_+ and i_+ are the ones after the passage. This study will be considered the planar case, $i_- = i_+ = 0$. The Table 2 last column shows that the results follow the Tisserands criterion, because the values before and after the close approach are very similar for both dynamics, which means that the models are working correctly.

3 Results

The space debris starts in a given orbit around the Sun which is specified by its apoapsis and periapsis distances. Table 1 shows resonant orbits for the spacecraft, given the number of periods of Saturn before the following approach, the number of orbits of the fragment (debris) in this same period of time, the period of the orbit of the fragment (in days), the semi-major axis (km) of the orbit, and the order of the resonance. Then, it is possible to organize the orbits to put them in order of crescent values of the energy. Next, it is possible to find the values of rap for every passage. The assumptions used here are:

1. The close approach occurs at the point A (Fig.1);
2. No perturbations affect the space debris;
3. The two-body (Sun-fragment) energy is constant after and before the passage by Saturn;
4. The angular momentum (C) and the energy (E) are measured before and after the maneuvers.

It is then necessary to remove orbits that have periapsis below the surface of the Sun. Table 2 shows the useful orbits. It shows the number of the maneuver, the period (days), the distance of the closest approach (in units of radius of Saturn), semi-major axis (km), eccentricity, energy (km^2/s^2), periapsis distance (km), apoapsis distance (km), half of the deflection angle (degree), angle of approach (degree), order of the resonance and the time elapsed since the start of the maneuvers (days). The initial orbit for the fragment is assumed to have a periapsis of $155,000,000$ km, which is near the orbit of the Earth around the Sun, and apoapsis of $1,858,220,000$ km, that is a little bit higher than the orbit of Saturn around the Sun. Then, it is built Table2, where the resonant orbits are organized in crescent values of the energy. There is also no problem of having values for the periapis that is inside the Sun. Some of the orbits (the first 6) have periapis below the initial value, so they also intercept the orbit of the Earth around the Sun and there is a potential risk of collision, which is neglected in the present study. It is clear that there are decreasing values for the distance of the closest approach to compensate the increase of the velocity of approach. The sequence is limited to 17 revolutions before a situation where a value below the surface of Saturn is found. This situation is also a characteristic of the sequence of orbits shown and initial conditions where there is an escape orbit can be found for Saturn.

Those results show the evolution of the resonant orbits encountering Saturn. Still based on that results, it is possible to see that the variations of the orbital elements are larger when the space debris has a close approach at lower velocities and closer to the planet Saturn. These results are possible due to the fact that the gravitational force is larger at low altitudes. The energy, the angular momentum, the semi-major (axis) and the apoapsis distance increase after each close approach as forced by the objective of the sequence. This results can see in the Fig.2-5. The energy goes from -89.56 km^2/s^2 after the first passage until -18.48 km^2/s^2 after the last one, in crescent steps. This variation in energy causes the semi-major axis to go from $27,793,200$ km to $729,803,000$ km, which corresponds to a variation of the apoapsis from $1,457,280,000$ km (a little above the orbit of Saturn) to $6,466,380,000$ km.

It is a very large interval, so the spacecraft travels in different regions of the Solar System. The ec-

Table 1: Resonant orbits for the space debris passing by Saturn.

Number of revolutions of the Saturn between successive close approaches	Number of revolutions of the space debris successive close approaches	Period of the space debris (years)	Semi-major axis of the space debris (km)
1	1	29.4571	$14.279 \cdot 10^8$
	2	14.7286	$8.995 \cdot 10^8$
2	1	58.9142	$22.666 \cdot 10^8$
	3	19.6381	$10.897 \cdot 10^8$
	5	11.7828	$7.751 \cdot 10^8$
3	1	88.3713	$2.970 \cdot 10^9$
	2	44.1857	$1.871 \cdot 10^9$
	4	22.0928	$1.178 \cdot 10^9$
	5	17.6743	$1.015 \cdot 10^9$
	7	12.6245	$8.116 \cdot 10^8$
	8	11.0464	$7.425 \cdot 10^8$
4	1	117.828	$3.598 \cdot 10^9$
	3	39.2762	$1.729 \cdot 10^9$
	5	17.6743	$1.230 \cdot 10^9$
	7	16.8326	$9.832 \cdot 10^8$
	9	13.0921	$8.315 \cdot 10^8$
5	1	147.286	$4.175 \cdot 10^9$
	2	73.6428	$2.630 \cdot 10^9$
	3	49.0952	$2.007 \cdot 10^9$

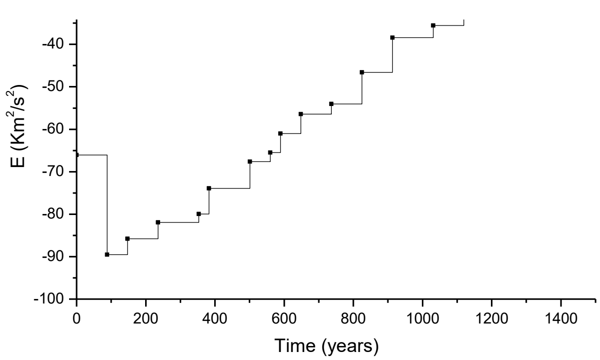

Figure 2: Energy of the spacecraft as a function of time.

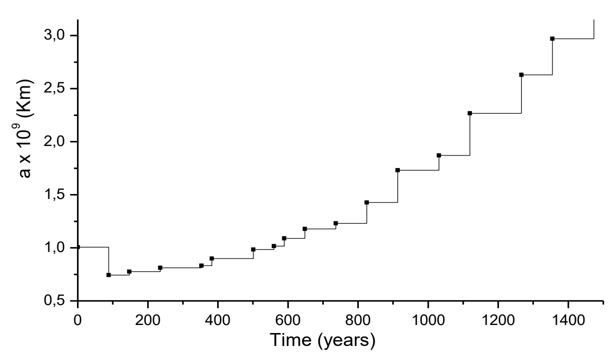

Figure 3: Semi-major axis of the fragment as a function of time.

centricity has an oscillating sequence, with a first decrease series and then an increasing sequence. The time span for this sequence is1472 years. The values of rap decrease from passage to passage, to compen-

Table 2: Sequence of orbits performing close approaches with Saturn

Man.	**orbital period (day)**	r_{ap}**(Radius of Saturn)**	**a** (10^6 **Km**)	**e**	**energy**	**resonance**	**time(days)**	**Tisserands Criterion**
0	6368.43	–	1006.61	0.8460	-66.06	-	0	1.1574
1	4034.77	941.86	742.54	0.9626	-89.56	8:3	88.37	1.1578
2	4303.76	2218.14	775.19	0.9474	-85.79	5:2	147.29	1.1577
3	4611.17	1281.58	811.67	0.9299	-81.93	7:3	235.66	1.1577
4	4781.96	955.96	831.59	0.9204	-79.97	9:4	353.49	1.1577
5	5379.70	699.75	899.52	0.8891	-73.93	2:1	382.94	1.1576
6	6148.23	457.55	983.27	0.8547	-67.63	7:4	500.77	1.1574
7	6455.65	348.93	1015.78	0.8427	-65.47	3:2	559.69	1.1575
8	7172.93	285.62	1089.69	0.8186	-61.03	1:1	589.14	1.1574
9	8069.58	216.64	1178.71	0.7950	-56.42	1:2	648.06	1.1572
10	8607.53	174.03	1230.53	0.7836	-54.04	5:3	736.43	1.1572
11	10759.40	126.69	1427.9	0.7540	-46.57	4:3	824.80	1.1571
12	14345.86	71.70	1729.78	0.7368	-38.44	5:3	913.17	1.1569
13	16139.08	44.20	1871.08	0.7357	-35.54	3:4	1031.00	1.1569
14	21518.80	27.36	2266.65	0.7440	-29.34	2:3	1119.37	1.1567
15	26898.52	13.56	2630.22	0.7579	-25.28	2:5	1266.66	1.1567
16	32278.26	7.07	2970.16	0.7722	-22.39	1:3	1355.03	1.1565
17	43037.63	2.79	3598.09	0.7972	-18.48	1:4	1472.86	1.1565
18	53796.97	0.54	4175.21	0.8171	-15.93	1:5	1620.14	1.1565

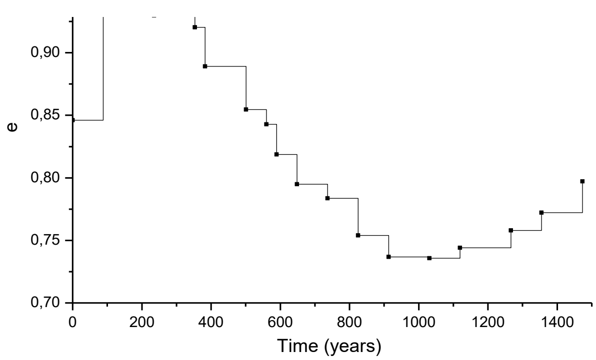

Figure 4: Eccentricity of the fragment as a function of time.

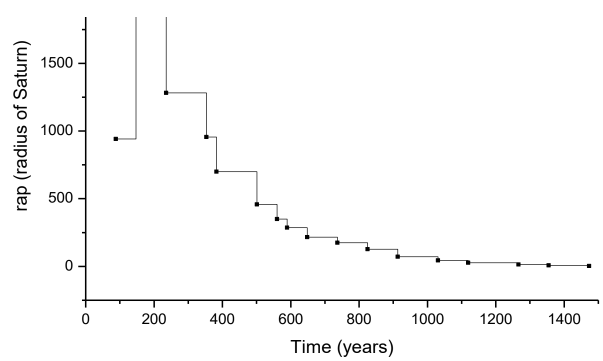

Figure 5: rap distance of the spacecraft as a function of time.

sate the increasing values of the velocity of approach. A series of resonant orbits with Saturn that has increasing values for the apoapsis to cover a large area of the space around the orbit of Saturn which the fragment

4 Conclusion

This study was made to show the evolution of the trajectories, as well as the amplitudes of the variations of the velocity, energy and angular momentum of an orbit due to a series of close approaches with Saturn. Analytical equations are used to make the calculation of the distance of the closest approach that generates a specified orbit. Then, a series of resonant orbits with

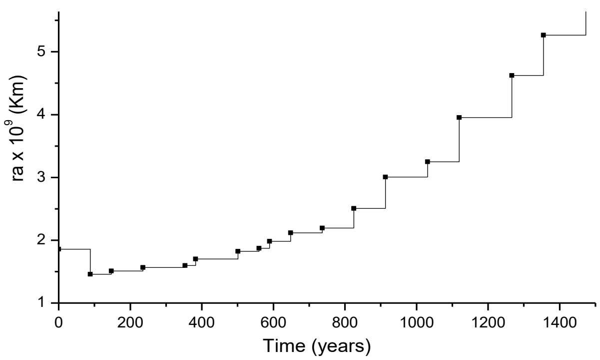

Figure 6: Apogee distance of the spacecraft as a function of time.

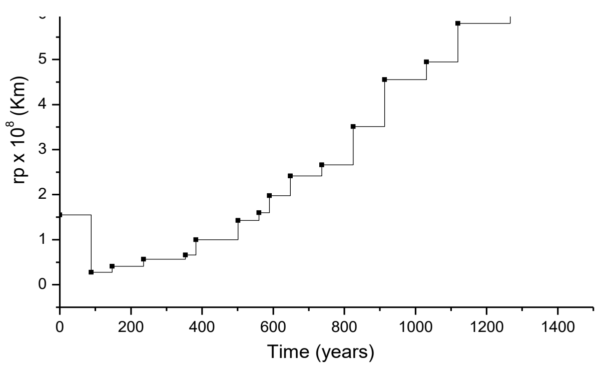

Figure 7: Perigee distance of the spacecraft as a function of time.

Saturn that has increasing values for the apoapsis to cover a large area of the space around the orbit of Saturn is found. Using these equations it is possible to establish a sequence of close approaches that meets the goals. The results showed that it is possible to find useful sequences of close approaches to study the space near Saturn by using these natural changes of orbits to pass by different positions in the space without the expenses of applying a control to the spacecraft.

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References:

[1] J.A.V. Allen, Gravitational assist in celestial mechanicsa tutorial, *American Journal of Physics*, 71, 2003, pp. 448–4515.

[2] A.E. Petropoulos, J.M. Longuski and N.X. Vinh, Shape-based analytic representations of low-thrust trajectories for gravity-assist applications, *American Astronautical Society.*, 1999, AAS Paper 99-337.

[3] N.J. Strange and J.M. Longusk, Graphical Method for Gravity-Assist Trajectory Design, *Journal of Spacecraft and Rockets.* 39, No. 1, 2002.

[4] R.H. Battin, An introduction to the mathematics and models of astrodynamics, *AIAA, New York.* New York, 1987.

[5] R.A. Broucke, The Celestial mechanics of gravity assist, *In:AIAA/AAS Astrodynamics Conf., Minneapolis, MN, August.* 1988, (AIAA paper 88-4220), 1988.

[6] V.M. Gomes, A.F.B.A. Prado and H.K. Kuga, Low thrust maneuvers for artificial satellites. *Transactions on Applied and Theoretical Mechanics.* 03, 2008, pp.10.

[7] A.F.B.A. Prado, A comparison of the patched-conics approach and the restricted problem for swing-bys.*Advances in Space Research.* 40, 2007, pp. 113-117.

[8] A.F.B.A. Prado and R.A. Broucke, Transfer orbits in restricted problem. *Journal of Guidance Control and Dynamics.* 18(3), 1995, pp. 593-598.

[9] A.F.B.A. Prado, Traveling between the Lagrangian points and the Earth. *Acta Astronautica.* 39(7), 1996, pp. 483-486.

[10] A.F.B.A. Prado and R.A. Broucke, Transfer orbits in the Earth-Moon system using a regularized model.*Journal of Guidance, Control and Dynamics.* 19(4), 1996, pp. 929-933.

[11] G. Flandro, Fast reconnaissance missions to the outer Solar System utilizing energy derived from the gravitational field of Jupiter.*Astronautical Acta.* 12(4), 1966.

[12] L.A. D'Amario, D.V. Byrnes and R.H. Stanford, R.H., Interplanetary Trajectory Optimization with Application to Galileo.*Journal of Guidance, Control, and Dynamics.* 5(5), 1982, pp. 465–471.

[13] S.A. Striepe and R.D. Braun, Effects of a Venus Swing by Periapsis Burn During an Earth-Mars Trajectory.*Journal of the Astronautical Sciences* 39(3),1991,pp.299-312.

[14] R.L. Dowling, W.J. Kosmann, M.A. Minovitch and R.W. Ridenoure, Gravity Propulsion Research at UCLA and JPL.*In: 41st Con. of the IAF, Dresden, GDR.* 6-12 Oct. 1991, pp. 1962-1964.

[15] B.L. Swenson, Neptune Atmospheric Probe Mission.*In: AIAA7AAS Astr. Conf., Hilton Head, SC, Aug. 10-12.* 1992,AIAA paper pp.92-4371.

[16] S.S. Weinstein, Pluto Flyby Mission Design Concepts for Very Small and Moderate Spacecraft.*In: AIAA/AAS Astr. Conf., Hilton Head, SC, Aug. 10-12.* ,1992, AIAA paper pp. 92-4372.

[17] A.E. Petropoulos and J.M. Longuski, Automated design of low-thrust gravity-assist trajectories, AIAA Paper 2000-4033, 2000.

[18] A.F.B.A. Prado, Powered Swingy. Journal of Guidance, Control and Dynamics, 19(5), 1142-1147, 1997a.

[19] A.F.B.A. Prado and R.A. Broucke, Effects of atmospheric drag in Swing-By trajectory. Acta Astronautica, 36(6), 285-290, 1995b.

[20] A.F.B.A. Prado, Close-approach trajectories in the elliptic restricted problem. Journal of Guidance, Control and Dynamics, 20(4), 797-802, 1997b.

[21] V.M. Gomes and A.F.B.A. Prado, A Study of the Impact of the Initial Energy in a Close Approach of a Cloud of Particles. WSEAS Transactions on Mathematics, Vol. 9, 2010, pp. 811-820.

[22] V.M. Gomes and A.F.B.A. Prado, Swing-by maneuvers for a cloud of particles with planets of the solar system. WSEAS Transactions on Applied and Theoretical Mechanics, Vol. 03, 2008, pp. 11-20.

[23] V.M. Gomes and A.F.B.A. Prado, Effects of the variation of the periapsis velocity in a swing-by maneuver of a cloud of particles. Proceedings: Recent Advances in Applied and Theoretical Mechanics, WSEAS Press, ISBN 978-960-474-140-3, Tenerife, Spain, 2009, pp. 106-108.

[24] V.M. Gomes and A.F.B.A. Prado, A Study of the Close Approach Between a Planet and a Cloud of Particles, Recent Advances in Signal Processing, Robotics and Automation, 2009, pp. 126-131, WSEAS Press, ISBN 978-960-474-054-3, Cambridge, England.

[25] F.C.F. Venditti, E.M. Rocco, A.F.B.A. Prado and A.A. Sukhanov, Gravity-assisted maneuvers applied in the multi-objective optimization of interplanetary trajectories?. Acta Astronautica, v. 67, p. 1255-1271, 2010.

[26] R.V. Moraes, A.F.B.A. Prado, N.R. Domecildes, On the scattering of comets by a planet. Advances in Space Research, v. 37, p. 169-173, 2006.

[27] J.C. Liou, An active debris removal parametric study for LEO environment remediation.*Advances in Space Research.* 47,2011,pp. 1865-1876.

[28] C. Colombo and C. McInnes, Evolution of swarms of smart dust spacecraft.*New Trends in Astrodynamics and Applications.* VI, New York, 2011.

[29] C.Pardini and L. Anselmo, Review of past on-orbit collisions among cataloged objects and examination of the catastrophic fragmentation concept.*Acta Astronautica.* 100,2014,pp. 3039.

[30] F. Letizia, C. Colombo, H. Lewis, C. R. Mcinnes, et al., Space debris cloudevolution in low earth orbit.*in: 64th International Astronautical Congress(IAC), Curran Associates.* 2013, pp. 25072517.

[31] A.E. White and H.G. Lewis, The many futures of active debris removal,*Acta Astronautica.* 95, 2014, pp 189-197, ISSN 0094-5765, https://doi.org/10.1016/j.actaastro.2013.11.009.

[32] C.R. McInnes, Simple analytic model of the long-term evolution of 360 nanosatellite constellations.*Journal of Guidance, Control, and Dynamics.* 23(2),2000,pp. 332 -338. doi:10.2514/2.4527.

[33] J.K.S. Formiga, A.F.B.A. Prado, Studying sequences of resonantorbits to perform successive close approaches with the moon.*Journal of the Brazilian Society of Mechanical Sciences and Engineering.* 3(4),2015,pp. 13911404, doi:10.1007/s40430-014-0254-8.

[34] D.P.S. Santos, A.F.B.A. Prado and E.M. Rocco, The study of the asymmetric multiple encounters problem and its applicationto obtain jupiter gravity assisted maneuvers, Mathematical Problems in Engineering, 2013. Article ID 745637. Doi:10.1155/2013/745637.

[35] N. Gorkavyi, M. Ozernoy,T. Taidakova.Quasi-stationary States of Dust Flows Under Poynting-Robertson Drag: New Analytical and Numerical Solutions.*The Astrophysical Journal.* 488,1997.

[36] H. G. Lewis,G. Swinerd, N. Williams,G. Gittins. DAMAGE: a dedicated GEO debris model framework.*EUROPEAN SPACE AGENCY-PUBLICATIONS-ESA SP.* 473,2001,pp.373–378.

[37] R. H. Battin, An introduction to the mathematics and models of astrodynamics.*AIAA, New York.* New York, 1987.