Variance of the Vertical Accelerations due to the Suspended Masses in the System "Railway Vehicle-Track": Simulation Technique, Differential Equation, Fourier Transform and Approximants

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Abstract: - In this paper the case of the system "railway vehicle rolling on a railway track with defects/faults" is investigated based on a simulation technique of an ensemble of springs and dashpots leading to the formation of the differential equation of this system. The application of the Fourier Transform on this second order differential equation of motion leads to its solution for the case of the Suspended/Sprung Masses of the railway vehicle and the approximants of the standard deviation of the vertical acceleration which appears on the vehicle'scar-body.

Key-Words: - Static/Dynamic Stiffness Coefficient, Sprung Masses, Unprung Masses, Fourier Transform, Spectral Density, Variance, Standard Deviation, Dynamic Component of Actions.

1 "Railway Vehicle-Track" System

The Railway Vehicle and the Railway Track, on which the Vehicle is running, constitute a unified system which functions (with actions and reactions between these two "elements") and oscillates as an ensemble. This system with its dashpots and springs is depicted -in a simplified form- in Fig. 1.

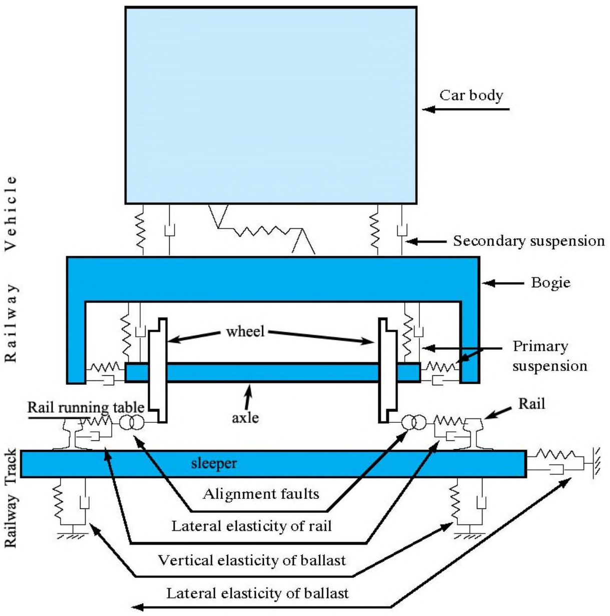

Fig. 1 The system "Railway Vehicle-Track".

The motion of a railway vehicle on the rail running table/surface or the motion of a road vehicle on the road, the response of the structures to earthquakes, etc, is a forced oscillation with a forcing excitation (force), and damping expressed by a random, non-periodic function.

In the railway vehicle there are two levels of suspension, the primary and the secondary, which define the Non-Suspended/Unsprung and the Suspended/Sprung Masses. The simulation technique, the implied differential equation and the computing of the variance of the vertical accelerations due to the Non-Suspended Masses (located under the primary suspension of the vehicle) was presented in [1], [2]. The present paper presents a simulation technique, the implied differential equation and the computing of the variance of the vertical accelerations due to the Suspended Masses (located over the primary suspension of the vehicle) [see relevantly [3]].

2 Simulation Technique

2.1 Simulation of the System "Railway Vehicle-Railway Track"

In Fig. 2 a schematic model of the system "Railway Vehicle-Track" is depicted; in practice it is an ensemble of springs and dashpots. The railway track is represented by the resultants of springs and dashpots, as described in [1] and [2]. It has to be noted that a part of the track mass is also added to the Non-Suspended Masses, which participates in their motion ([4], [5]).

The rail running table has the shape of a wave, that is not completely "rectilinear", consequently it does not form a perfectly straight line but contains faults/defects, varying from a few fractions of a millimeter to a few millimeters, and imposes forced oscillation on the railway vehicles that circulate on it. The faults/defects are represented by the ordinate n in Fig. 2. Moreover, during the rolling of the wheel, a deflection y of the rail running table appears (see [6]), since the support (track) is not undeflected.

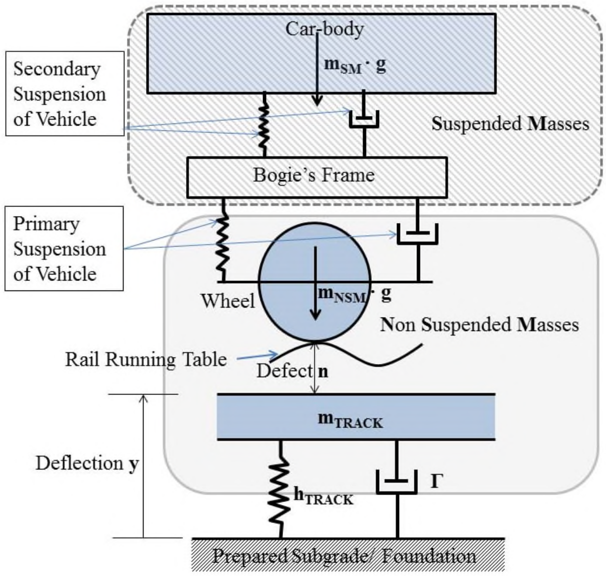

Fig. 2 Model of a Vehicle running the Rail Running Table; the Non-Suspended Masses (NSM) and the Suspended Masses (SM), the primary and secondary suspensions are depicted.

2.2 Differential Equation for the System "Railway Vehicle-Track"

The simulation technique leads to the simplified depiction of the track as an elastic media with damping (Fig. 2) circulated -on the rail running table- by a wheel (see relevantly also [7]). This system "Wheel-Track" is subjected to a forced oscillation by the irregularities of the rail running table (like an input random signal) –which are represented by n–, in a gravitational field with acceleration g. If the random excitation is given, it is difficult to derive the response, unless the system is linear and invariable, where the input signal can be defined by its spectral density, which can lead to the computing of the spectral density of the response. The theoretical results confirm and explain the experimental verifications ([8], p.39, 71). The differential equation for the system "wheel-track" becomes ([1], [3], [6], [7]):

$$\left(m_{NSM}+m_{TRACK}\right)\cdot\frac{d^2y}{dt^2}+\Gamma\cdot\frac{dy}{dt}+h_{TRACK}\cdot y= \\ =-m_{NSM}\cdot\frac{d^2n}{dt^2}+\left(m_{NSM}+m_{SM}\right)\cdot g \quad (1)$$

where: m_{NSM} the Non-Suspended Masses (NSM) of the vehicle in tonnes-mass, m_{TRACK} the mass of the track that participates in the motion of the NSM (for its calculation see Ref. [4] and [5]), m_{SM} the Suspended Masses (SM) of the vehicle, Γ damping constant of the track, h_{TRACK} the total dynamic stiffness coefficient of the track (its calculation Eqn. 3 below), n the fault ordinate of the rail running table, g the acceleration of gravity and y the total deflection of the track. Furthermore:

$$\frac{1}{\rho_{total}}=\frac{1}{\rho_{rail}}+\frac{1}{\rho_{pad}}+\frac{1}{\rho_{sleeper}}+\frac{1}{\rho_{ballast}}+\frac{1}{\rho_{subgrade}} \quad (2)$$

where: ρ_i the static stiffness coefficients of the constitutive layers of the track, the quasi spring constants of the layers and ρ_{total} the resultant total static coefficient of the track and h_{TRACK} the total dynamic stiffness coefficient of the track given by:

$$h_{TRACK}=\rho_{\ dyn-total}=\frac{1}{2\cdot\sqrt{2}}\cdot\sqrt[4]{E\cdot J\cdot\frac{\rho_{total}}{\ell}} \quad (3)$$

with E, J the modulus of elasticity and the moment of inertia of the rail (steel) and ℓ the distance among the sleepers.

The phenomena of the wheel-rail contact and of the wheel hunting, particularly the equivalent conicity of the wheel and the forces of pseudo-glide, are non-linear. In any case the use of the linear system's approach is valid for speeds lower than the $V_{critical}$≈500 km/h. The integration for the non-linear model (wheel-rail contact, wheel-hunting and pseudoglide forces) is performed through the Runge Kutta method ([8], p.94-95, 80, [9], p.98, see also [10], p.171, 351).

In Fig. 2 the rail running table depicts a longitudinal fault/defect of the rail surface. In the above equation, the oscillation of the axle is damped after its passage over the defect. Viscous damping, due to the ballast, enters the above equation under the condition that it is proportional to the variation of the deflection dy/dt. To simplify the investigation, if we ignore the track mass (for its calculation Ref. [4] and [5]) in relation to the much larger Vehicle's Non-Suspended Mass and bearing in mind that y+n is the total subsidence of the wheel during its motion (since the y and n are added algebraically).

2.3 Applying the Fourier Transform on the Differential Equation of a Track Loaded by a Running Railway Vehicle

We begin from the hypothesis of a cosine-form defect on the rail running table of the form:

$$\eta = a \cdot \cos\omega t = a \cdot \cos\left(2\pi \cdot \frac{V \cdot t}{\lambda}\right) \quad (4)$$

where: η the ordinate of the defect along the track (abscissa x), V the speed of the vehicle, t the time and λ the wavelength of the defect, so:

$$T = \frac{2\pi}{\omega} \Rightarrow \omega t = \frac{2\pi}{T} t \Rightarrow \omega t = \frac{2\pi V t}{\lambda} \quad (5a)$$

since the wheel overpasses the wavelength λ of the defect, in:

$$T = \frac{\lambda}{V} \Rightarrow \lambda = T \cdot V \quad (5b)$$

If we set:

$$y = z + \frac{m_{SM} + m_{NSM}}{h_{TRACK}} \cdot g \Rightarrow \frac{dy}{dt} = \frac{dz}{dt}$$

its second derivative will be:

$$\frac{d^2 y}{dt^2} = \frac{d^2 z}{dt^2}$$

where the quantity $\frac{m_{NSM} + m_{SM}}{h_{TRACK}} \cdot g$ represents the subsidence due to the static loads only, and z random (see [21]) due to the dynamic loads. Eqn (1) becomes:

$$m_{NSM} \frac{d^2 z}{dt^2} + \Gamma \cdot \frac{dz}{dt} + h_{TRACK} \cdot z = -m_{NSM} \cdot \frac{d^2 n}{dt^2} \Rightarrow \quad (6a)$$

$$\Rightarrow m_{NSM} \left(\frac{d^2 z}{dt^2} + \frac{d^2 n}{dt^2}\right) + \Gamma \cdot \frac{dz}{dt} + h_{TRACK} \cdot z = 0 \quad (6b)$$

Since, in this case, we are examining the dynamic loads only (derived from the actions of the Suspended and Non-Suspended Masses) , in order to approach their effect, we could narrow the study of equation (6b), by changing the variable:

$$u = n + z \Rightarrow \frac{d^2 u}{dt^2} = \frac{d^2 n}{dt^2} + \frac{d^2 z}{dt^2}$$

Equation (6) becomes:

$$m_{NSM} \frac{d^2 u}{dt^2} + \Gamma \cdot \frac{dz}{dt} + h_{TRACK} \cdot z = 0 \Rightarrow \quad (7a)$$

$$\Rightarrow m_{NSM} \frac{d^2 u}{dt^2} + \Gamma \cdot \frac{d(u-n)}{dt} + h_{TRACK} \cdot (u-n) = 0 \quad (7b)$$

where, u is the trajectory of the wheel over the vertical fault (of ordinate n) in the longitudinal profile of the rail.

If we apply the Fourier transform to the equation (6a) (see relevantly Ref. [11] for solving second order differential equations with the Fourier transform):

$$(i\omega)^2 \cdot Z(\omega) + \frac{\Gamma \cdot (i\omega)}{m_{NSM}} \cdot Z(\omega) + \frac{h_{TRACK}}{m_{NSM}} \cdot Z(\omega) = = -(i\omega)^2 \cdot N(\omega) \Rightarrow \quad (8a)$$

$$H(\omega) = \frac{Z(\omega)}{N(\omega)}, \quad (8b)$$

$$|H(\omega)|^2 = \frac{m_{NSM}^2 \cdot \omega^4}{\left(m_{NSM} \cdot \omega^2 - h_{TRACK}\right)^2 + \Gamma^2 \cdot \omega^2} \quad (8c)$$

H(ω) is a complex transfer function, *called frequency response function* [11], that makes it possible to pass from the fault n to the subsidence Z. If we apply the Fourier transform to equation (7a):

$$(i\omega)^2 \cdot U(\omega) + \Gamma \cdot (i\omega) \cdot Z(\omega) + h_{TRACK} \cdot (i\omega)^0 \cdot Z(\omega) = 0 \Rightarrow$$

$$G(\omega) = \frac{U(\omega)}{Z(\omega)}, \quad |G(\omega)|^2 = \frac{h_{TRACK}^2 + \Gamma^2 \cdot \omega^2}{m_{NSM}^2 \cdot \omega^4} \quad (9)$$

G(ω) is a complex transfer function, *the frequency response function,* that makes it possible to pass from Z to Z+n.

If we name U the Fourier transform of u, N the Fourier transform of n, *p=2πiν=iω* the variable of frequency and ΔQ the Fourier transform of ΔQ and apply the Fourier transform at equation (7b):

$$Eq.(7) \Rightarrow m_{NSM} \frac{d^2 u}{dt^2} + \Gamma \cdot \frac{du}{dt} + h_{TRACK} \cdot u = \Gamma \cdot \frac{dn}{dt} + h_{TRACK} \cdot n \Rightarrow$$

$$\left(m_{NSM} \cdot p^2 + \Gamma \cdot p + h_{TRACK}\right) \cdot U = \left(\Gamma \cdot p + h_{TRACK}\right) \cdot N \Rightarrow$$

$$U(\omega) = \underbrace{\frac{\Gamma \cdot p + h_{TRACK}}{m_{NSM} \cdot p^2 + \Gamma \cdot p + h_{TRACK}}}_{B(\omega)} \cdot N(\omega) \quad (10a)$$

where:

$$|B(\omega)|^2 = \frac{\Gamma^2 \cdot \omega^2 + h_{TRACK}^2}{\left(m_{NSM} \cdot \omega^2 - h\right)^2 + \Gamma^2 \cdot \omega^2} \quad (10b)$$

B(ω) is a complex transfer function, *the frequency response function,* that makes it possible to pass from the fault n to the u=n+Z. Practically it is verified also by the equation:

$$\left|B(\omega)\right|^2 = \left|H(\omega)\right|^2 \cdot \left|G(\omega)\right|^2 = = \frac{h_{TRACK}^2 + \Gamma^2 \cdot \omega^2}{\left(m_{NSM} \cdot \omega^2 - h_{TRACK}\right)^2 + \Gamma^2 \cdot \omega^2} \quad (10c)$$

passing from n to Z through H(ω) and afterwards from Z to n+Z through G(ω). This is a formula that characterizes the transfer function between the wheel trajectory and the fault in the longitudinal level and enables, thereafter, the calculation of the transfer function between the dynamic load and the track defect (fault).

The transfer function B(ω) allows us to calculate the effect of a spectrum of sinusoidal faults, like the undulatory wear. If we replace $\omega/\omega_n=\rho$, where ω_n= the circular eigenfrequency (or natural cyclic frequency) of the oscillation, and:

$$\omega_n^2 = \frac{h_{TRACK}}{m_{NSM}}, \quad \omega = \frac{2\pi V}{\lambda}, \quad 2\zeta\omega_n = \frac{\Gamma}{m_{NSM}}, \quad \beta = \frac{\omega}{\omega_n}$$

where ζ is the damping coefficient. Eqn (10b) is transformed:

$$\left|B(\omega)\right|^2 = \left|B_n(\beta)\right|^2 = \frac{1+4\zeta^2 \cdot \beta^2}{\left(1-\beta^2\right)^2 + 4\zeta^2 \cdot \beta^2} \quad (10d)$$

The transfer function C(ω) of the second derivative of (Z+n) in relation to time: $\frac{d^2(Z+n)}{dt^2}$, –that is the acceleration γ–, will be equal to *ω·B(ω)*:

$$p^2 \cdot U(\omega) = p^2 \left|B(\omega)\right| \cdot p^2 \cdot N(\omega) \Rightarrow$$
$$\Rightarrow U(\omega) = \underbrace{p^2 \left|B(\omega)\right|}_{C(\omega)} \cdot N(\omega) \quad (11a)$$

that is:

$$\left|C(\omega)\right| = \omega^2 \cdot \left|B(\omega)\right| = \omega_n^2 \cdot \frac{\omega^2}{\omega_n^2} \cdot \left|B(\omega)\right| \Rightarrow$$
$$\Rightarrow \left|C(\omega)\right| = \omega_n^2 \cdot \beta^2 \cdot \left|B(\omega)\right| \quad (11b)$$

The increase of the vertical load on the track due to the Non-Suspended Masses, according to the principle *force = mass x acceleration*, is given by:

$$\Delta Q = m_{NSM} \cdot \frac{d^2 u}{dt^2} = m_{NSM} \cdot \frac{d^2(n+Z)}{dt^2} \quad (12)$$

If we apply the Fourier transform to Eqn. (12):

$$\hat{\Delta}Q = m_{NSM} \cdot p^2 \cdot U(\omega) = m_{NSM} \cdot p^2 \cdot \hat{f}_{Z+n}(\omega) \Rightarrow \text{(13a)}$$

$$\left|\hat{\Delta}Q\right| = m_{NSM} \cdot \left|C(\omega)\right| \cdot \left|N(\omega)\right| = m_{NSM} \cdot \left|p\right|^2 \cdot \left|B(\omega)\right| \cdot \left|N(\omega)\right|$$

$$\left|\hat{\Delta}Q\right| = m_{NSM} \cdot \beta^2 \cdot \omega_n^2 \cdot \left|B(\omega)\right| \cdot \left|N(\omega)\right| \quad (13b)$$

In [3] the solution of the differential Eq. (1) is presented as far as the input and output Variance of the vertical acceleration of the Suspended Masses of the vehicle.

In order to pass from the defect n to n+Z [3]:

$$s_E(\omega) = \left|B(\omega)\right|^2 \cdot s_\upsilon(\omega) \Rightarrow$$

$$\Rightarrow s_E(\omega) = \frac{1+4\zeta^2 \cdot \frac{\omega^2}{\omega_n^2}}{\left[1-\frac{\omega^2}{\omega_n^2}\right]^2 + 4\zeta^2 \cdot \frac{\omega^2}{\omega_n^2}} \cdot \frac{AV^2}{\left(BV+\omega\right)^3} \quad (14)$$

where: $s_E(\omega)$ is the power spectrum density of the excitation, ω_n is always the eigenfrequency of the Non-Suspended Masses, ζ the damping coefficient of the track, $s_\upsilon(\omega)$ the spectrum of the excitation of the wheel due to the track defects/faults and ┃B(ω)┃the modulus of the transfer function of the motion of the wheel.

C(ω) is the transfer function of the second derivative of (Z+n) in relation to time: $\frac{d^2(Z+n)}{dt^2}$, that is the acceleration γ and it is equal to *ω·B(ω)*.

In [12] the results of a sensitivity analysis are presented, specifically for the case of long wave-length defects in relation to the vertical acceleration z′′(t) (as a percentage of the gravitational acceleration g) multiplied by a (constant) factor $[(m_{NSM}+m_{TRACK})/(m_{NSM} \cdot a)]$. In the case of the Suspended Masses ┃C(ω)┃is the modulus of the transfer function of the accelerations of the car-body. For wavelengths inferring cyclic-frequencies higher than two times the eigenfrequency of the wheel [see 12], the vertical acceleration of the NSM becomes very small, almost negligible (consequently the dynamic component of the load), compared to the results presented in [1], [2], for smaller values of ω_n/ω_1 (0.5 till 2). In the cases of the long wavelength defects, the influence of the SM should be examined.

For the railway vehicles the eigenfrequencies ω'_n of the car-body are in the area of 1 Hz, since with the development of high-speeds it could arrive 10 Hz. For the damping coefficient of the car-body of the railway vehicles two characteristic values of ζ′ could be used with reliability: 0,15 and 0,20 (see relevantly [13] and [8]).

$$s_\gamma(\omega) = |C(\omega)|^2 \cdot s_E(\omega) \Rightarrow s_\gamma(\omega) = \omega_n^4 \cdot \frac{AV^2}{(BV+\omega)^3} \cdot$$
$$\cdot \frac{1+4\zeta^2 \cdot \beta^2}{(1-\beta^2)^2 + 4\zeta^2 \cdot \beta^2} \cdot \frac{1+4\zeta'^2 \cdot \beta'^2}{(1-\beta'^2)^2 + 4\zeta'^2 \cdot \beta'^2} \qquad (15)$$

The variance of the accelerations of the car-body of the railway vehicles is given [12]:

$$\sigma(\gamma)^2 = \frac{1}{\pi} \cdot \int_0^\infty s_\gamma(\omega) \cdot d\omega \qquad (16)$$

which converges for ω infinite. Consequently, the variance of the part of the dynamic component of the load due to the Suspended Masses of the vehicle is given by [12]:

$$\Delta Q_{SM} = m_{SM} \cdot \Delta\gamma \Rightarrow \sigma^2(\Delta Q_{SM}) = m_{SM} \cdot \sigma^2(\gamma) \qquad (17)$$

Finally, an approximation could be used for the calculation of the variance of this part of the dynamic component of the load (see [6], [7]):

$$\sigma(\Delta Q_{SM}) = \frac{V-40}{1000} \cdot N_L \cdot Q_{wheel} \qquad (18)$$

where: Q_{wheel} is the static wheel load, V is the operational speed, and the coefficient N_L is the mean standard deviation of the longitudinal level condition of the track, on a 300 m length approximately, for both rails is the mean standard deviation of the longitudinal level condition of the track, on a 300 m length fluctuating between 0,7–1,5 mm or more (see [7]; [13], p. 335–336); for the Greek network N_L is estimated to fluctuate –mainly– between 1 and 1,5 [6].

In more details N_L, the average of the brutal signal on a basis of approximately 300 m for the vertical and horizontal defects of the two rails, is the convolution:

$$N_L(x_0) = \frac{1}{300} \int_{-\infty}^{+\infty} \eta l(x) e^{\left(\frac{x-x_0}{300}\right)} dx \qquad (19)$$

where *ηl(x)* is the value of the primary signal; in practice a weighted average index which "crashes" less the isolated defects than one classic average and simulates roughly the "memory" of the vehicle (see relevantly [13]).

3 Approximants

This computing technique is complicated in practice for every-day use -especially- on work-sites. There is a need for the use of approximation methods -in simpler computers- with mathematical equations which could be compared to the measured values on real systems "railway vehicles-tracks".

The findings of an investigation, performed during a research program of the Greek railways in collaboration with the French state railways (SNCF), with the author as Coordinator and further research performed by the author, included the presented in Fig. 3 below data. In real conditions, according to measurements of the French State Railways (SNCF), the standard deviation σ(γ) of the vertical accelerations [as a percentage of the gravitational acceleration g] due to the Suspended Masses of the vehicles, in relation to the running speed is depicted in Fig. 3, as presented by professor J. Alias(†) [8] and A. Prud'Homme(†) [13]. The measurements were performed in tracks under operation in the French network (SNCF) and obviously, the measured values of σ(γ) have been influenced by the variation of the real values of $\rho_{subgrade}$ and ρ_{track}, as it existed along the measured tracks (consequently, the values of ρ were not constant), since the variability of the track stiffness e.g. due to imperfect sleeper support and inhomogeneities of the track structure is an inherent property. Fig.3 is the final product of the measurements performed along the tracks of the French network, and it is given in the publications above, as of general validity.

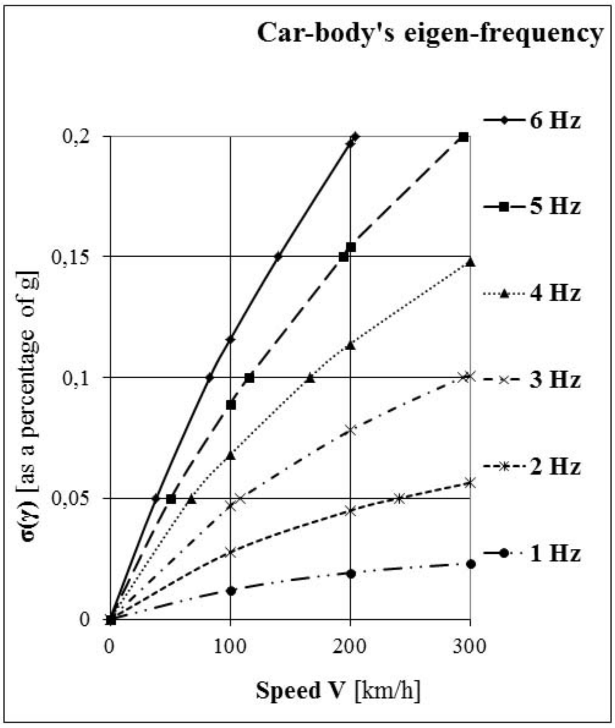

Fig. 3 Measured values of the standard deviation of the vertical accelerations σ(γ) [of railway vehicles] in relation to the running speed and the eigenfrequencies of the railway vehicles (see [8], [13], [12]).

Six curves are presented for eigenfrequencies of the car-body 1 to 6 Hz. More analytically, the coupled system car-body-bogie-axles (a two-floor system) [Fig. 1] presents an eigenfrequency, that of the axle on the track, approximately 30-40 Hz, in an attenuated form, corresponding to the track defects which will become much more important when the speed in consideration will provoke a frequency very close to the coupled frequency of the car-body with the axles. This coupled low frequency of the car-body-bogie-axles, which mainly affects the car-body and it is specifically interesting to us in this analysis, is approximately 1 Hz for the passenger wagons and higher for the freight wagons (see relevantly [8, p. 47]). This implies that the curve for 1Hz eigenfrequency represents the passenger wagons and the higher frequencies the freight wagons, running at lower speeds.
According to the analysis cited above –about the coupled system car-body-bogie-axles– it is clear that finally in this coupled motion (of 1 Hz frequency) all the parts of the vehicle participate, so we can approach the equations using the total Q_{wheel} instead of m_{SM}.

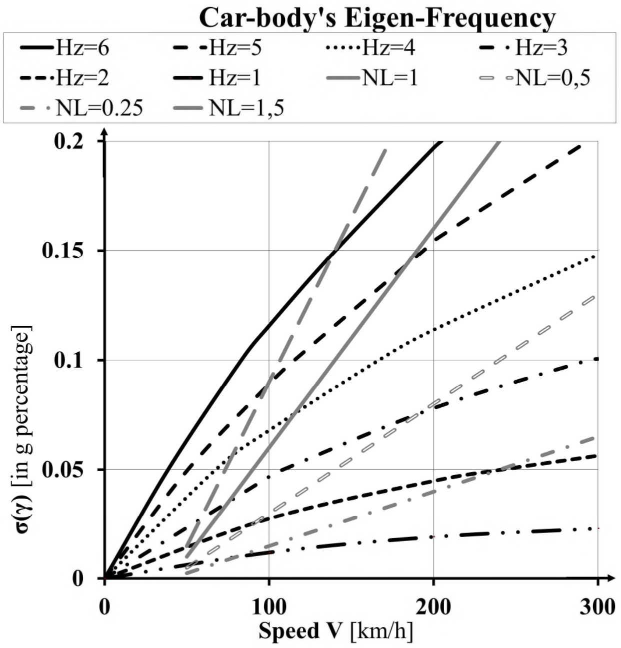

Fig. 4 Comparison of (1) the measured values of the standard deviation of the vertical accelerations σ(γ) [of railway vehicles] in relation to the running speed and the eigenfrequencies of the railway vehicles (see [8], [13]) and (2) the values of the empiric Eqn (18) for four values of the coefficient N_L=0.25, 0.50, 1.00 and 1.50.

This Figure is used for verification and comparison of the empiric Eqn (18) presented in [6] and the real conditions of the system "Railway Vehicle-Track". In Fig.4 the curves of the eigenfrequencies of the vehicles as measured by the SNCF are depicted and, also, the Eqn (18) for four values of N_L: 0.25, 0.5, 1 and 1.5. The approximation is close enough.
Some first remarks lead to the conclusion that small values of N_L approach the passenger vehicles (with small eigenfrequencies) since larger values of N_L approach the freight vehicles (with larger eigenfrequencies).
We must underline that the system "Railway Vehicle-Track" is subjected to random excitations and responses along the track and the derived approximants according to Eqn (18) have been based on Railway Track's approaches and data since the measured values of the eigenfrequencies of the Railway Vehicles -on which this approximation is based- are derived from the domain of the Vehicles' proper motion. Therefor a further research and sensitivity analysis should be performed in the future.

4 Conclusions

In this paper, after the application of the Fourier Transform on the second order differential equation of motion -formulated for the system "Railway Vehicle-Track"- led to the solution of the differential equation for the case of the Suspended/Sprung Masses of the railway vehicle. The approximants of the standard deviation of the vertical acceleration which appears on the vehicle's car-body have been investigated through a sensitivity analysis in relation to the railway vehicles' eigenfrequencies and the operational speed. A real Railway Track with defects/faults have been taken into account. The solution is verified from findings from a research program performed by the Greek railways in collaboration with the French state railways (SNCF) and further research performed by the author.

References:
[1] Giannakos K. Modeling the Influence of Short Wavelength Defects in a Railway Track on the Dynamic Behavior of the Non-Suspended Masses, *Journal of Mechanical Systems and Signal Processing (Mech. Syst. Signal Process.)*, Elsevier, 2015, Vol. 68-69, 2016, pp. 68-83.
[2] Giannakos K. Track Defects and the Dynamic Loads due to Non-Suspended Masses of Railway Vehicles, *NAUN International Journal of Mechanics*, Vol.7, Issue 3, 2013, pp. 180-191.

[3] Giannakos K. Second Order Differential Equation of Motion in Railways: the Variance of the Dynamic Component of Actions due to the Sprung Masses of the Vehicles, International Journal of Theoretical and Applied Mechanics pp.30-37.

[4] Giannakos K. Theoretical calculation of the track-mass in the motion of unsprung masses in relation to track dynamic stiffness and damping, *International Journal of Pavement Engineering (IJPE)*, Special Rail Issue "*High-Speed Railway Infrastructure: Recent Developments and Performance*", Vol. 11, Issue 4, August 2010, pp. 319-330.

[5] Giannakos K. Influence of the track's damping on the track mass participating in the motion of the Non-Suspended Masses of railway vehicles – theoretical calculation and comparison to measurements, *volume published in honour of (fs) professor George Giannopoulos*, Aristotle University of Thessaloniki, 2012.

[6] Giannakos K. *Actions on the Railway Track*, Papazissis publ., www.papazisi.gr, Athens, 2004.

[7] SNCF/Direction de l' Equipement, *Mecanique de la Voie*, 1981.

[8] Alias J. *La Voie Ferree – Techniques de Construction et Entretien*, deuxieme edition, Eyrolles, Paris, 1984.

[9] Fortin J. La Deformee Dynamique de la Voie Ferree, *Revue Generale des Chemins de Fer (RGCF)*, 02/1982.

[10] Thompson D. *Railway Noise and Vibration*, Elsevier, 2009.

[11] Wylie C.R. and Barrett L.C. *Advanced Engineering Mathematics*, sixth edition, McGraw-Hill, Inc., USA, 1995.

[12] Giannakos K., Track Defects and the Dynamic Loads due to Suspended (Sprung) Masses of Railway Vehicles, in Loizos A/Al-Qadi I./T., Scarpas, (eds), Intl. Conference *Bearing Capacity of Roads, Rails, Airfelds*, Athens, proceedings, Taylor & Francis group, 2017, pp.1911-1919.

[13] Prud'Homme A. La Voie, *Revue Generale des Chemins de Fer (RGCF)*, Janvier, 1970, extrait de RGCF.