Restricted Optimum Design of Reinforced Concrete Retaining Walls

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Abstract: -The optimum design of reinforced concrete (RC) members is subjected to several limitations resulting from safety measures and construction opportunities. In this study, the cost optimization of RC retaining walls was investigated for considering the restrictions on construction sites. Generally, the footing dimensions may not be applied if the distance were blocked with other structures. In that case, the optimum design may be different than the most economical results. In order to investigate this factor, the upper limit of the solution range is reduced under the optimum values of a problem case. The investigation was done for different cases with different limitations of the heel and toe of the retaining wall. In the optimum design, teaching-learning based optimization (TLBO) was employed and the retaining wall is both subjected to static and dynamic loads resulting from earthquakes. As a conclusion, the proposed method is effective to find the restricted optimum design of RC retaining walls.

Key-Words: Restricted optimum design, teaching-learning based optimization, structural optimization, reinforced concrete, retaining walls.

1 Introduction

The main aim of an engineer in the design process is to find a design ensuring stability under the loads, displacement constraints, stress capacity of elements and the other variables. In addition to that, an engineering design must be economical. For that reason, the design process of an engineering design can be defined as an optimization process. In this process, metaheuristic algorithms such as genetic algorithm (GA), particle swarm optimization (PSO), ant colony optimization (ACO), big bang-big crunch algorithm (BB-BC), simulated annealing (SA), harmony search (HS), firefly algorithm (FA), charged system search (CSS), and bat algorithm (BA) have been employed.

Reinforced concrete (RC) members or structures, especially RC retaining walls, are an important optimization practice. The optimization RC retaining walls dates back to 1980s. In order to obtain the optimum shape, structural stability, minimization of flexural moments and optimum orientation, several studies have been conducted [1-8]. In addition to these studies, several metaheuristic based methods have been proposed for RC retaining walls. The employed algorithms include SA [9-10], PSO [11], HS [12], BB-BC [13], GA [14], FA [15] and CSS [16]. The RC retaining walls were investigated on static loads in these studies. The dynamic loads resulting from earthquakes are considered by Kaveh and Soleimani [17] employing colliding bodies optimization and democratic particle swarm optimization. The teaching-learning based optimization (TLBO) is a relatively new algorithm. Temur and Bekdaş [18] optimized RC retaining walls by employing TLBO and the optimum shape, dimensions and design was investigated for the minimization of the cost of the wall. Kayabekir et al. [19] considered dynamic earthquake loads according to TEC2007 (Turkish Earthquake Code for Buildings, Specification for Buildings to be Built in Earthquake Areas) [20] in the optimum design of RC retaining walls employing TLBO. In the present study, the investigations were enlarged by considering multiple design cases considering the limitation of the heel and the toe of the RC retaining wall. This limitation can occur in the construction yards because of the existing structures or limitation of the construction yard.

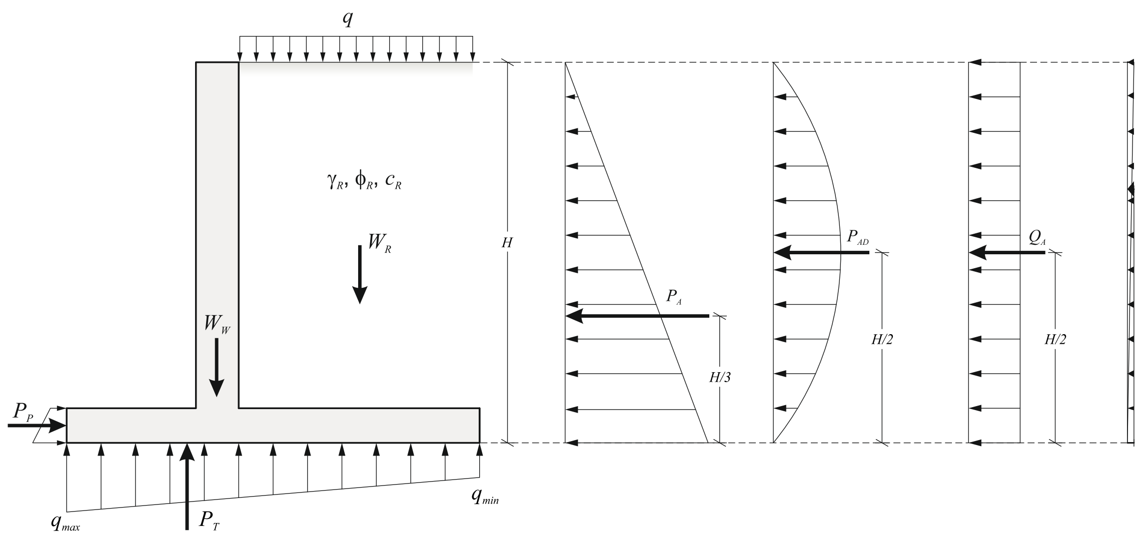

Figure 1. A cantilever retaining wall and load conditions

2 Design of RC retaining walls

A retaining wall with the resulting forces are presented in Fig. 1.

The definition of symbols in the Fig. 1 are as follows:
W_v: The weight of the retaining wall
W_T: The weight of backfill on the heel
q: The surcharge loads
P_A: The active earth pressure
P_{AD}: The active earth pressure resulting from earthquake loads
P_P: The passive earth pressure
P_T: The bearing stress forces
Q_A: The active earth pressure resulting from surcharge loads.
Q_{AD}: The additional active earth pressure resulting from surcharge and earthquake loads.
q_{min}: The minimum stress under the footing
q_{max}: The maximum stress under the footing
H: The height of the wall
γ_R: Specific gravity of soil
ϕ_R: Internal friction angle
c_R: Cohesion of retaining soil

A retaining wall is designed according to the geotechnical and structural constraints. The geotechnical constraints are about overturning, sliding and bearing capacity. If the total moments that resist for overturning and the sum of the moment resulting overturning forces are respectively defined with $\sum M_R$ and $\sum M_O$, the safety factor for overturning (SF_o) is defined as follows:

$$SF_O = \frac{\sum M_R}{\sum M_o} \quad (1)$$

The passive pressure is not considered in the resisting of overturning because these load may not exist in time. The coefficient used in calculation of active earth pressure (k_a) is defined according to the Rankine theory as seen in Eq. (2).

$$k_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\theta}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\theta}} \quad (2)$$

In the Eq. (2), β and θ symbolize the slope and internal friction angle of backfill, respectively.
The safety factor for sliding failure is defined as

$$SF_S = \frac{\sum F_R}{\sum F_D} \quad (3)$$

where the resisting forces are equal to

$$\sum F_R = \left(\sum W_W\right)\tan\left(\frac{2\phi_{base}}{3}\right) + \frac{2Bc_{base}}{3} + P_P \quad (4)$$

and the sliding forces are defined as

$$\sum F_D = P_a \cos\beta\ . \quad (5)$$

In the equations, ϕ_{base}, B and c_{base} are internal friction angle of the base soil, length of the base slab and adhesion of the base soil, respectively.
The passive earth pressure is defined as follows;

$$P_P = \frac{1}{2}\gamma_{base} D_1^2 k_p + 2c_{base} D_1 \sqrt{k_p} \quad (6)$$

where γ_{base} and D_1 are specific gravity and depth of the base soil, respectively.
Finally, the safety factor of the retaining wall for bearing failure is defined as Eq. (7).

$$SF_B = \frac{q_u}{q_{\max}} \tag{7}$$

q_u is the bearing capacity of the soil. The maximum and minimum soil pressure are found as follows;

$$q_{\min,\max} = \frac{\sum V}{B}\left(1 \mp \frac{6e}{B}\right). \tag{8}$$

Where the eccentricity of the moments is defined as

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_O}{\sum V} \tag{9}$$

$\sum V$ is the sum of the vertical loads.

2.1 The design variables

The engineering problem has eight design variables. Four of them are about the geometry of the wall as seen in Fig. 2. The other ones are about reinforcements.

2.2 The design constraints

All design constraints are presented in Table 2. The first four constraints are the geotechnical constraints and other ones are for structural constraints defined in TS500 (Requirements for Design and Construction of Reinforced Concrete Structures) [21]. In the Table 2, M_u, V_u, A_s and S are flexural moment in critical sections, shear forces in critical section, the reinforcement area in critical sections and spacing of bars, respectively.

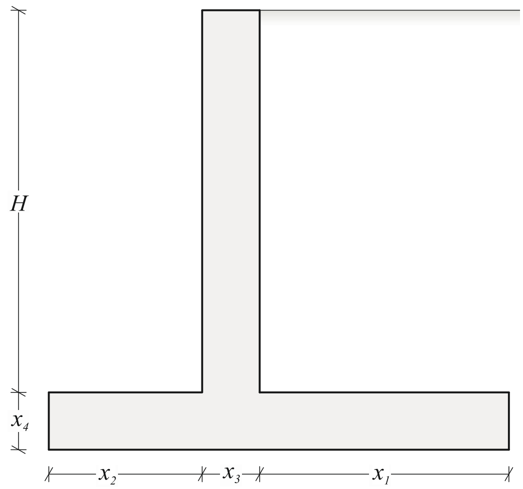

Figure 2. Geometrical design variables of a retaining wall.

TABLE I. THE DEFINITION OF DESIGN VARIABLES

	Definition	Design variables
Geometrical design variables	The length of the heel	X_1
	The length of the toe	X_2
	The thickness of the stem	X_3
	The height of the footing	X_4
Design variables about reinforcements	The size of reinforcements of stem	X_5
	The spacing of the bars in stem	X_6
	The size of reinforcements of toe	X_7
	The spacing of the bars of toe	X_8
	The size of reinforcements of heel	X_9
	The spacing of the bars of heel	X_{10}

TABLE II. THE DEFINITION OF DESIGN CONSTRAINTS

Definition	Constraints
Safety for overturning	$g_1(X)$: $SF_{O,design} \geq SF_O$
Safety for sliding	$g_2(X)$: $SF_{S,design} \geq SF_S$
Safety for maximum bearing capacity	$g_3(X)$: $SF_{B,design} \geq SF_B$
Safety for maximum bearing capacity, q_{min}	$g_4(X)$: $q_{min} \geq 0$
Flexural moment capacity in critical sections, M_d	$g_{5\text{-}6}(X)$: $M_d \geq M_u$
Shear force capacity in critical sections, V_d	$g_{7\text{-}8}(X)$: $V_d \geq V_u$
Minimum reinforcement area in critical sections, A_{smin}	$g_{9\text{-}10}(X)$: $A_s \geq A_{smin}$
Maximum reinforcement area in critical sections, A_{smax}	$g_{11\text{-}12}(X)$: $A_s \leq A_{smax}$
Maximum spacing of bars in critical sections, S_{max}	$g_{13\text{-}14}(X)$: $S \leq S_{max}$
Minimum spacing of bars in critical sections, S_{min}	$g_{15\text{-}16}(X)$: $S \geq S_{min}$
Minimum clear cover, c_c	$g_{17}(X)$: $c_c \geq 40$ mm

2.3 The objective function

The objective function is the total material cost of the RC retaining wall, as seen in Eq. (10).

$$\min f(X) = C_c \cdot V_c + C_s \cdot W_s + C_f \cdot A_f \qquad (10)$$

In this equation, C_c is the cost of the concrete per m^3 in Turkish liras (*TL*) including transportation and forming. C_s is the cost of the steel per m^3 in *TL* including transportation and labor work. C_f is the cost of the formwork per m^2 in *TL* including transportation and labor work. V_c is the volume of the concrete, W_s is the total weight of the reinforcements, and A_f is the total area of the formworks.

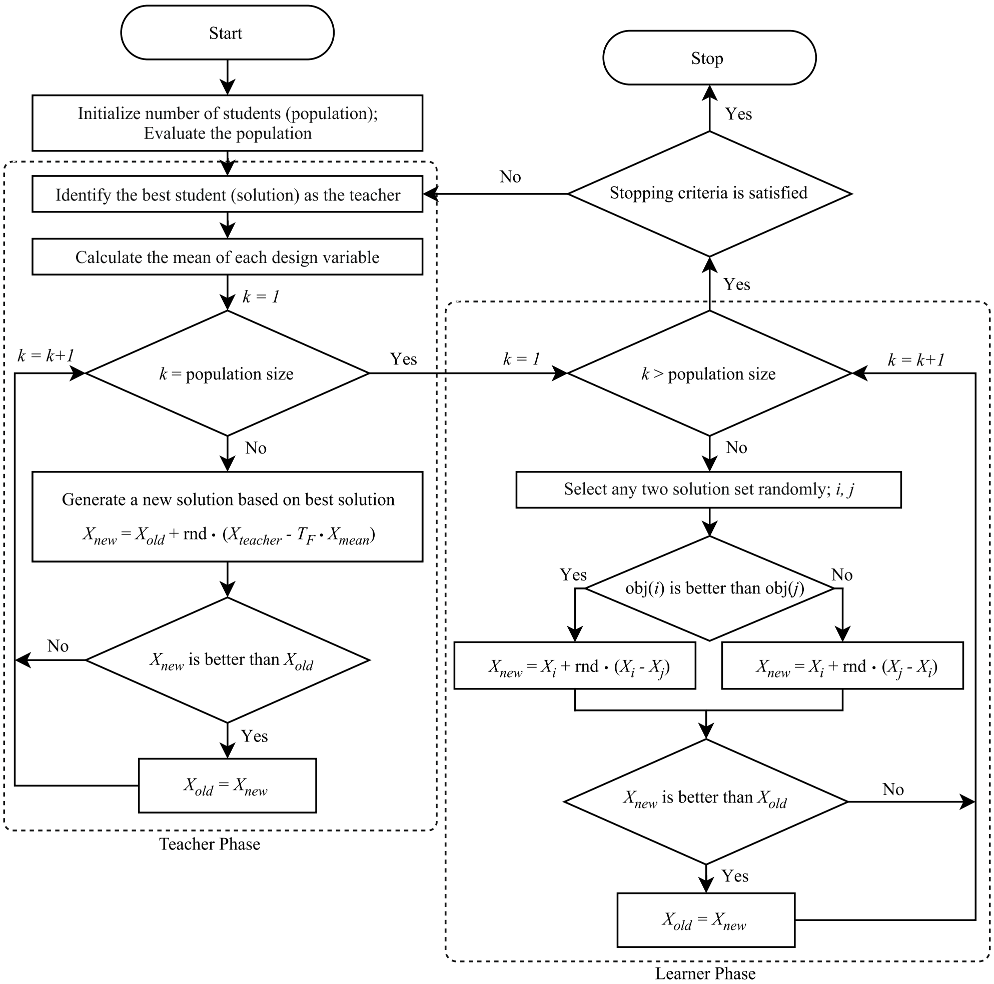

Figure 3 Flowchart of the methodology [18]

3 The design methodology

TLBO is an algorithm inspired from the education process. It is developed by Rao et al. [22]. In TLBO, two stages are consequently used. These stages are teacher and learner phases. In the teacher phase, the inspiration is the education of the teacher. The self-study of students is the inspiration of the learner phase.

The proposed methodology employing TLBO can be explained in five steps.

Step 1: The algorithm has two parameters such as population number of students (*np*) and the maximum iteration number. It is an easy algorithm since there are no specific parameters for the algorithm. The two parameters are defined together with the problem constants in this step. The design constants are shown in the section 4. Also, the ranges for the design variables are defined. The problem has discrete variables. The geometrical variables are assigned with the multiples 50 *mm* for practical construction. Also, the diameters of steel bars are produced with even sizes. The spacing of bars are the multiples of 10 *mm*.

Step 2: In this step, an initial solution matrix is generated. This matrix is defined as a class (*CL*). The generation is randomly done by the range bounded by $X_i^{\min}$, and X_i^{max} as seen in Eq. (11).

$$X_i^{\min} \leq X_i \leq X_i^{\max} \qquad i = 1, vn \tag{11}$$

TABLE III. THE DESIGN CONSTANTS AND DESIGN VARIABLES

Definition	Symbol	Unit	Value
Height of stem	H	*m*	5.6
Yield strength of steel	f_y	*MPa*	420
Compressive strength of concrete	f'_c	*MPa*	25
Concrete cover	c_c	*mm*	60
Max. aggregate diameter	D_{max}	*mm*	16
Elasticity modulus of steel	E_s	*GPa*	200
Specific gravity of steel	γ_s	t/m^3	7.85
Specific gravity of concrete	γ_c	kN/m^3	23.5
Cost of concrete per m^3	C_c	*TL*	111
Cost of steel per ton	C_s	*TL*	1400
Cost of formwork per m^2	C_f	*TL*	14.05
Surcharge load	q	kN/m^2	10
Effective Ground Acceleration Coefficient	A_0	-	0.3
Backfill slope angle	β	°	0
Internal friction angle of retained soil	ϕ_R	°	30
Internal friction angle of base soil	ϕ_B	°	0
Unit weight of retained soil	γ_R	kN/m^3	18
Unit weight of base soil	γ_B	kN/m^3	18
Cohesion of retained soil	c_R	*kPa*	0
Cohesion of base soil	c_B	*kPa*	0
Depth of the soil in front of wall	D	*m*	0
Bearing capacity of the soil	q_U	*kPa*	250
Safety for overturning stability	$SF_{O,design}$	-	1.5
Safety for sliding stability	$SF_{S,design}$	-	1.5
Safety for overturning stability under earthquake	$SF_{O,design}$	-	1.3
Safety for sliding stability under earthquake	$SF_{S,design}$	-	1.1
Safety for bearing capacity	$SF_{B,design}$	-	1.0
Range of stem thickness	X_3	*m*	0.2-1.0
Range of heel and toe projection for case 1	X_1-X_2	*m*	0.2-4.0
Range of heel projection for case 2	X_1	*m*	0.2-3.0
Range of toe projection for case 2	X_2	*m*	0.2-1.5
Range of heel projection for case 3	X_1	*m*	0.2-3.0
Range of toe projection for case 3	X_2	*m*	0.2-1.0
Range of footing thickness	X_4	*m*	0.2-1.0
Range of diameter of reinforcing bars of stem	X_5	*mm*	16.0-50.0
Range of diameter of reinforcing bars of footing	X_7 , X_9	*mm*	16.0-50.0

The solution matrix is shown in Eq. (12). In this matrix, *vn* represents the total number of design variables.

$$CL = \begin{bmatrix} X_{1,1} & X_{1,2} & . & . & . & X_{1,vn} \\ X_{2,1} & X_{2,2} & . & . & . & X_{2,vn} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ X_{pn-1,1} & X_{pn-1,2} & . & . & . & X_{pn-1,vn} \\ X_{pn,1} & X_{pn,2} & . & . & . & X_{pn,vn} \end{bmatrix} \tag{12}$$

For all set of design variables in the solution matrix, the objective function vector is generated as shown in Eq. (13).

$$f(X) = \begin{bmatrix} f(X_1) \\ f(X_2) \\ . \\ . \\ f(X_{pn-1}) \\ f(X_{pn}) \end{bmatrix} \tag{13}$$

Step 3: In this step, the teacher phase is applied. The best design variables with the minimum objective function ($X_{\min f(x)}$) is chosen as the teacher;

$$X_{teacher} = X_{min\, f(X)} \,. \tag{14}$$

Then, the existing results ($X_{old,i}$) are modified by using Eq. (15) according to mean of all existing solutions (X_{mean}), $X_{teacher}$, a random number between 0-1 (*rnd*) and teaching factor (T_F). Thus, the new solution ($X_{new,i}$) is generated.

$$X_{new,i} = X_{old,i} + rnd \cdot (X_{teacher} - T_F \cdot X_{mean}) \quad (15)$$

Teaching factor is a random number which is only rounded to 1 or 2 as seen in Eq. (16).

$$T_F = round\left[1 + rnd\right] \rightarrow \{1-2\} \quad (16)$$

Step 4: This step is about the learner phase. The new solution is generated according to Eq. (17) by using two randomly chosen existing solutions (X_i and X_j).

$$X_{new,i} = \begin{cases} X_{old,i} + rnd \cdot (X_i - X_j); & f(X_i) > f(X_j) \\ X_{old,i} + rnd \cdot (X_j - X_i); & f(X_i) < f(X_j) \end{cases} \quad (17)$$

Step 5: In this step, the maximum iteration number is checked. The iterative analyses continue from the Step 3 until the maximum iteration number.

4 Numerical examples

As a numerical example, three cases of design variable ranges were investigated. In the second case, the maximum limits are lower than the first case for the heel and toe of the wall. The design constants and design variables ranges are presented in Table 3. The optimum results for geometrical variables and reinforcements are given in Table 4 and 5, respectively. The optimum costs are also given in Table 4. The analyses are done for unit length of the wall.

TABLE IV. OPTIMUM GEOMETRICAL DESIGN VARIABLES AND OBJECTIVE FUNCTION

Design variables	Optimum values of design variables (*m*)		
	Case 1	Case 2	Case 3
X_1	1.90	2.00	2.45
X_2	1.95	1.40	1.00
X_3	0.50	0.45	0.45
X_4	0.25	0.35	0.35
Optimum cost (*TL*)	1085.17	1086.73	1140.86

TABLE V. OPTIMUM REINFORCEMENTS

	Case 1	Case 2	Case 3
	Bar size (*mm*) / spacing (*mm*)	Bar size (*mm*) / spacing (*mm*)	Bar size (*mm*) / spacing (*mm*)
X_5/X_6	ϕ24/185	ϕ18/90	ϕ18/90
X_7/X_8	ϕ12/260	ϕ14/165	ϕ14/225
X_9/X_{10}	ϕ14/70	ϕ 14/80	ϕ24/170

5 Conclusions

In the first case, the maximum bound of the heel of the retaining wall is 4 *m*. In that case, the optimum cost is 1085.17 *TL* and the optimum value of heel and toe are 1.90 *m* and 1.95 *m*, respectively. In the second case, the maximum limit of the toe is reduced below the optimum value of Case 1. In that situation, the length of the heel increase to 2 *m* for the optimum results. For the last case, the toe value is limited with 1 *m* while the heel is limited with 3 *m*. In that case, the increase of the total cost is clearly seen from the results. The proposed method employing TLBO is a feasible approach for the problem for restricted optimum design.

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