



















$$= \frac{1}{\bar{a}_1^2} \left[ \frac{\partial p_1}{\partial r} \frac{\partial T_1}{\partial r} + p_1 \left( \frac{m/r^2}{\omega - mPe} \frac{\partial T_1}{\partial \varphi} + \frac{k}{\omega - kPe} \frac{\partial T_1}{\partial x} \right) \right],$$

where  $\delta_1^2 = k^2 - (\lambda + ikPe)/\bar{a}_1^2$ ,  $\delta_2^2 = k^2 - \lambda/\bar{a}_2^2$ . Here  $\bar{\kappa}_n = const$ , therefore heat diffusivity coefficients  $\bar{a}_i$  are constant too. For the amplitudes of velocity and pressure perturbations the same designations, as for the perturbations themselves, are kept.

As observed from (36), at high melt flow velocities ( $Pe \gg 1$ ) when convective heat transfer prevails over molecular heat transfer, the amplitudes of the perturbed velocities are small, except the waves with  $\omega = kPe$ . In the latter case strong instability of system can take place. These waves at any  $Pe$  are resonant and lead to considerable fluctuations of melt temperature. Distribution of temperature in a solid phase doesn't depend on  $Pe$  but depends on  $k, m$  and frequency  $\omega$ .

Boundary conditions for (37) with account (35):

$$r = 0, \quad u_1 = \theta_1 = 0; \quad (38)$$

$$r = 1, \quad u_1 = (\rho_{21} - 1)\lambda\zeta, \quad \theta_j = -\zeta \left( \frac{\partial T_j}{\partial r} \right)_{r=1},$$

$$\bar{\kappa}_2 \frac{d\theta_2}{dr} - \frac{d\theta_1}{dr} = \zeta \frac{Pe}{\bar{a}_1^2} \left( \frac{\partial T_1}{\partial x} \right)_{r=1} - \lambda R_\lambda \zeta; \quad (39)$$

$$r = s, \quad d\theta_2 / dr = -Bi_{k,m} \theta_2. \quad (40)$$

Using (40) and accounting the correlation

$$\sin(\alpha_{2nk}s) = \frac{J_0(\alpha_{2nk})Y_0(\alpha_{2nk}s) - J_0(\alpha_{2nk}s)Y_0(\alpha_{2nk})}{J_1(\alpha_{2nk})Y_0(\alpha_{2nk}) - J_0(\alpha_{2nk})Y_1(\alpha_{2nk})},$$

results in

$$\left( \frac{\partial T_1}{\partial r} \right)_{r=1} = \sum_{j=1}^2 \sum_{n=1}^4 \sum_{k=1}^{\infty} \alpha_{1k} J_1(\alpha_{1k}) d_{jk} \exp(q_{3-j} \alpha_{2n} x), \quad (41)$$

$$\left( \frac{\partial T_2}{\partial r} \right)_{r=1} = \sum_{j=1}^2 \sum_{n=1}^4 \sum_{k=1}^{\infty} \frac{\alpha_{2nk} b_{jnk}}{\sin(\alpha_{2nk}s)} \exp[(-1)^{j+1} \alpha_{jnk} x].$$

### 3.2.3 Calculation of the Eigen values for the task

Parameter  $R_\lambda$  as it was noted, for crystalline solids is big owing to what there is an opportunity to define Eigen values from the last boundary condition (39) solving (37)-(40), taking into account (41) by means of asymptotic decomposition of required functions in a series by  $\lambda$ . From (37), using  $\theta_j = \theta_j^0 + \lambda \theta_j^1 + \lambda^2 \theta_j^2 + \dots$ ,  $\bar{v}_j = \bar{v}_j^0 + \lambda \bar{v}_j^1 + \lambda^2 \bar{v}_j^2 + \dots$ ,  $p_1 = p_1^0 + \lambda p_1^1 + \lambda^2 p_1^2 + \dots$ , yields

$$u_1^0 = -\frac{i}{k} \frac{dw_1^0}{dr}, \quad \frac{d^2 \theta_2^0}{dr^2} + \frac{1}{r} \frac{d\theta_2^0}{dr} - \left( k^2 + \frac{m^2}{r^2} \right) \theta_2^0 = 0,$$

$$\frac{d^2 w_1^0}{dr^2} + \frac{u_1}{r} \frac{dw_1^0}{dr} - \left[ \frac{m^2(\omega - kPe)}{(\omega - mPe)r^2} + k^2 \right] w_1^0 = 0, \quad (42)$$

$$\frac{d^2 \theta_1^0}{dr^2} + \frac{1}{r} \frac{d\theta_1^0}{dr} - \left( \delta_{10}^2 + \frac{m^2}{r^2} \right) \theta_1^0 = \frac{1}{\bar{a}_1^2} \left( u_1^0 \frac{\partial T_1}{\partial r} + w_1^0 \frac{\partial T_1}{\partial x} \right),$$

where  $\delta_{10}^2 = k^2 - ikPe/\bar{a}_1^2$ . Obviously even in zero approach the solutions exist only when the melt temperature gradients by  $r$  and  $x$  in equilibrium state are functions only of  $r$ . General solution of the differential equation array (DEA) (42):

$$u_1^0 = -i \left[ c_1 I_q'(kr) + c_2 K_q'(kr) \right], \quad w_1^0 = c_1 I_q(kr) + c_2 K_q(kr),$$

$$\theta_1^0 = I_m(\delta_{10}r) \left[ c_3 - \int_0^r B_1^0(r) K_m(\delta_{10}r) dr \right] + \quad (43)$$

$$+ K_m(\delta_{10}r) \left[ c_4 + \int_0^r B_1^0(r) I_m(\delta_{10}r) dr \right], \quad \theta_2^0 = c_5 I_m(kr) + c_6 K_m(kr),$$

where dash means derivative by independent variable in a brackets,  $c_j$  ( $j = \overline{1,6}$ )- constants computed from boundary conditions. Here:

$$B_1^0 = \frac{1}{\bar{a}_1^2 \delta_{10} A_1^0} \sum_{j=1}^2 \sum_{n=1}^4 \sum_{k=1}^{\infty} \left\{ \alpha_{2n} q_{3-j} J_0(\alpha_{1k}r) \left[ c_1 I_q(kr) + c_2 K_q(kr) \right] + \right.$$

$$\left. -i \alpha_{1k} J_0'(\alpha_{1k}r) \left[ c_1 I_q'(kr) + c_2 K_q'(kr) \right] \right\} d_{jk} \exp(q_{3-j} \alpha_{2n} x),$$

$$A_1^0 = I_m(\delta_{10}r) K_m'(\delta_{10}r) - I_m'(\delta_{10}r) K_m(\delta_{10}r), \quad (44)$$

$q = m \sqrt{\frac{\omega - kPe}{\omega - mPe}}$ . From the equations obtained is seen that in a resonance  $\omega = kPe$  is  $q = 0$ , and by  $k = m$  results  $q = m$ . By  $\omega = mPe$  due to properties of  $I_q$  follows  $w_1^0 \approx 0$  by all values  $kr$ , except  $kr = \infty$ .

$B_1^0(r)$  can be only slowly changing function  $x$ , then for example at  $Pe \gg 1$  it is necessary to keep in (44) only two values of  $\alpha_2$ :  $\alpha_2 = \pm 0,5 \left( Pe - \sqrt{Pe^2 + 4\alpha_1^2} \right)$ , having equated zero coefficients at two other values of  $\alpha_2$  ( $T_{1x} = 0$  in expression (41)).

Boundary conditions (38)-(40) in zero approach:

$$r = 0, \quad u_1^0 = \theta_1^0 = 0; \quad (45)$$

$$r = 1, \quad u_1^0 = 0, \quad (46)$$

$$\theta_2^0 = \chi_2^0 \zeta = \zeta \sum_{j=1}^2 \sum_{k=1}^{\infty} \sum_{n=1}^4 \frac{\alpha_{2nk} b_{jnk}}{\sin(\alpha_{2nk}s)} \exp[(-1)^{j+1} \alpha_{2nk} x],$$

$$\theta_1^0 = \chi_1^0 \zeta = -\zeta \sum_{j=1}^2 \sum_{k=1}^{\infty} \sum_{n=1}^4 \alpha_{1k} d_{jk} J_1(\alpha_{1k}) \exp(q_{3-j} \alpha_{2n} x),$$

$$\bar{\kappa}_2 \frac{d\theta_2^0}{dr} - \frac{d\theta_1^0}{dr} = \zeta \frac{Pe}{\bar{a}_1^2} \left( \frac{\partial T_1}{\partial x} \right)_{r=1} - \lambda R_\lambda \zeta;$$

$$r = s, \quad d\theta_2^0 / dr = -Bi_{k,m} \theta_2^0 \quad (47)$$

Analysing the equations and boundary conditions one must note that physically substantiated

