Stability Analysis of a Geometrically Imperfect Structure using a Random Field Model

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Abstract: - There exist two main categories of structural uncertainties which include spatial correlation, and require the application of random fields. These categories are material and geometrical characteristics. In the presented studies, the random field is applied to the modelling of initial curvature of a slender member axis. Load-carrying capacity of the compress member is calculated by using geometrically nonlinear solution in the programme ANSYS. The solution was carried out by using the beam element BEAM188. The specimen application includes the static random response of an imperfect system generated by geometrical imperfections of member axes with open and hollow cross-sections. The Latin Hypercube Sampling Method was applied.

Key-Words: Load carrying capacity, Initial curvature, Incision, Random field, Limit state, Stress, Warping

1 Introduction

One half wave of the function sinus is a frequently applied idealization of real curvature of the axis of a slender hot-rolled steel member, although the real geometry can be substantially more complicated. Model approaches based on one half-wave make possible to calculate the solution of the state of stress in an analytical form by a close-form formula [1,2]. On these assumptions, it is not difficult to calculate the load carrying capacity, if the amplitude of initial curvature e_0 is known.

The amplitude of initial curvature is a random quantity for the identification of which the source materials can be found in literature [2, 3].

A more detailed method assumes a spatial distribution of geometrical or material properties and models of this random distribution by a continuous field called a random field [4, 5]. In combination with the finite element method, this approach is generally called the stochastic finite element approach [6].

Modelling of the initial curvature with applying the random field is more complicated, and requires more input parameters to be identified [7]. However, it is not the rule that the initial axis curvature must have the shape of sinus function half-wave or of random field. Both cases are the idealization of real axial generally spatial curvature. An explicit solution cannot be obtained for the general spatial curvature.

The present paper deals with modelling the member axial curvature based on random fields. The random spatial curvature is considered as the initial geometrical imperfection which influences the load carrying capacity of the member.

Two symmetrical thin-walled cross sections were chosen for computer modelling, namely the open and the hollow ones. The stress state of an imperfect member under compression is given by pressure, bending, and by torsion. The torsion can significantly contribute to the increase of the stress state, and to the decrease of load carrying capacity for slender members with open cross section in particular [8]. The warping torsion the influence of which on stress state, and load carrying capacity are studied, is connected with the stress of members having open cross sections. The load carrying capacity of compress members was calculated by geometrically nonlinear solution in the ANSYS software.

2 Computational Model

A two-hinge member was considered, see Fig. 1. The length of the member is L = 2.798 m and non-dimensional slenderness $\bar{\lambda}$ = 1.0. Non-dimensional slenderness $\bar{\lambda}$ is the parameter applied to design and dimensioning of member, defined in the standard EUROCODE 3. The parameter $\bar{\lambda}$ is sometimes considered to be a quantity more transparent than the member length, because it (the parameter) is the function also of the radius of gyration about the relevant axis, determined using the properties of the gross cross section.

To the load carrying capacity analysis of the compress member, the member element BEAM188 with seven degrees of freedom was applied which is a part of the offer, and is accessible in the ANSYS software. Seven degrees of freedom include three degrees of freedom corresponding to translations along axes x, y, z, other three ones, to rotations around these axes, and the 7^{th} degree of freedom corresponds to warp.

The member scheme is in Fig. 1. In the node a, translations in directions of all three axes, and the rotation about axis x are prevented.

In the node b, the translations in the direction of axes y and z and rotation around axis x are prevented. Further on, it is assumed that both end cross sections in nodes a and b cannot warp; by this, the model approaches a real laboratory experiment. The model is subsequently loaded by translation in the node b in the direction of axis x. In node a, the value of the reaction in direction x is followed to determine the load carrying capacity.

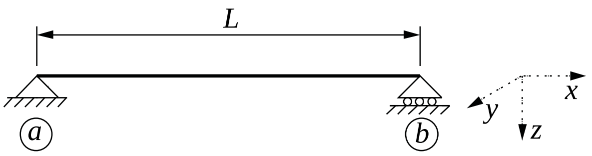

Fig.1: Model of steel member.

2.1 Cross Section

In the given problem, two symmetrical square thin-walled cross sections were used, the first being hollow, and the second, open. In the middle of one side of open cross section, there is an incision. Both cross sections are presented in Fig. 2. The member is modelled so that all the points of the centre of gravity of cross section lie on the member axis.

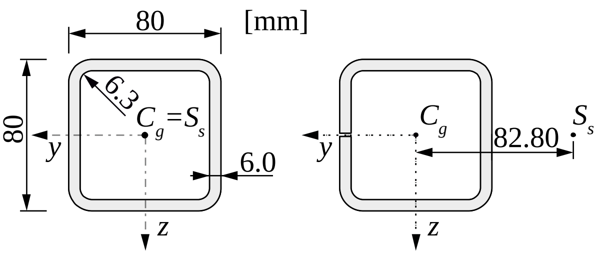

Fig.2: Cross sections: a) hollow, b) open.

The hollow cross section is symmetrical along both axes, and thus the position of the centre of gravity C_g corresponds to the position of shear centre S_s. In open cross section, the position of shear centre was shifted to the right, according to Fig. 2 b). Cross-section characteristics of both cross sections are given in Table 1.

Table 1: Cross section characteristics.

Characteristic	Hollow	Open
Area A	$1.68.10^{-3}$	$1.68.10^{-3}$
Second moment of area I_y	$1.49.10^{-6}$	$1.49.10^{-6}$
Second moment of area I_z	$1.49.10^{-6}$	$1.49.10^{-6}$
Second warping moment I_ω	$2.83.10^{-12}$	$6.96.10^{-9}$

These cross sections are then applied to each realization of spatial curvature of member axis (see Chapter 2.2). In view of the fact that the initial curvature is here independent of the applied profile, the open cross section is; rotated into four positions. The load carrying capacity of member of the given realization is then calculated for five cross sections according to Fig. 3.

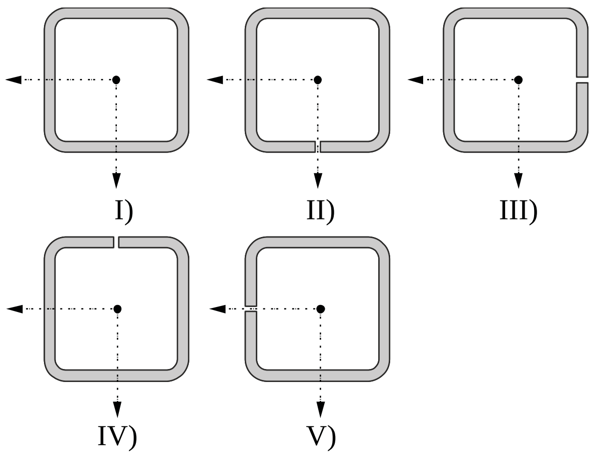

Fig.3: Hollow and open cross sections.

2.2 Random Input Variables, Random Field

The Gauss probability density distribution with mean value 297.3 MPa, and with standard deviation 16.8 MPa was considered for yield point f_y [9]. The initial curvature of member axis was modelled using eleven nodes through which the spline was interlaid, see Fig. 4. Each of these nodes had the Gauss probability density distribution with zero mean value, and with standard deviation 0.0015248 $\sin(\pi \cdot x_i/L)$ m, where x_i is the position on the member axis. The value 0.0015248 was calculated, based on the assumption that 95 % realizations of initial spatial deformation lay within the tolerance limits

±0,15 % L. The values of coordinates in each of the two planes are mutually correlated by means of the correlation matrix. The correlation matrix defines the random field with correlation length L_{cor} = 1.442 m. The correlation is considered among the values of coordinates of nodes lying on one plane. It means that the curvature on one plane is independent of the curvature on the other one. Random realizations of yield strengths and initial curvature were simulated by applying the method Latin Hypercube Sampling [10, 11] using Freet software, see http://www.freet.cz/.

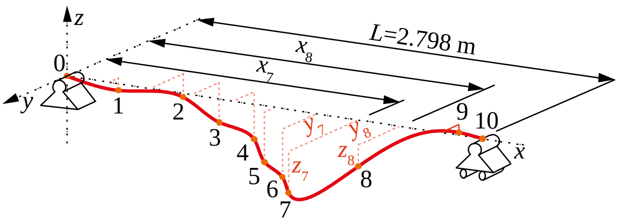

Fig.4: Definition of spatial curvature of the axis.

The other parameters of the model were considered as deterministic ones, and were taken by their nominal values. The Young's modulus of steel E=210GPa and geometrical characteristics of the cross section, e.g., are concerned. By loading using the method step-by-step, the strength was searched for which the maximum value of the von Mises stress of the member would be equal to yield strength. 60 random realizations of axial curvatures of members and 60 random realizations of yield strength were simulated. Each realization of axial curvature of the member forms a pair with one realization of yield point. An example of one realization of initial curvature of member axis is illustrated in Fig. 5 and Fig. 6.

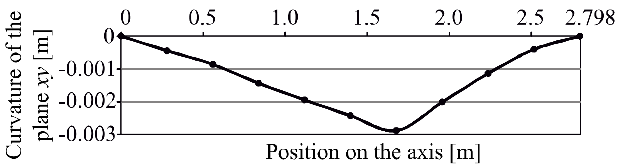

Fig.5: Axis curvature of 1 random run - plane *xy*.

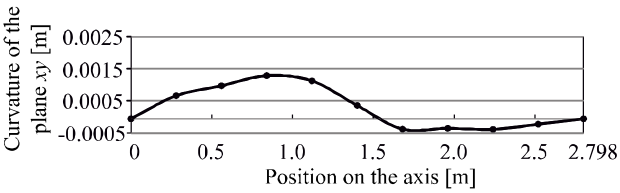

Fig.6: Axis curvature of 1 random run - plane *xz*.

3 Stress State and Limit State

Building structures are usually designed so that maximum expected stresses were within the limits of linear elasticity, it means that deformation caused by internal stress is directly proportional to them. At loading, the real imperfect member is in the state of spatial stress. For the assessment, it is necessary to know when the stress state is approaching the limit state of stress in a material. Yield strength f_y is usually considered to be the limit state of stress for structural steel. Combined stresses cannot be described by a single vector. For the limit condition, there was used the von Mises (Huber, Hencky) condition of plasticity in the form.

$$f = \bar{\sigma} - \sigma_0 = 0 \tag{1}$$

where $\bar{\sigma}$ is equivalent stress [Pa], which is

$$\bar{\sigma} = \sqrt{\frac{1}{2}\left[\begin{array}{l}(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + \\ + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) + \end{array}\right]} \tag{2}$$

σ_0 corresponds to f_y, elements σ_x, σ_y, σ_z are called orthogonal normal stresses (relative to the chosen coordinate system), and τ_{xy}, τ_{xz}, τ_{yz}, orthogonal shear stresses. σ_x is longitudinal stress in direction of axis x, σ_y is longitudinal stress in direction of axis y, σ_z is longitudinal stress in direction of axis z. τ_{xy} is shear stress on the plane x in direction of y, τ_{xz} is shear stress on the plane x in direction z, τ_{yz} is shear stress on the plane y in direction z. The analysis of stress can be considerably simplified for members that are subjected to moderate compressing, bending and twisting. When the solution is carried out by member model, it is assumed that σ_y=σ_z=τ_{yz}=0. The formula (2) is so reduced to the form

$$\bar{\sigma} = \sqrt{\sigma_x^2 + 3(\tau_{xy}^2 + \tau_{zx}^2)} \tag{3}$$

So, the limit state occurs, when yield point f_y is reached in the maximum stressed point of member.

$$\bar{\sigma} = f_y \tag{4}$$

4 Load Carrying Capacity

The load carrying capacity values of all sixty random realizations of the member are presented in Table 2. By Chi-quadrate test of good agreement on the significance level 5 %, the hypothesis on normality of distribution for no solved cross-section types is rejected. Based on the standard EN1990, the design load carrying capacity is calculated as 0.1 percentile. It corresponds to design reliability index $\beta_d = 3.8$. 0.1 percentile was calculated on assumption that the load carrying capacity had the Gauss probability density function.

Table 1: Cross section characteristics.

Cross section	Mean value [kN]	Variation coefficient	0.1 percentile [kN]
I	326.20	7.69	248.63
II	272.00	6.19	219.98
III	270.95	6.25	218.59
IV	271.94	6.34	218.66
V	271.07	6.32	218.10

The design load carrying capacity of the member with hollow cross section (cross section I) is – according to EUROCODE 3 – 262.29 kN. The EUROCODE 3 does not state the design load carrying capacity of open cross sections (cross sections II, III, IV).

5 Conclusion

The statistical modeling of a structure is usually carried out for the purpose of finding the probabilistic response, or for reliability assessment. In the paper, there was presented the application of a random field for modelling initial curvature of the member axis.

Statistical characteristics of load carrying capacity evaluated for two cross-section types are the result of the study. The Table 1 contains load carrying capacities of members with several variants of open cross sections, completed by the variant with hollow cross section. Mean values and standard deviations of members with open cross sections are similar. The design load carrying capacity of these cross sections is not stated by the standard EUROCODE 3 expressly, but in compliance with standard proceedings, the curve of buckling strength *b* can be used, and the value of design load carrying capacity 235 kN can be obtained. The value 235 kN 0.1 is higher than all 0.1 percentiles of Gauss probability density function, by which the values of load carrying capacity of members with open cross sections were approximated.

The design load carrying capacity of members with hollow cross section according to EUROCODE 3 is 262.29 kN. This value is higher than 0.1% quantile of normal distribution, as well. The comparison of design value according to EUROCODE 3 with the design value obtained as 0.1 percentile represents the basic principle for verification of design reliability. The statistical analysis points out that it is necessary to introduce more strict standard design values. However, it must be verified by experimental research and a further probabilistic analysis according to EN1990. Random fields are an appropriate instrument for research into this phenomenon.

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