material equal to $(E/\rho)1/2$ where E is the modulus of elasticity of material, the term ρc is referred as the acoustic impedance of the material, making use of equation (3) in equation (2) and solving for equation (1) for σ_I and σ_R gives:

$\sigma_T=(2A_1\rho_2C_2/(\rho_1C_1A_1+\rho_2C_2A_2)$ (4)

$\sigma_R=(\rho_2C_2A_2-\rho_1C_1A_1)/(\rho_1C_1A_1+\rho_2)$ (5)

It can be concluded from equation (5) that for a simple change in cross sectional area i.e. when $\rho_{1=}$ ρ_2 and C1=C2, the incident and reflected waves will have the same or opposite signs according to the increase or decrease in size of the cross sectional area. Also the intensity of the transmitted stress falls below or exceeds the intensity of the incident stress. Furthermore, equations (4) and (5),if $\rho_1C_1= \rho_2C_2$ the following points may be concluded:

(a) If $A_2 / A_{1=} 0$, i.e. the end of the rod is effectively free, equation (5) gives $\sigma_{R\,=}$ $-\sigma_{I,}$ and $= \sigma_T = 2\sigma_I\sigma_{T=} 2\sigma_{I,}$

(b) If A_2 / $A_1 = \infty$, i.e the end of the rod is effectively fixed, $\sigma_{R\,=}$ $\sigma_{I,}$ and $\sigma_{T\,=}$ 0.These results indicate that a very small shaft attached to the end of one larger in cross sectional area will act as a momentum trap for a pulse or blow on the far end of the large shaft.

$\sigma_T=(2k_1/(1+k1))\sigma_I$ (6)

$\sigma_R=((k_1-1)/(k_1+1))\sigma_I$ (7)

and referred to as the impedance mismatch factor, and if the discontinuity involves ; sectional area, the case of conical rod, then equations (4) and (5are reduced to:

$\sigma_T=(2A_2/(A_1+A_2))\sigma_I=(2k_2/(k_2+1))\sigma_I$ (8)

and $\sigma_{R=}((A_2-A_1)/(A_2+A_1)\ \sigma_I = (1-k_2)/(1+k_2))$ (9)

where $k_2 = A_1/A_2$ i.e. the end of the rod is effectively fixed, $\sigma_{R\,=}$

$\sigma_{T(n-1)} = [\ 2A_n/(A_n+A_{(n-1)})]\ \sigma_I = [2n^2/(n^2+(n-1)^2]\ \sigma_I$ (10)

and the transmitted stress at segment 1 at the apex σ_{T1} is given

$\sigma_{T1}=[2n^2/(n^2+(n-1)^2)*2(n-1)^2/((n-1)^2+(n-2)^2)\ldots..2(3)^2/(3^2+2^2)*2(2)^2/(2^2+1^2)]\ \sigma_I$ (11)

The reflected stress σ_R is determined using eq. 9

$\sigma_{R(n-1)} = [\ (A_{n-1}-A_n)/(\ A_{n-1}+A_n)]\ \sigma_I = [\ (n-1)^2 – n^2/((n-1)^2+n^2)]$ (12)

This reflected stress is added to other arriving stresses resulting from the multi- reflections at the interface.

Special cases:

i) If the rod is made of the same material but encounters only change in cross sectional area, Fig.1 (a), the incident and reflected waves will have the same or opposite signs, compression or tension, according to the increase or decrease in size of the cross sectional area: and at the same time the intensity of the transmitted stress, σ_R, falls below or exceeds the intensity of the incident stress, σ_I. This is of course only true inside the material but not at its end surfaces. ii). If the rod is of uniform cross sectional area but made of two materials have different acoustic impedances ,ρC,, Fig.1 (b), then the intensity of the transmitted stress value exceeds or falls below the incident stress level according to whether ρ2C2 is higher or lower than ρ1C1 respectively. Furthermore the incident and reflected waves will have the same sign in this case, whereas the reflected wave will have opposite sign to the incident wave if ρ2C2 is lower than ρ1C1. It can also be noted that the transmitted wave has the same sign as the incident wave.

iii) If A2/A1 is about zero i.e. the end of the rod is free end then $\sigma_R = -\sigma_I$.

iv) If A2/A1 is &i.e the end of the rod is effectively fixed then $\sigma_R = \sigma_I$ and $\sigma_T= 0$. It can be concluded from equations(4) and (5) that if a small shaft is placed at the end of a shaft of larger cross sectional area, it will act as a momentum trap to the stress wave or pulse. This was utilized in treatment of scabbing caused at the end of a mild steel plate partially cut by a linear explosive shaped charge as will be discussed later in the paper.

2.2 Applications of elastic stress waves

Elastic stress waves were applied successfully in the following:

1) Determining the mechanical behaviour of materials under dynamic loading

The Hokinson and the split Hopkinson pressure bars which utilize the concepts of elastic stress waves transmission and reflection were successfully used in determining the mechanical behaviour under compressive, tensile, and torsional. The description of these methods is beyond the scope of the paper, and can be found in Refs.,[ ].

2) Producing multi-layered and laminated sheets
Multi-layered and laminated sheets (either reinforced with carbon or glass fibers or without reinforcement) by explosive welding.
3) Production of laminated or duplex cylinders
Duplex and laminated cylinders of identical or different materials were successfully produced using either explosive or implosive welding by the aid of a wave shaper to generate a plane wave along the length of the cylinders.
4) Production of solid cylinders from thin cylindrical rods. This a recent application of utilizing stress waves in producing a solid cylinder from thin cylindrical rods made of four different materials namely: mild steel, stainless steel, copper and brass in the middle carried out by the author using the implosive welding process. The produced cylinders were successfully extruded and drawn without any separation of the rods. Other blanking tests were carried out on specimens of 3 mm thickness and no sign of separation or fracture on any of the welding interfaces. It is very interesting to mention that the above explosively welded parts are produced by the same mechanism of jet formation. The most interesting feature is that the mechanism of jet formation which was used in the welding process is the same one which will be used in perforation and cutting by lined cavity shaped charge as will be discussed in another paper in this conference, [18 (a) and (b)].
5) Separation of welded multi-layered sheets and laminated cylinders. These products were produced by the transmitted compressive stress waves produced from the explosive charge and are separated by either the reflected tensile waves or the interaction of the transmitted and reflected waves. Zaid, [18 (a) and (b)] analyzed the transmission, reflection and interaction of stress waves in explosively welded multi-layered sheets and discussed the separation between the different layers and showed how it can be avoided. The separation is caused by the reflection of the compressive waves as tensile waves from the surface of the inside cylinder.

2.3 Fractures caused by stress waves

Three types of fracture are caused by longitudinal stress waves:
a) Due to incident and transmitted stress waves
b) Due to reflected stress waves, and the interaction of the transmitted and reflected stress waves.

The criterion used for the fracture of materials in this section is: material fractures when the stress level at any section in the material reaches the dynamic fracture stress of this material, it will fracture depending on the type of stress e.g. tension, compression, or shear. Table 2 gives the dynamic fracture stresses of some materials.

Table 2: Dynamic fracture stress of some materials

Materials	Dynamic fracture stress N/m^2
Copper	$2.282*10^9$
Brass	$2.13*10^9$
4130 steel	$3.03*10^9$
1020 steel	$1.10*10^9$

In this paper, only fractures caused by both longitudinal waves are discussed.

2.4 Different fractures caused by stress waves

Three types of fracture are caused by longitudinal stress waves:
Due to incident stress waves. If the stress intensity of the incident pulse is higher than the corresponding dynamic fracture stress of the material which it impinges on, the material will fracture if the material is perfectly elastic, homogeneous, isotropic and there is no attenuation, change in shape, or variation in the velocity of propagation. If the intensity of the incident stress is lower than the dynamic fracture stress of the material and it meets a stress discontinuity, (either due to change in cross sectional area or acoustic impedance such that it will cause amplification of its value it will also cause fracture. Fig.3 shows amplification of a compressive stress impinging on the large end of a conical rod.

The transmitted stress, $\sigma_{T,}$ at any section determined using equation 10. which enables the transmitted stress at any section along the rod length to be determined. The results are plotted in Fig.3 for different values of n. it can be seen from Fig.3 that the incident compressive stress gets amplified as it proceeds towards the apex. Furthermore, it can be seen from the figure that the approximation converges for values of n larger than 96, as little difference exists between curves 5 and 6, for 96 and 192 steps, therefore 96 steps is considered adequate. Depending on the magnitude of the incident compressive stress, if the amplified transmitted stress at any section along the rod just exceeds the dynamic fracture compressive strength of the rod material it will cause the rod fracture at this section although the incident stress, $\sigma_{I,}$ may be less than the elastic limit stress of the rod. Following the same approximation Zaid has also given the variation of the compressive pulse at the tip of a triangular projectile, [22(a) and (b)].

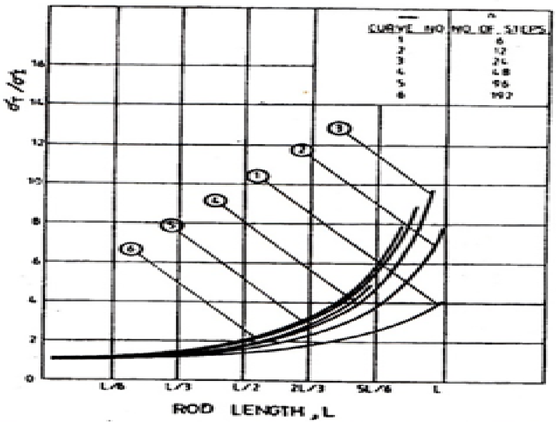


Fig.3: Amplification of the stress

If an incident triangular compressive stress pulse is transmitted along anisotropic conical rod, Fig.4, without meeting a stress discontinuity, when it reaches the free end it will be reflected due to reflected stress wave.

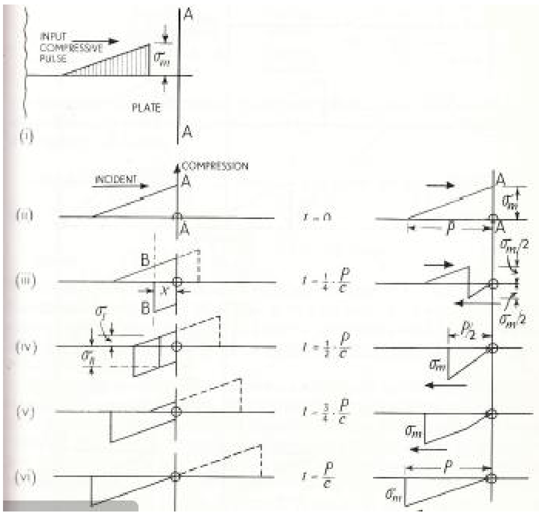


Fig. 4: Interaction of the transmitted compressive wave and the reflected tensile wave, [9].

2.4.1 Scabbing and multiple scabbing

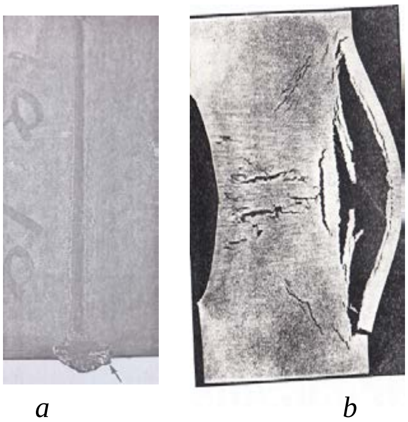
Whilst long recognized as a phenomenon in Hopkinson reported work on the use of the pressure bar further to the measurement of the compressive pressure wave resulting from the detonation of the guncotton, the destructive effects of the latter when placed in contact with a thick steel plate were also considered. The fracture of the plate was shown to be brittle, or "short" with little plastic flow. Although the fractures were not associated with reflected stress waves at that time and were described as "Hopkinson fractures". Rinehart was the first to report a quantitative investigation into the fracturing of metals due to reflected waves. The following assumptions were made:

(i) The compressive wave is plane and approaches the free surface normally and suffering reflection without change in intensity.

(ii) The material is perfectly elastic, homogenous, and isotropic and there is no attenuation, change in shape, or variation in the propagation velocity. Regardless of the stress level.

(iii) The fracture forming the scab r the spall occurs instantaneously.

A further fracture reported by Rinehart, is the multi scabbing. It occurs if the incident compressive wave is of sufficient intensity, then after the first scab, the remaining part of the compressive wave will also suffer at the newly created free surfaces and further scabs will be produced. This is made clear by reference to the multi scabs reported in Refs. [9, 11, 12]. Ever since work on scabbing and multiple scabbing continued. In general, the experimental results of Obrien and Davis substantiate the mechanism advanced for scabbing and multi scabbing, although they reported that their observations of multi-scabbing were in qualitative agreement with those of Rinehart but the position of the second scab was closer to the first than would be predicted from the profile of the pressure wave.



a *b*

Fig.5: Scabbing and multiple scabbing of a mild steel plate, (a) at the end of a cutting jet from aliened cavity angle shaped charge, (b) From a contact high explosive charge

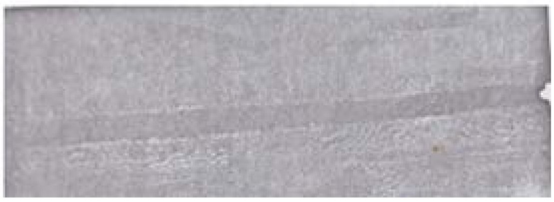


Fig.6: The mild steel plate in (a) after treating the scab by using a momentum trap

2.4.2 Momentum traps

Fig.7 shows a uniform cylindrical bar (1), of length L1 and cross-sectional area A_1, to the end of which is lightly attached a bar (2) of length L2 and cross sectional area A_2, with $A_1 > A_2$; also L1 >L2. The mating surfaces of the two bars are carefully scraped fit and a smear of greaese is applied to ensure close contact.

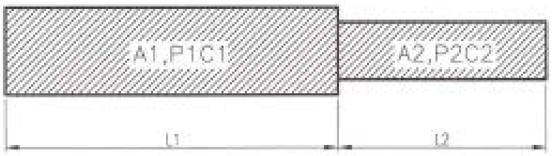


Fig.7: The general case of a plane of stress discontinuity

If at the end of bar (1) is applied a constant intensity compressive stress of magnitude σ, the incident compressive stress is transmitted by the attached bar as a compressive wave, and later reflected from its free end as a tensile wave. It then reaches the interface, the common plane to bar (1) and bar (2). The stress at any position is then found by the summing the incident and reflected waves. As the interface is unable to withstand tension, hence immediately contact ceases; bar (2) moves off processing a certain kinetic energy and having trapped some of the momentum in the incident pulse. It is the same concept which was used by Hopkinson to determine the general form of the incident compressive pulse from an explosive or from an impact blow using pieces of different thicknesses, and measuring the momentum entrapped within each piece, [2]. The same method was used by the author to avoid scabbing of the plate in Fig.6

3 Conclusions

In this paper, the following points are given:

i) The old and recent literatures are reviewed which includes: the history of the phenomena and its Theoretical aspects.

ii) The useful applications of stress waves are given and discussed.

iii) The damages caused by their transmission, reflection and interaction (scabbing and multiple scabbing) are given and discussed. Finally, the method for their avoidance using the momentum trap is also given and discussed.

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