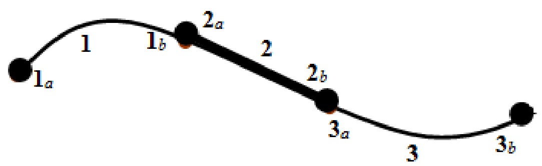

Fig. 3 Collision avoidance path decomposed in 3 finite elements

Each finite element is parameterized using two variables: time span t_{nspan} and the highest order constrained state variable. Time span t_{nspan} may be uniformly chosen by decomposing the total maneuvering time in n segments or by considering other parameters such as change of tire-road friction coefficient μ and road curvature. In this formulation, angular acceleration is the highest order constrained state variable and assumed constant in each segment for $t_n \in [0, t_{nspan}]$. In this context, angular velocity r_n and position θ_n are:

$$\dot{r}_n = a_{2n} \quad (12)$$

$$r_n = \int_0^{t_{n,span}} \dot{r}_n \cdot dt = a_{2n} \cdot t + a_{1n} \quad (13)$$

$$\theta_n = \int_0^{t_{n,span}} r_n \cdot dt = 0.5 \cdot a_{2n} \cdot t^2 + a_{1n} \cdot t + a_{0n} \quad (14)$$

where $t_n \in [0, t_{nspan}]$.

The states $y_n = \left[\dot{r}_{n,a} \quad r_{n,a} \quad \theta_{n,a} \quad \dot{r}_{n,b} \quad r_{n,b} \quad \theta_{n,b}\right]^T$ at the boundaries of the finite element are expressed in matrix form as:

$$\mathbf{y_n} = \mathbf{A_n} \cdot \mathbf{x_n} \quad (15)$$

$$\mathbf{y_n} = \left[\, r_{n,a} \quad \theta_{n,a} \quad r_{n,b} \quad \theta_{n,b} \right]^T \quad (16)$$

$$\mathbf{x_n} = \left[\, a_{2n} \quad a_{1n} \quad a_{0n} \right]^T \quad (17)$$

$$\mathbf{A}_n = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ t_{nspan} & 1 & 0 \\ 0.5 \cdot t_{nspan}^2 & t_{nspan} & 1 \end{bmatrix} \quad (18)$$

The finite element matrix A_n constitutes the basis for joining subsequent elements and deriving the system's solution. For a detailed description the reader is referred to [13].

3.2 Solution methodology

The path is decomposed in N=3 uniform finite elements with the same time span . The EP is computed by solving the following linear system of equations:

$$\mathbf{y}_{bc} = \mathbf{A} \cdot \mathbf{x}_u \quad (19)$$

$$\mathbf{y}_{bc} = \left[r_{1,ades} \quad \theta_{1,ades} \quad \cdots \quad r_{n,bdes} \quad \theta_{n,bdes} \right]$$

$$\mathbf{x}_u = \left[a_{21} \quad a_{11} \quad a_{01} \quad \cdots \quad a_{2n} \quad a_{1n} \quad a_{0n} \right]$$

$$\sum_{i=1}^{N} t_{nspan} = T$$

where $\mathbf{y}_{bc}$ is the vector of boundary conditions, $\mathbf{x}_u$ is the vector of unknown coefficients and **A** the system's matrix.

Vectors $\mathbf{x}_u$ and $\mathbf{y}_{bc}$ as well as system matrix **A** are formed by joining subsequent elements. In particular, we use the desired conditions at beginning (*t=0*) and end (*t=T*) of the EP:

- $r(t=0) = r_{1,ades}$ and $\theta(t=0) = \theta_{1,ades}$
- $r(t=T) = r_{N,bdes}$ and $\theta(t=T) = \theta_{N,bdes}$

To assemble system matrix **A** we use the continuity equations between subsequent elements

$$r_{n,b} = r_{n+1,a}\, , \theta_{n,b} = \theta_{n+1,a} \quad (20)$$

and the desired lateral displacement Y_{des} at the end (*t=T*) of the EP:

$$\sum \delta Y_n = Y_{des} \quad (21)$$

where δY_n is the lateral displacement of a finite element:

$$\delta Y_n = \int_0^{t_n s_{pan}} u_f \cdot \sin(\theta_n) \cdot dt \approx u_f \cdot \int_0^{t_n s_{pan}} \theta_n \cdot dt = \left(\frac{1}{6} \cdot \alpha_{2n} \cdot t_n^3 + \frac{1}{2} \cdot \alpha_{1n} \cdot t_n^2 + \alpha_{0n} \cdot t_n \right) \cdot u_f \quad (22)$$

In Equation (22) the incremental lateral displacement δY_n is linearized by assuming $\sin(\theta_n) \approx \theta_n$. The proposition is valid only for small angles $\theta_n \leq 5^o$. For larger angular displacement θ_n the path has to be decomposed into a greater number of finite elements.

It is obvious that different path decomposition would lead to a different system matrix **A** and subsequently a different solution. Actually, there are infinite EPs that satisfy the boundary conditions and that can be computed using the FE method. This is exactly the reason why we are interested in comparing the two different formulations.

4 Numerical results

The finite element formulations have been tested for an extensive number of driving scenarios in Matlab simulation environment. The numerical examples are based on the vehicle data listed in Table 1 and tire parameters listed in Table 2. One driving scenario which highlights their features is presented and discussed.

In the scenario considered it is assumed that the vehicle moves in a straight line road segment with a speed $u_f = 30\,m/s$. The road surface is dry $\mu = 1$ and an obstacle at distance $d = 54\,m$ suddenly appears in its direction of travel. To avoid the collision the vehicle has to displace laterally by $Y_{des} = 3\,m$.

We solve the problem by decomposing the path in uniform road segments and apply the solution methodology described in the previous section and in [13]. The numerical results using the second order finite element methods are shown in Figs 4-6, while those with the 3rd order method in Figs 7-9.

5 Conclusions

When an obstacle suddenly appears in the trajectory of a vehicle a path has to be designed in real time to avoid the collision. A vast number of path planning methods for ground vehicles have been proposed until now. A comparative evaluation of the different methods is necessary to illustrate their advantages and disadvantages and ease their selection. In this paper, two different finite element formulations for collision avoidance are presented and compared for a case study.

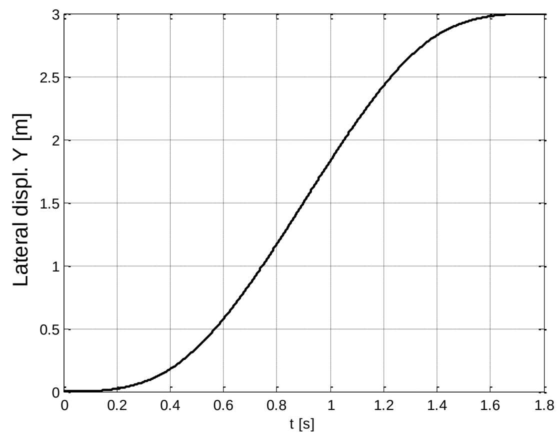

Fig. 4 Lateral displacement using the 2nd order FE method

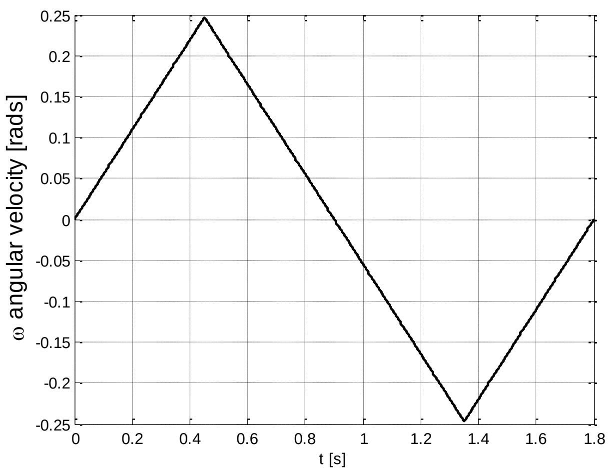

Fig. 5 Angular velocity using the 2nd order FE method

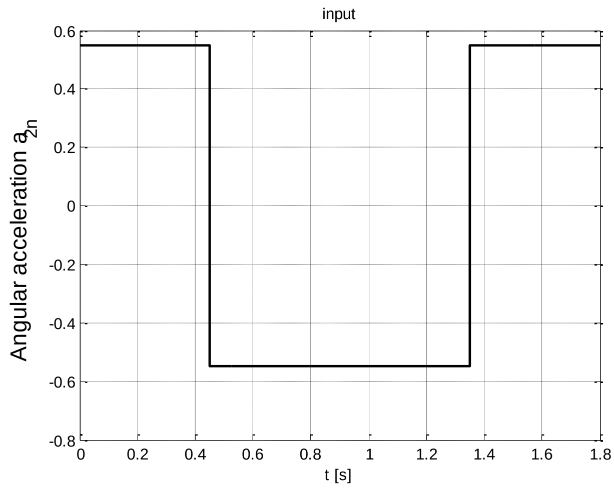

Fig. 6 Angular acceleration using the 2nd order FE method

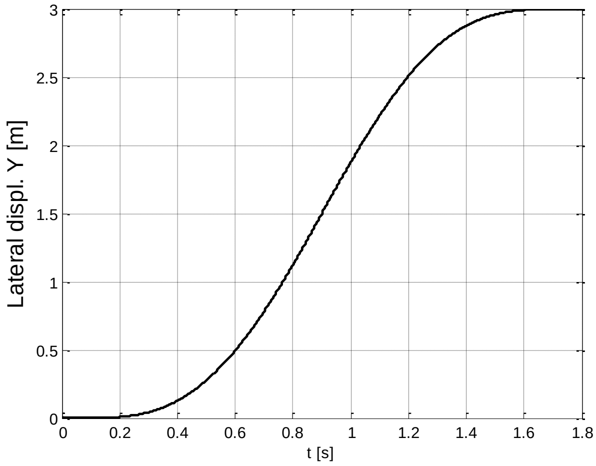

Fig. 7 Lateral displacement using the 3rd order FE method

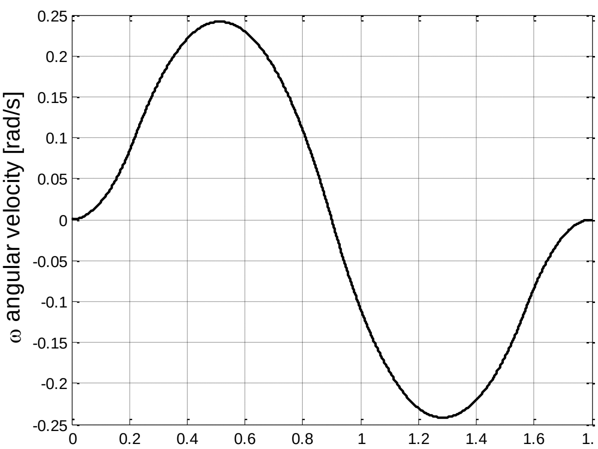

Fig. 8 Angular velocity using the 3rd order FE method

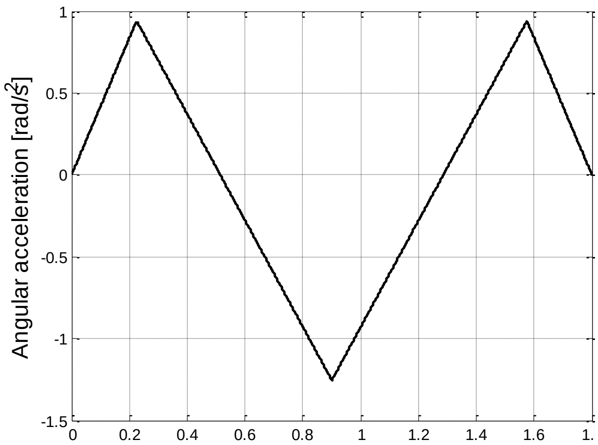

Fig. 9 Angular acceleration using the 3rd order FE method

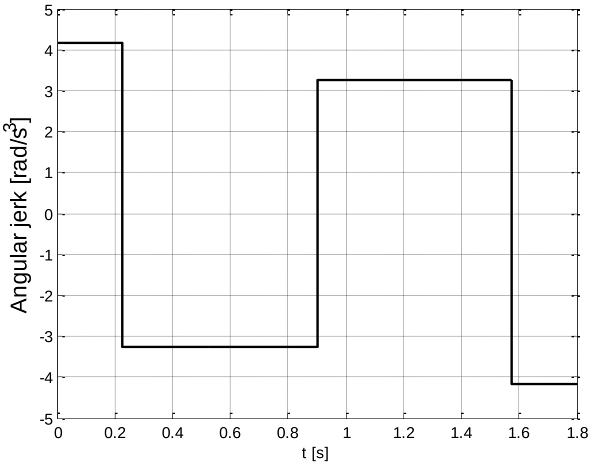

Fig. 10 Angular jerk using the 3rd order FE method

A finite element (FE) method has been developed based on a reformulation of Pontryagin's Maximum Principle to plan collision avoidance-time optimal paths. Two different formulations, which differ in the order of approximation, have been presented and evaluated in this study for a typical collision avoidance scenario.

From the numerical results it becomes clear that both methods a) improve the path dynamics compared to the solution obtained with a uniform time mesh and b) satisfy the maneuverability requirements of the vehicle with respect to the maximum admissible yaw rate However, the methods differ in the achievable maximum acceleration and maximum jerk. In the 2nd order method the maximum acceleration is smaller (44%) compared to the one obtained with the 3rd order method. However, the angular jerk is infinite which is negative in terms of comfort. Furthermore, if an active steering system is used to guide the vehicle it is a wrong assumption since the steering dynamics isn't negligible. In case a differential braking system is used then it is an acceptable solution.

In the future the design of collision avoidance paths should become standard and available through the communication protocols between vehicles. The development of a simple but powerful method like the one presented in this paper is considered to be a contribution in this direction.

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