Model of Fatigue Crack and Effect of Triaxiality

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Abstract: - These article is devout to the fatigue crack growth and it's simulation. The special problem of planar stress is discussed. The effectiveness of three-dimensional and two-dimensional crack is discussed. Both models are compared. In the article is shown that the simulation based only on the stress intensity factor K is much less accurate than the model based on combination of stress intensity factor *K* and stress triaxiality *T*. The stress triaxiality *T* can describe local influence of constrains and so improve crack model.

Key-Words: - Fatigue crack, stress intensity factor *K*, triaxiality *T*, constrain, plane stress, plane deformation.

1 Introduction

This article is devout to the problem of simulation of fatigue crack in the specimen made from aluminum alloys.

In the case of engineering structure, its failure is often caused by materials fatigue. The material fatigue is cumulative damage of material under cyclic loading [1,2,3]. The fatigue fracture typically initiates in the region of maximal stress, i.e. in the zone of stress concentrations [3]. Corners and wholes in structure are acting as such stress raisers. These crack initiation sites are subjected to the same loading spectrum as other parts of structure.

In the work are discussed options for simulations and prediction of fatigue crack [4,5,6]. The model of fatigue crack was improved by stress triaxiality *T* [7,8,9]. The stress triaxiality *T* was used to describe crack growth, primarily to determine crack velocity.

2 Experimental Procedure

Each theoretical model must be verified by comparison with experimental data. An experiment is needed in which the crack will propagate from the corner of the square hole in the flat plate. The geometry of specimen is shown in Fig. 1. The specimen is subjected to cyclical loading (pull – push) in the direction parallel to the plate. The fatigue crack propagates from the corners of central hole. Results of theoretical simulations are compared with experimental results obtained from fractographycal analysis of fracture surface [10,11,12,13]. The fractographycal analysis allows to track changes in the geometry of crack forehead. The experimental sample was prepared from aluminium alloy 7075.

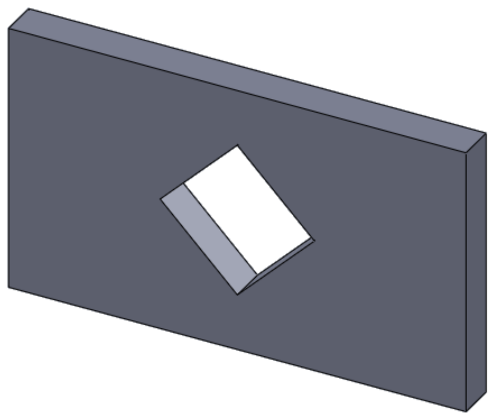

Fig. 1 Sample geometry

3 Computional Model Based On Stress Intensity Factor

3.1 Relationship between crack velocity *v* and stress intensity factor *ΔK*

In an effort to obtain functional model of planar cracks several approaches have been proposed. Most of them are based on first amplitude of stress-intensity factor *ΔK*, which is understood as a variable along the forehead the crack. In engineering practice different types of computer programs are used, these programs take into account loading and material characteristics. Control algorithms of these programs are based on concept of interacting plastic zones. Also, algorithms based on analytic assessment of crack closure and effective amplitude of ΔK_{eff} are used. The latter is used only rarely. Simple phenomenological hypothesis was postulated. This hypothesis says:

The velocity *v* of crack growth (at a certain point on the forehead, the velocity is perpendicular to the forehead of the crack) is controlled by local value of relative stress intensity factor K'_r. The relative stress intensity factor is specific for given loading mode. The value of relative stress intensity factor K'_r is related to the some nominal stress value such as nominal stress 1 MPa. The stress intensity factor K'_r is function of loading and specimen's material. Function $v = f(K')$ is independent on the shape of specimen. This function can be determined experimentally.

However, this hypothesis has obvious shortcomings. Firstly, the fatigue experiments used to produce function $v = f(K')$ are relatively complex. For each loading mode (or it's combination), it is necessary realize separate set of fatigue experiments. Also, detailed fractographyc analysis of crack surface is necessary. The assessment of *K'* is also considerably complex because radius of crack's forehead changes over time.

3.2 Results of simulation based on stress intensity factor conception

Quantitative analysis of the material on the fatigue crack forehead requires complicated calculations. These calculations require 3D-model of specimen with very dense mesh (high number of cells). Moreover, the deformations on the cracks forehead are elasto-plastic, i.e. finite element model used in calculation is non-linear. Whole problem is complicated by requirement to simulate the process in a longer time.

However, basic trends can be obtained relatively easily by 2D-simulation. These 2D-simulation are based by assumption of small deformations and plane stress or plane strain.

The results of such simulations are shown in Fig. 2 to Fig. 5. All graphs are related to the point of maximal loading. Horizontal axes display the *x* coordinate in the spreading direction. Vertical lines indicate the current positions of crack forehead. The shift of crack forehead u_y (Fig. 2) is engraved on the vertical axis *y*, changes in plastic deformation $\Delta\varepsilon^p$ (Fig. 3) during one loading half cycle, increment of plastic zone thickness u_{yr} (Fig. 4) and normal stress σ_n (Fig. 5). These graphs are demonstrating significant effect of out-of–plane constraint. Greater opening of the crack u_{yr} (Fig. 2) and significantly smaller residual plastic deformation (Fig. 3) behind the forehead leads to the decrease of constraint at the crack tip. The crack is not closed at all. Plastic deformation in the loading direction is going at the state of significant triaxial stress. This state is characterized by high value of normal stress σ_{yy}, whose maximum is three times higher than in the case plane stress (Fig. 4).

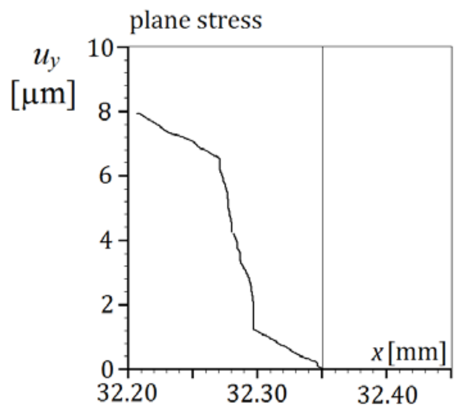

Fig. 2 Results of simulation: shift of crack forehead u_y.

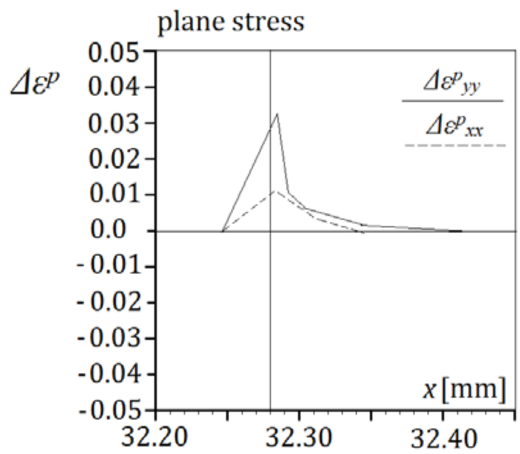

Fig. 3 Results of simulation: plastic deformation $\Delta\varepsilon^p$

In the case of general 3D-object, the crack constrain along the curved forehead is variable. The crack constrain is function of series of variables such as plastic zone shape, plastic zone thickness, specimens shape or forehead profile. With increasing penetration depths of crack, the effect of crack constrain is growing.

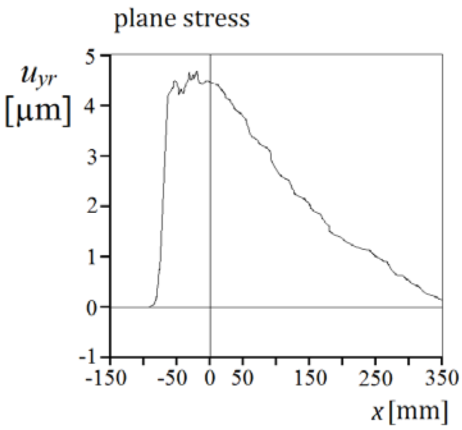

Fig. 4 Results of simulation: shift of crack forehead $u_{y.}$

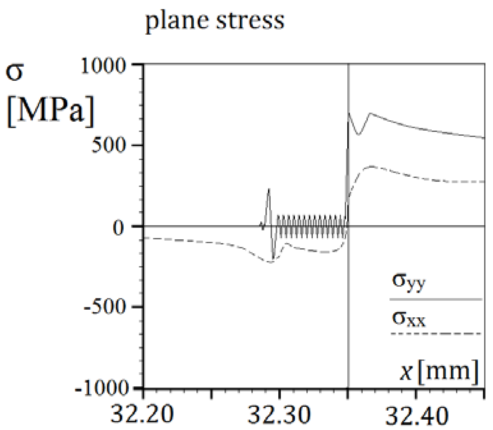

Fig. 5 Results of simulation: shift of crack forehead $u_{y.}$

4 Triaxiality and *K*-factor As Interacting Controling Parameters

Previous results confirm fact that the driving effect of *K* factor must be taken in the account during the simulation of motion of the fatigue cracks forehead. Another fact which must be taken into account during simulations is local effect of constraint variations. As a parameter of local constraint, the stress triaxiality *T* was used. The stress triaxiality is defined as ratio between medium stress

$$\sigma_m = \left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right)/3 \tag{1}$$

and Von Mises effective stress $1/\sqrt{2}.\sqrt{\sigma_{\Sigma q}}$, $\sigma_{\Sigma q}$ is defined as sum of multiplied differences between principal elements of stress tensor:

$$\sigma_M = \left[(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{xx} - \sigma_{zz})^2 + (\sigma_{yy} - \sigma_{zz})^2 + 6.\phi\right] \tag{2}$$

where

$$\phi = \left(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2\right) \tag{3}$$

.

If the triaxility *T* is growing, the effect of constrain is growing too. As a result of these facts the controlling function $v = f(K')$ mast be written as

$$v = f(K',T) \tag{4}$$

This new model needs to be tested. Testing will be based on assumptions: (1) the state of stress is changing along the whole length of cracks forehead; (2) stress is changing with growing distance from cracks forehead. The model assumed cyclic pull-push loading.

The state of stress triaxility will be determined by 3D static calculation of triaxility *T* in the distance 45μm from cracks forehead. The state of deformation is elasto-plastic during whole computing cycle. Computing cycles are repeated roughly until the moment when one third of maximum stress is reached.

The value of velocity *v*, *K′* factor and triaxiality *T* were calculated along two fractographycaly identified foreheads. These two foreheads correspond to two symmetrical cracks in the specimen with central opening. This hole determines central plane of symmetry and plane of crack. Both of cracks are initiated from central hole.

The foreheads velocity *v* is growing according to increase of both quantities *K′* and also triaxiality *T*. This way you can get a substitution for function $v = f(K',T)$ Eq.1 as count of two independent functions. These two functions are *K′* factor and triaxiality *T*. These functions can be written as:

$$v = f_1(K') + f_2(T) \tag{5}$$

This modified function can be used for simulation of two different models. These two models represents fatigue crack in the corner of the structure with distinct geometry.

Both models are subjected to the same loading with the same time course. In bout cases the crack initiated in the corner of the specimen. The results of simulations are shown in the Fig. 6. Full lines in the Fig. 6 correspond to the smoothed foreheads of fatigue crack. For each of these cracks (respectively for each stopping of forehead) the method of border elements was used. In these manner *K'* was correlated. For every third forehead the triaxiality *T* was determined by finite element method and using the results obtained by border element analysis.

Then the time course of perpendicular velocity v_P can be determined. For each new location of crack the procedure is repeated. In the Fig. 6 are rectangles with a number of cycles *N*. Places marked with rectangles correspond to the experimentally determined positions of cracks forehead. These positions therefore correspond to the points determined by fractographycal analysis on the fracture surface. The experimentally determined points on the forehead are then marked with black dots. These black dots form lines, which indicate shape of real cracks forehead.

Both pairs of simulations and tests lead to a good match of the results. Theoretically obtained positions of foreheads correspond to the points on the fracture surface. This match of the theoretical curves and points will stand out in comparison with results obtained by calculation based only on *K'* (look at the Fig. 5 and compare results in the case (a) with results in the case (b)). This agreement is the consequence of reduction of driving effect of *K*-factor at the first stage of crack growth. The *K*-factor will fall only lightly after the introduction of triaxiality, but the effect on the shape and velocity of forehead is considerable.

Utilization of triaxiality in the fatigue crack growth simulations leads also to the good agreement between theoretical a real velocity of forehead. Only in the case of first model the theoretical velocity is considerable higher than in the reality (compare Fig. 5 (a) and (b)).

Causes of this distortion are not conclusively clearly. The fractographical analysis of real specimen shows that in the reality the fatigue crack initiate in multiple places. For example in the studied specimen the crack initiate on three places. These three particular cracks grow on the three different planes. These three particular planes are very slightly diverted. These three cracks merge together after changing the crack propagation mechanism. The development of the fracture could thus be hampered in the initial stage of the process. The model did not count on the existence of three initiations. Distortion could also be caused due to the fact that the bulk of the growth of the through-crack was controlled by *v*, *K'*, *T*. These three values are obtained for crack growing from the corner of flat specimen. All these values were determined for symmetrically deployed pairs of cracks. If the symmetry of the cracks were not used, the function would be dependent on specimens shape.

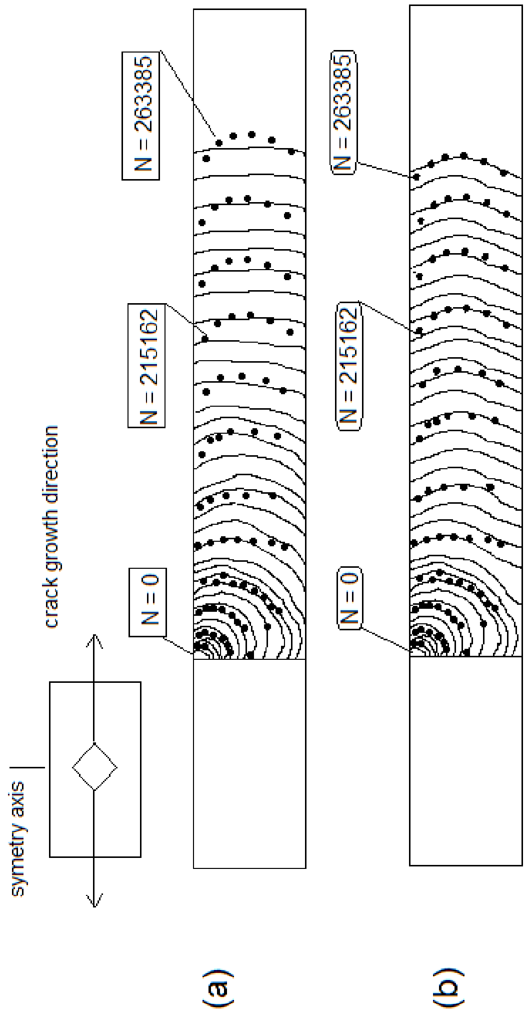

Fig. 6 Fractographycal analysis and simulations. (a) model base on K-factor conception; (b) model based on combination of *K* and *T*. Black dots at are experimental points obtained by fractography.

5 Conclusion

Simulation of planar fatigue crack growth with curved forehead in 3D-body should be based on assumptions: the calculations must take not only stress intensity factor *K* in mind, but also constrain at the crack tip and its course. The value of crack constrains affects value of cyclic stress in the critical region before crack. The local value of constrains can be characterized by stress triaxiality. The stress triaxiality is determined by ratio between medium value of normal stress and Von Misess effective stress. The theory was verified in practice. This theory says that the local value of forehead velocity is dependent on the local value of stress intensity factor for normal stress normalized to stress 1 MPa and local value of triaxiality. The triaxiality is estimated by calculations in elasto-plastic region. Parameters used in this calculation are obtained from fatigue test on the specimen with simple geometry. In this case numerical simulations lead to the satisfactory results.

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