Stability and Stabilization of the Solidification Front for Melt Flow in Cylindrical Channel with Phase Change on a Wall. Part 2

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Abstract: - Based on the mathematical model developed and analyzed in the Part 1, further analysis and computer simulation is performed as concern to peculiarities of the stability and possible stabilization of the unstable modes for interfacial boundary in the cylindrical channel, where melt is moving with solidification on the walls. Thin solid film on the walls is called garnissage, which is useful for the walls' protection against destroying from high-temperature and chemically aggressive melts and for the keeping the transported melt in a pure state, without pollution from the particles of the walls. The results may be of interest both theoretically (in stabilization of the processes in continua), as well as practically (in metallurgical aggregate machines).

Key-Words: - Control, Garnissage, Instability, Stabilization, Solidification, Melt, Wall, Protection, Channel

1 Introduction

More complex physical situations comparing to the ones considered in the Part 1, in which suppression of the parametric oscillations of the system by considered simple approaches is impossible, is analyzed in this paper. As an example the results of calculations by the model obtained for the following values of parameters:
$\kappa_1 = 4,33\ \mathrm{cal}/(m \cdot s \cdot K)$, $\rho_2 = 7,8 \cdot 10^3$ kg/m^3, R_0 =0,1m, $\lambda_{21} = 65$ kcal/kg are presented in the Table 1:

Table 1 Dependence of exponential (e^{ω}) by different crystallization temperatures:
values decrease exponentially e^{ω} times

k	ω, Hz by T_*, K 293	333	373
10	0,17	0,51	0,85
10^2	1,70	5,10	8,50
10^3	17,0	51,0	85,0

From the table it is visible, which is the influence of physical statements of the problem on the fading rate of parameters' perturbations of the studied system. So, at $k = 10^3$ in time, equal to one second, the perturbation amplitude of parameters decreases in e^{85} time if temperature of crystallization makes 373K. With increase of T_* (decrease of entropy of system), stability of system increases.

According to the Table 1 it is possible to estimate characteristic fading time of fluctuations of the parameters of physical system (technological process) in each case. Even when the system is steady, at slow fading of casual (or regular) perturbations of its parameters it is expedient to use the automatic heat flux control systems for acceleration of the process of parametric fluctuations' suppression. Because the technological mode in some cases demands maintenance of characteristics of process in strictly set limits (special metallurgy, protection of lining of metallurgical units against thermal and chemical destruction, etc.).

As shown in the Part 1, general information about the nature of spreading of parametric oscillations on the boundaries of melt crystallization and stability of system can be received from the analysis of the differential equations making mathematical model of the physical phenomenon. In more complex systems considered below such analysis isn't always possible, but application of asymptotic decompositions of the functions sought in a series by the small parameter λ (Eigen values of a task) is effective.

In a lack of experimental data about physical process it is difficult to estimate an adequacy to the constructed mathematical model and solutions of a task received on its basis to the real physical object. Therefore especially important is the question of reliability of the applied technique of researches, which is closely connected with adequacy of mathematical model. In the case considered the reliability of the applied technique can be estimated as follows. The studied problem of stability of the front of crystallization in the cylindrical channel allows analyzing the properties of system on the basis of the differential equations describing it by means of the stated artificial approach, without solution of the equations. The same problem can be solved using the asymptotic decompositions of the functions sought by small parameter. Comparison of the two approaches allows estimating efficiency and reliability of the applied research technique for stability of fronts of crystallization.

2 The results obtained and the problem statement

Though the harmonic perturbations of parameters were considered, the conclusions made are fair for any small perturbations as far as owing to the principle of superposition similarly it is possible to carry out the analysis of system stability for a case of non-harmonic perturbations decomposing them in Fourier series by harmonic components.

2.1 Solutions with application of the asymptotic decompositions

Consider the asymptotic decompositions of the type

$$\theta_j = \theta_j^0 + \lambda\theta_j^1 + \lambda^2\theta_j^2 + \lambda^3\theta_j^3 + ... , \qquad (1)$$

where j=1,2. Solving the equation array

$$u_1 = \frac{-1}{\bar{\rho}_1\lambda}\frac{dp_1}{dr},\ v_1 = \frac{mp_1}{\bar{\rho}_1\omega r},\ w_1 = \frac{kp_1}{\bar{\rho}_1\omega}, \qquad (2)$$

$$\frac{du_1}{dr} + i\left(m\frac{v_1}{r} + kw_1\right) + \frac{u_1}{r} = 0,\ \frac{d^2\theta_n}{dr^2} + \frac{1}{r}\frac{d\theta_n}{dr} - \left(\delta_n^2 + \frac{m^2}{r^2}\right)\theta_n =$$

$$= \frac{2-n}{\bar{a}_n^2}\left[\frac{1}{\lambda}\frac{\partial T_1}{\partial r}\frac{\partial p_1}{\partial r} + \frac{p_1}{\omega}\left(\frac{m}{r^2}\frac{\partial T_1}{\partial \varphi} + k\frac{\partial T_1}{\partial x}\right)\right].$$

With the boundary conditions:

$$r = s_0,\ \vec{v} = 0,\ r = 1,\ \frac{dp_1}{dr} = \left(1-\rho_{21}\right)\bar{\rho}_1\omega\zeta,$$

where $\{u_1, v_1, w_1\}, p_1, \theta$- amplitudes of perturbations of velocity, pressure and temperature, correspondingly, and $\delta_n^2 = k^2 - \lambda / a_n^2$, the Eigen values λ were computed based on the fact that value R_λ for crystalloid matters is usually rather big [1] and therefore λ can be calculated already in a zero approach. The Eigen values of the perturbed system obtained in the previous subchapter for the case of immovable melt are the following:

$$\lambda = \frac{-k}{R_\lambda \ln s_0}\left(\frac{A_1 Bi_{k,m} + A_2 k}{A_3 Bi_{k,m} + A_4 k} + \frac{A_5}{A_6}\right). \qquad (3)$$

After computing λ by the method above, from solution (3) of the problem in a zero approach the condition of oscillations' grow at the boundary of phase transition in time is written as follows (instability condition):

$$\frac{A_1 Bi_{k,m} + A_2 k}{A_3 Bi_{k,m} + A_4 k} + \frac{A_5}{A_6} < 0, \qquad (4)$$

where are:

$$A_1 = K_m(ks)I_m'(k) - K_m'(k)I_m(ks) > 0,$$
$$A_2 = K_m'(ks)I_m'(k) - K_m'(k)I_m'(ks) > 0,$$
$$A_3 = K_m(k)I_m(ks) - K_m(ks)I_m(k) > 0,$$
$$A_4 = K_m(k)I_m'(ks) - K_m'(ks)I_m(k) > 0,$$
$$A_5 = K_m'(k)I_m(ks_0) - K_m(ks_0)I_m'(k) < 0,$$
$$A_6 = K_m(k)I_m(ks_0) - K_m(ks_0)I_m(k) < 0,$$

where dash means derivative by argument kr. The inequalities are satisfied due to the known properties of the Bessel and Hankel functions. Using the last equations, one can show that condition (4) is not satisfied by any values of the parameters, so that the Eigen oscillations of the system are absent in this model statement. An analysis shows that oscillations of the front crystallization of constant amplitude are impossible too.

Thus, in this model statement there can be only fading oscillations in a system, decrease's rate with time of which depends on the value λ. If characteristic time of course of technological process considerably exceeds time of fading of casual parameters' perturbations, the control system isn't necessary. Otherwise it is necessary in order that corresponding change of parameter $Bi_{k,m}$ to achieve the maximum value λ for the purpose of increase of fading intensity for system's perturbations.

Further, in view of complexity of tasks about stability of fronts of crystallization in systems with existence not only the perturbed but also the melt's average flow, we will apply generally the technique described here, which efficiency is confirmed on a concrete example.

2.2 Mutual influence of the system's parameters and basic features of its behavior

The influence of physical properties of the melt and channel wall, thickness of the layer of a solid phase formed of melt on a wall of the channel, type of a wall and other factors on stability of the system and, in particular, on stability of the front of crystallization was shown above. Because in engineering applications such tasks are interesting in connection with protection of walls of channels and linings of metallurgical units against thermal and chemical destruction, and melt - against pollution by outsider admixture, the absolute importance has also a question of a choice of optimum ratios between parameters of system.

For example, for protection of the channels with garnissage [1-3] the automatic control of a form and position of the boundary of garnissage phase transition (thin layer of solid phase solidified on the wall from melt) must be done the thickness of solid film must be kept in the range stated. The question about an optimal garnissage thickness was discussed in [3,4]. As far as unperturbed state of the system has substantial influence on the system's behaviors, it is necessary to consider concrete features of the available unperturbed states. The considered solution belongs to a case when the equations of thermal balance are written for $\kappa_n = const$ and the thermal resistance of the channel wall can be neglected, owing to what boundary conditions also become significantly simpler and the solution of the corresponding boundary task has the form:

$$T_1 = 1 + \frac{\ln r}{\ln s_0}, \qquad T_2 = 1 + \frac{\ln r}{\bar{\kappa}_2 \ln s_0}. \tag{5}$$

If the requested thickness of solid layer on the channel wall is stated, from (5) the temperature of channel wall is:

$$T_w = 1 + \frac{\ln s}{\bar{\kappa}_2 \ln s_0}. \tag{6}$$

If the temperature of channel wall is given, the thickness of the garnissage layer with account of (6) is computed as

$$s = s_0^{\bar{\kappa}_2 (T_w - 1)}. \tag{7}$$

The analysis of (7) shows that it is possible to receive thin garnissage in case when temperature of a wall is close to a melting temperature (at any s_0) and in case of wide axial area of constant temperature ($s_0 \sim 1$) with a sharp gradient on a channel wall. Thus, in the second case irrespective of other conditions any perturbations of system fade almost instantly ($\lambda \gg 1$), whereas in the first case the rate of fading perturbations of system significantly depends on width of axial area with a constant temperature (parameter's s_0 value). It is also necessary to notice that as $A_j > 0$ ($j = \overline{1,4}$), the modified Biot number $Bi_{k,m}$ has limited influence on the rate of development of the system's perturbations at $Bi_{k,m} > 0$, only at $Bi_{k,m} \sim -A_4 / A_3$ this influence can be any big. The last is possible only in case of application of special control systems of heat fluxes, however then there can be incorrect a zero approach based on the assumption of a small λ (a consequence of small gradients of temperature on the boundary of phase transition).

At $s_0 \to 0$ (the axial area of constant temperature transforms in a cord) we receive $\lambda \to 0$, therefore without the operating system any perturbations spread without fading and increase. Only application of the special control systems of heat fluxes on the channel wall can achieve increase or fading in time for the corresponding perturbations (for achievement of spatial recognition to each harmonic with wave numbers k, m there has to be the regulation channel for value $Bi_{k,m}$).

For a choice of optimum thickness of a garnissage layer in each case it is necessary to proceed from the analysis of extreme values of expressions of type (5)-(7). For example, research of expression (7) shows that function $s(s_0)$, generally speaking, has no extreme because in general case $ds / ds_0 < 0$. Therefore it is necessary to define $extr\lambda(s)$, $s \in (1, s_m)$, where s_m - physically maximal attainable value s. Generally for determination of extreme Eigen values of a task taking into account (3) requires solving the transcendental equation. For analysis we consider later on two limit cases.

2.2.1 The short-wave perturbations

By $k \gg 1$ functions $I_m(z), K_m(z)$ and derivatives $I'_m(z), K'_m(z)$ have the characteristic orders of value:

$$I_m(z) \sim \frac{e^z}{\sqrt{2\pi z}}, \quad K_m(z) \sim \frac{\pi I_m(z)}{e^{2z}},$$
$$K'_m(z) \sim -\sqrt{\frac{\pi}{2z}} e^{-z}, \; I'_m(z) \sim I_m(z),$$

therefore: $A_3 \sim A_2$, $A_4 \sim A_1$, $\dfrac{A_2}{A_1} \sim \dfrac{e^{-2k} - e^{-2ks}}{e^{-2k} + e^{-2ks}}$,

$$A_1 \sim \frac{e^{k(s+1)}}{2k\sqrt{s}} \left(e^{-2k} + e^{-2ks} \right), \; A_5 \sim A_6 \sim \frac{-1}{2k\sqrt{s_0}} e^{k(1-s_0)},$$

where from with account (3) follows:

$$\lambda = \frac{-k}{R_\lambda \ln s_0} \left[1 + \frac{Bi_{k,m} + k \cdot thk(s-1)}{Bi_{k,m} thk(s-1) + k} \right]. \tag{8}$$

By $k(s-1) \gg 1$ from (8), accounting $thk(s-1) \approx 1$:

$$\lambda = \frac{-2k}{R_\lambda \ln s_0},$$

where from follows that short-wave perturbations of the boundary of melting thick garnissage layer are not controllable with the method described above, they are decreasing with time by a rate proportional to the wave number k. By $k(s-1) \ll 1 \ (s \ll 1+1/k)$ an approximate correlation $thk(s-1) \approx k(s-1)$ can be considered, which yields:

$$\lambda = \frac{-1}{R_\lambda \ln s_0}\left[k + \frac{Bi_{k,m} + k^2(s-1)}{Bi_{k,m}(s-1)+1}\right].$$

Analysis of the expression obtained shows that a thin garnissage allows controlling the stability of front crystallization. With account of the assumptions made the following approximate estimations of the parameters follow:

$$s > 1+1/k, \quad Bi_{k,m} > Bi^*_{k,m}, \quad \lambda < 0; \qquad (9)$$

$$s < 1+1/k, \quad Bi_{k,m} < Bi^*_{k,m}, \quad \lambda < 0;$$

where $Bi^*_{k,m} = -k$ is the critical modified Biot number corresponding to a loss of stability. As seen from (8), the short-wave perturbations of the boundary of crystallization of thin garnissage layer can grow in time by big negative $Bi_{k,m}$, so that in absence of the heat flux control system it is impossible.

In other cases, investigating function (8) on an extreme, we receive that $d\lambda/ds$ has the sign determined by value $Bi_{k,m}$: at $|Bi_{k,m}| < k$ the Eigen numbers $\lambda(s)$ increase and, therefore, owing to the carried-out estimates, the garnissage is stable, and at $|Bi_{k,m}| > k$ the function $\lambda(s)$ decreases, but $d\lambda/ds$ doesn't change a sign anywhere and as $\lim\limits_{s\to\infty} \lambda(s) = -2k/(R_\lambda \ln s_0) > 0$, it also corresponds to stability. Thus, short-wave perturbations of garnissage in a linear statement don't lead to violation of its stability.

2.2.2 The long-wave axisymmetric perturbations

More dangerous as concern to breaking the garnissage instability is another limit case – long-wave perturbations of the parameters of physical system. By $k \ll 1, m = 0$ approximations of the functions $I_0(z), K_0(z)$ and their derivatives are applied, which are correct for $z \ll 1$: $I_0(z) \sim 1$, $I_0'(z) \sim 0$, $K_0(z) \sim \ln(2/(\gamma' z))$, $K_0(z) \sim -1/z$, where γ' - the Euler constant. This gives: $A_1 \sim 1/k$, $A_2 \sim 0$, $A_3 \sim \ln s$, $A_4 \sim 1/(ks)$, $A_5 \sim -A_1$, $A_6 \sim \ln s_0$, afterwards from (8) follows

$$\lambda = \frac{1}{R_\lambda \ln s_0}\left(\frac{1}{\ln s_0} - \frac{Bi_{k,m} s}{1 + s Bi_{k,m} \ln s}\right). \qquad (10)$$

In contrast to the short-wave perturbations, as seen from (10), the long-wave perturbations have the Eigen values independent of the wave number. Therefore for $\forall k \ll 1$ garnissage is stable. Instability may be provoked by the system of automatic heat flux control if $Bi_{k,m}$ satisfies

$$\frac{-1}{s \ln s} < Bi_{k,m} < \frac{-1}{s(\ln s - \ln s_0)}.$$

Investigation of the function (10) shows that by $Bi_{k,m} > 0, \ s < 1/Bi_{k,m}$ with increase of garnissage thickness the fading rate of perturbations is growing, and by $s > 1/Bi_{k,m}$ - the fading rate of perturbations is falling down. This means that $s = 1/Bi_{k,m}$ is the maximum point of function $\lambda(s)$.

Also the question of a ratio of fading rates for short-wave and long-wave perturbations of a garnissage is of interest. We consider it based on formulas (8), (10), assuming $k(s-1) \gg 1$ in the first case and $k \ll 1$- in the second, $s_0 \ll 1$. Then we get for the ratio of values $\lambda(s)$ in specified cases

$$\Delta_\lambda = 2k\frac{1 + Bi_{k,m} s \ln s}{s Bi_{k,m}}. \qquad (11)$$

The obtained formula (11) is analyzed by $Bi_{k,m} \sim 1$, $s \sim 1$, where $\Delta_\lambda \sim 2k$, e.g. $\Delta_\lambda \gg 1$, and condition $\Delta_\lambda \sim 1$ requires by $s \sim 1$ the following value of the parameter $Bi_{k,m} \sim 2k$. From here it is possible to draw a conclusion that in absence of a control system for heat fluxes on an external surface of the channel the short-wave perturbations fade much faster than the long-wave perturbations. And commensurable fading rates of these perturbations can be only in the presence of the powerful operated heat fluxes from the channel into surroundings (almost instant freezing of a melting wave for a solid layer on a channel surface that is practically very problematic to provide).

The revealed features of behavior of the boundaries of phase transition in cylindrical channels and interference of various parameters of physical system allow organizing optimum technological process in each case. Further the considered questions will be investigated for channels, in which melt is flowing in the channel along its axis in unperturbed state (transportation of melt in channel with garnissage protection of a wall).

3 Effect of liquid phase flow on the instability of front crystallization

The main regularities of oscillations of the boundaries of phase transition in systems with the cylindrical channels containing the motionless melt solidifying (crystallizing) near the walls having temperature below solidification temperature of melt were considered above. Thus, we investigated mutual influence of various parameters of physical system, including influence of a wall of the channel which, generally speaking, can be multilayered. By comprehensive consideration of a task on a number of the simplified physical and mathematical models application of a highly effective technique of approximate solution of this sort of tasks by means of asymptotic decomposition of the functions sought in a series by small parameter λ was shown.

3.1 Channel with multilayered wall

Now the described technique is applied for the systems, which structural scheme is shown in Fig.1:

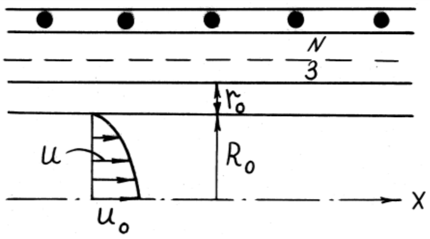


Fig.1 Structural scheme of the cylindrical front crystallization of melt flowing in a channel

The channel generally may contain a multilayered wall from N various layers. On an external surface the system of automatic control of heat fluxes for excitation or suppression of oscillations in system can be installed. Physical properties of a multilayered wall of the channel have impact on thermal-hydraulic stability of system at any mode of a melt flow in the channel. However the considered rather simple model task can seem too rough approach of complex real physical system and besides perhaps essential influence of hydrodynamic instability on the general course of technological process may happen.

Parameters of unperturbed system can play a significant role in the course of appearance and distribution of oscillations on the boundary of phase transition and oscillations of all characteristics of system. Taking into account the above it is necessary to pay attention to adequacy of the mathematical description of unperturbed system and to reveal features of influence of its characteristics on the perturbed state. We consider melt as incompressible liquid and viscosity in the majority of the considered cases is neglected due to its negligible value for many metal melts. We assume that in a state of dynamic balance of system a melt moves with a speed, almost constant on all cross section of the channel (viscous friction of melt with a wall is neglected). Then the equations of thermal balance of the described system possessing axial symmetry are written in the form:

$$\rho_1 c_1 u_0 \frac{\partial T_1}{\partial x} = \kappa_1 \left(\frac{\partial^2 T_1}{\partial x^2} + \frac{1}{r}\frac{\partial T_1}{\partial r} + \frac{\partial^2 T_1}{\partial r^2} \right),$$

$$\frac{\partial^2 T_2}{\partial x^2} + \frac{1}{r}\frac{\partial T_2}{\partial r} + \frac{\partial^2 T_2}{\partial r^2} = 0. \qquad (12)$$

Here ρ_1, c_1, u_0 are, respectively, density, heat capacity coefficient and melt flow velocity along the channel axis, *T*- temperature of unperturbed system. Similarly – the second equation of the system (12) is the heat conductivity equation for solid phase. If the channel's wall is under consideration as well, then corresponding equation is also added for it.

The boundary conditions are similar to the above considered, only the difference is that temperature of the wall T_w is in general function of *x*, and, besides, two boundary conditions must be stated by *x*, for example, temperature profile $T_0(r)$ and heat flux $T_{1x}^0(r)$ of a melt in cross-section *x*=0 may be stated. Here $T_{1x}^0(r) = \partial T_1 / \partial x$ by x =0.

3.1.1 Dimensionless mathematical model

In dimensionless form with account of the above the following boundary-value problem is got:

$$Pe\frac{\partial T_1}{\partial x} = \frac{\partial^2 T_1}{\partial x^2} + \frac{1}{r}\frac{\partial T_1}{\partial r} + \frac{\partial^2 T_1}{\partial r^2}, \quad \frac{\partial^2 T_2}{\partial x^2} + \frac{1}{r}\frac{\partial T_2}{\partial r} + \frac{\partial^2 T_2}{\partial r^2} = 0; \qquad (13)$$

$$r = 1, \quad T_j = 1 \ (j = 1, 2), \quad \frac{\partial T_1}{\partial r} = \bar{\kappa}_2 \frac{\partial T_2}{\partial r}; \qquad (14)$$

$$r = s, \ T_2 = T_w(x); \ x = 0, \ T_1 = T_1^0(r), \ \frac{\partial T_1}{\partial x} = T_{1x}^0(r), \qquad (15)$$

where $Pe = w_0 R_0 a_1^2$- Peclet number, $T_w(x), T_1^0(r)$, $T_{1x}^0(r)$- given functions. The criterion $Pe = \mathrm{Re}\cdot\mathrm{Pr}$ characterizes heat similarity in a moving medium and reflects the ration of convective and conductive (molecular) heat transfer in the channel.

3.1.2 Solution of the dimensionless boundary task
The boundary task (13)-(15) may be solved by Fourier method introducing instead of $T_j(r)$ function $T_j(r)$-1, for which system (13) preserves its view but the first boundary condition (14) becomes uniform. Let $\tilde{T}_j = T_j - 1$, $\tilde{T}_j = \tilde{X}_j(x)\tilde{R}_j(r)$, the with account of the above yields

$$\frac{d^2\tilde{R}_j}{dr^2}+\frac{1}{r}\frac{d\tilde{R}_j}{dr}+\alpha_j^2\tilde{R}_j=0,\ \frac{d^2\tilde{X}_j}{dx^2}+(j-2)Pe\frac{d\tilde{X}_j}{dx}-\alpha_j^2\tilde{X}_j=0,$$

where α_j are Eigen values. The solution of the equation array thus obtained is written as follows

$$\tilde{X}_1=d_1\exp\left[\left(Pe-\sqrt{Pe^2+4\alpha_1^2}\right)\frac{x}{2}\right]+d_2\exp\left[\left(Pe+\sqrt{Pe^2+4\alpha_1^2}\right)\frac{x}{2}\right],$$
$$\tilde{X}_2=b_1e^{\alpha_2 x}+b_2e^{-\alpha_2 x},\ \tilde{R}_j=c_{j1}J_0(\alpha_j r)+c_{j2}Y_0(\alpha_j r),\quad (16)$$

where J_0, Y_0 - Bessel functions of the first and second type of zero order, b_j, d_j, c_{jn} ($j=1,2,\ n=1,2$) are constants computed from conditions (14)-(15).

Because value $\tilde{R}_1$ must be limited, $c_{12}=0$ must be due to $Y_0(0)=\infty$. Without restriction on generality $c_{11}=1$ can be put, then from condition $J_0(\alpha_1)=0$ follows numbered set of positive values of parameters α_{1k}. Constants c_{2j} have the form

$$c_{21}=\frac{Y_0(\alpha_2)}{J_0(\alpha_2 s)Y_0(\alpha_2)-J_0(\alpha_2)Y_0(\alpha_2 s)},$$
$$c_{22}=\frac{J_0(\alpha_2)}{J_0(\alpha_2)Y_0(\alpha_2 s)-J_0(\alpha_2 s)Y_0(\alpha_2)}.\quad (17)$$

For calculation of b_j the following equation is used:

$$b_1e^{\alpha_2 x}+b_2e^{-\alpha_2 x}=A\left(d_1e^{\alpha_2 q_2 x}+d_2e^{\alpha_2 q_1 x}\right),$$

which must be satisfied identically, therefore expanding the functions in a Taylor series and comparing the equal coefficients by similar terms in the equation yields

$$b_1+b_2=A(d_1+d_2),\quad b_1-b_2=A(q_1d_1+q_2d_2),\quad (18)$$
$$(1-q_1^2)d_1+(1-q_2^2)d_2=0,\ q_1(1-q_1^2)d_1+q_2(1-q_2^2)d_2=0,$$

where are $q_{1,2}=1/(2\alpha_2)\left(Pe\pm\sqrt{Pe^2+4\alpha_1^2}\right)$, $s'=s-1$, $A=\alpha_{12}\kappa_{12}J_1(\alpha_1)\sin\alpha_2 s'$. The non-trivial solution of this uniform system of algebraic equations (SAE) exists only by $q_{1,2}=\pm1$ ($q_1\neq q_2$), therefore:

$$b_{1n}=A\left[0,5(1+q_{2,1})d_{2,1}\pm d_{1,2}\right],\ n=\overline{1,4},\quad (19)$$
$$b_{21}=b_{22}=0,5Ad_2(1-q_2),\quad b_{23}=b_{24}=0,5Ad_1(1-q_1).$$

where 4 pairs of values b_{jn} were got. In the first equation together with $n=\overline{1,4}$, all first indexes with signs «+» and «-»consecutively are taken one-by-one, then – similar the second indexes. Finally, accounting the laid out and expressions (16)-(19), solution of the boundary task (13)-(15) is written as:

$$T_1=1+\sum_{n=1}^{4}\sum_{k=1}^{\infty}\sum_{j=1}^{2}J_0(\alpha_{1k}r)d_{jk}\exp\left(q_{3-j}\alpha_{2n}x\right),$$
$$T_2=\sum_{n=1}^{4}\sum_{k=1}^{\infty}\sum_{j=1}^{2}b_{jnk}\exp\left[(-1)^{j+1}\alpha_{jnk}x\right]\cdot\quad (20)$$
$$\cdot\frac{J_0(\alpha_{2nk})Y_0(\alpha_{2nk}r)-J_0(\alpha_{2nk}r)Y_0(\alpha_{2nk})}{J_0(\alpha_{2nk})Y_0(\alpha_{2nk}s)-J_0(\alpha_{2nk}s)Y_0(\alpha_{2nk})},$$

where

$$(\alpha_2)_{1,2}=\pm1/2\left(Pe-\sqrt{Pe^2+4\alpha_1^2}\right),$$
$$(\alpha_2)_{3,4}=\pm1/2\left(Pe+\sqrt{Pe^2+4\alpha_1^2}\right)\cdot$$

Constants b_{jnk} computed by (19) through d_{jk}, which are obtained from boundary condition (15):

$$\sum_{j=1}^{2}\sum_{k=1}^{\infty}d_{jk}J_0(\alpha_{1k}r)=T_1^0(r),$$
$$\sum_{j=1}^{2}\sum_{k=1}^{\infty}d_{jk}\left(Pe+(-1)^j\sqrt{Pe^2+4\alpha_{1k}^2}\right)J_0(\alpha_{1k}r)=2T_{1x}^0(r).$$

Accounting these series $T_1^0(r), T_{1x}^0(r)$ by Bessel functions, expression for the coefficients yields

$$d_{1k}=\left(1+\frac{Pe}{\sqrt{Pe^2+4\alpha_{1k}^2}}\right)\frac{1}{J_1^2(\alpha_{1k})}\int_0^1(T_1^0-T_{1x}^0)J_0(\alpha_{1k}r)rdr,\quad (21)$$
$$d_{2k}=\frac{1}{J_1^2(\alpha_{1k})}\int_0^1\left[\left(1-\frac{Pe}{\sqrt{Pe^2+4\alpha_{1k}^2}}\right)T_1^0+\left(1+\frac{Pe}{\sqrt{Pe^2+4\alpha_{1k}^2}}\right)T_{1x}^0\right]J_0(\alpha_{1k}r)rdr$$

Mathematical model (17), (19)-(21) for heat equilibrium of the system despite the simplifications made for real physical situation is still substantially complex. It needs numerical solution on computer.

3.1.3 The case of linear dependence $T_1(x)$
One of the most simple is a case of a linear function $T_1(x)$ or $\partial T_1/\partial x=\Delta_1=const$ (temperature gradient of melt along the axis of channel is constant). Then accounting the type of boundary task (13)-(16), one can assume that $T_2(x)$ is linear function too, and solution of the task is written as follows:

$T_1=0,25Pe\cdot\Delta_1\cdot r^2+c_1(x)\ln r+c_2(x),\quad T_2=c_3(x)\ln r+c_4(x),$

where from the extremity condition of $T_1(x)$ by $r=0$ results $c_1\equiv0$, and from the assumptions made yields: $c_2(x)=\Delta_1 x+c_2,\ c_2,c_3=const$,

$$T_1=0,25Pe\cdot\Delta_1\cdot r^2+\Delta_1 x+c_2,\ T_2=c_3(x)\ln r+c_4(x)+c_5,$$

where $c_4, c_5 = const$. From here follows that in this model the boundary of phase transition $r = r_*$ changes by x from $r = 1$ by x=0, thus:

$$T_1 = 1 + 0,25Pe\cdot\Delta_1\left(r^2 - 1\right) + \Delta_1 x,$$

$$T_2 = 1 + 0,5Pe\cdot\Delta_1\kappa_{12}\ln r + \Delta_1\kappa_{12}x, \qquad (22)$$

with accuracy to linear terms by x. The approximate solution thus obtained (22) is correct if on the wall of channel the temperature distribution is kept as

$$T_w = 1 + Pe\cdot\Delta_1\kappa_{12}s^2\ln s + \Delta_1\kappa_{12}s^2 x.$$

The boundary of phase transition in this case is described by function $r_* = \sqrt{1 - 4x/Pe}$. This expression is the more precise, the less is value x/Pe, where from follows that it is valid for high Peclet numbers (strong convective heat transfer). By $Pe \ll 1$ the temperature profiles in liquid and solid phases are similar and in a first approach they can be considered independent of x for the short channel.

3.2 Region of the phase transition

By consideration of a problem of protection of the channels' walls in the steel-smelting units of ejector type by means of a garnissage two-phase area takes place also for the following reasons (see Fig.2). Slag-metallic melt moves along a channel axis, the area $r \in (0, R_1)$ is occupied by slag-metallic alloy, $r \in (R_1, R_2)$ - by metal melt with solid slag inclusions (the case, when slag is more refractory than metal considered), $r \in (R_2, R_2 + r_0)$ - a layer of a solid metal phase (garnissage).

Thus, based on a hypothesis of existence of phase transition area with zero width, we have: a surface $r = R_1$ - the boundary of slag phase transition having slag melting temperature $T = T_{ш*}$, $r = R_2$ - a surface of metal crystallization with a temperature $T = T_{м*}$, $r \in (R_1, R_2)$ - two-phase area. Even in the assumption T_{liq}=T_{sol} the described multiphase multi-component medium is non-uniform and has the complex mathematical description.

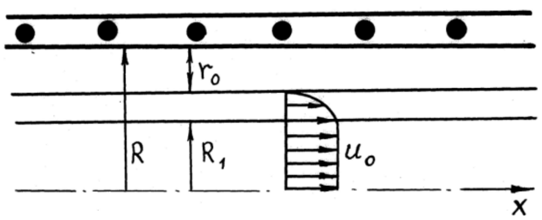


Fig.2 Structural scheme of the two-phase flow with crystallization of melt on the channel wall

3.2.1 Flow of two-phase medium with crystallization on channel's wall

In a first region of the described system flow of slag-metallic alloy can be assumed in first approach inviscid. In a second region viscosity of melt is substantial due to abrupt it's grow because of appearance of some solidified particles.

For simplification of the model we assume that the flow velocity profile in two-phase area is auto model: $u = u_0 f(r)$, where u_0 - flow velocity in the first area. We neglect viscous dissipation except influence in a form of velocity profile. Also we believe $\partial T_j/\partial x = \Delta_x = const$, j=1,2. Then the dimensionless boundary task is presented in a form

$$Pe\left[f(r)\right]^{j-1}\frac{\partial T_j}{\partial x} = \frac{\partial^2 T_j}{\partial x^2} + \frac{1}{r}\frac{\partial T_j}{\partial r} + \frac{\partial^2 T_j}{\partial r^2},\ \frac{\partial^2 T_3}{\partial x^2} + \frac{1}{r}\frac{\partial T_3}{\partial r} + \frac{\partial^2 T_3}{\partial r^2} = 0; \qquad (23)$$

where j=1,2. As the characteristic scales for length and temperature $R_2, T_{м*}$ are adopted. And the boundary conditions are stated as follows:

$$r = s_1,\quad T_1 = T_2 = T_{ш*},\quad \frac{\partial T_1}{\partial r} = \bar\kappa_2\frac{\partial T_2}{\partial r}; \qquad (24)$$

$$r = r_*,\quad T_3 = T_2 = 1,\quad \frac{\partial T_2}{\partial r} = \bar\kappa_{32}\frac{\partial T_3}{\partial r}; \qquad (25)$$

$$r = s_3,\quad T_3 = T_w(x); \qquad (26)$$

where $s_1 = R_1/R_2,\ s_2 = (R_2 + r_0)/R_2,\ \bar\kappa_{32} = \bar\kappa_3/\bar\kappa_2$.

Solution of the boundary task (23)-(26) yields

$$T_1 = T_{ш*} + 0,25Pe\cdot\Delta_x\left(r^2 - s_1^2\right) + \Delta_x x, \qquad (27)$$

$$T_2 = T_{ш*} + Pe\cdot\Delta_x\left[\Phi(r) + 0,5s_1^2\kappa_{12}\ln(r/s_1) + x/Pe\right],$$

$$T_3 = 1 + Pe\cdot\Delta_x\left[\kappa_{23}F(r_*) + 0,5s_1^2\kappa_{13}\right]\ln(r/r_*),$$

where $F(r) = \int_{s_1}^{r} f(r)rdr,\quad \Phi(r) = \int_{s_1}^{r} F(r)\frac{dr}{r}$. From the boundary conditions (24) correlation of parameters $s_1, T_{ш*}$ is: $s_1^2\ln s_1 = 2\bar\kappa_2\left[\Phi(1) + (T_{ш*} - 1)/(Pe\cdot\Delta_x)\right]$. Here $r = r_*$ is the boundary of phase transition different from the one at $r = 1$ by x=0 because of temperature gradient along the axis. Boundary of the phase transition is determined by

$$\Phi(r_*) + \frac{\kappa_{12}}{2}s_1^2\ln\frac{r_*}{s_1} + \frac{x}{Pe} = \frac{1 - T_{ш*}}{\Delta_x Pe}. \qquad (28)$$

On the wall following (27) we get temperature

$$T_w = 1 + Pe\cdot\Delta_x\left[\kappa_{23}F(r_*) + 0,5s_1^2\kappa_{13}\right]\ln(s_3/r_*). \qquad (29)$$

The profile $f(r)$ can be taken linear based on the conditions $f(s_1) = 1,\ f(r_*) = 0$, and in general case a view of this function is determined by the flow regime (with account of temperature distribution), content of slag phase, size of dispersions, etc.

The correlation (29) allows computing the required wall's temperature distribution by the given garnissage's thickness $r_0 = s_3 - r_*$. In a limit, by $s_1 \to 1$, $T_{ш*} \to T_{м*} = 1$ solution (27) transforms into the earlier obtained (22). But if in narrow region $r \in (1, r_*)$ due to abrupt increase of the melt flow velocity from 0 at the boundary of crystallization $r = r_*$ to 1 at the line $r = 1$, the regularity of the boundary evolution comparing to the case (22) changes. Let show it assuming due to small size of area $r \in (1, r_*)$ the linear approximation $f(r) = (1-r)(r_* - s_1)$. Then (28) accounting $\lim_{s_1 \to 1} \Phi(r_*) = \frac{5r_* - 4r_*^2 - 1}{36}$ with accuracy to linear terms by $(1 - r_*)$ gives

$$r_* = \frac{1}{8}\left[5 - 18\kappa_{12} + 3\sqrt{(1+6\kappa_{12})^2 - 64\frac{x}{Pe}}\right]. \qquad (30)$$

By small x/Pe the expression (30) can be simplified in a linear approach:

$$r_* = 1 - \frac{12x}{(1+6\kappa_{12})^2 Pe}, \qquad (31)$$

wheares earlier in a linear approach by x/Pe was got $r_* = 1 - 2x/Pe$. Here from follows that account of the abrupt melt velocity changes in a thin layer near surface of crystallization leads to dependence of function $r_*(x)$ from ratio of the heat conductivity coefficients for the phases. By $\bar{\kappa}_2 < (\sqrt{6}-1)/\sqrt{6} \approx 0,2416$, without account of $r_*(x)$, are underestimated, while by $\bar{\kappa}_2 > (\sqrt{6}-1)$ – inversely, overestimated.

In other limit case, $s_1 \to 0$, the solution (27) is

$$T_1 = T_0 + Pe \cdot \Delta_x\left[\Phi(r) + \frac{x}{Pe}\right],\ T_2 = 1 + \kappa_{12} Pe \cdot \Delta_x F(r_*) \ln\frac{r}{r_*}, \qquad (32)$$

where $T_0 = T_1(0,0)$. The expressions (32) present solution for viscous liquid flow, when dissipation can be neglected but must be accounted its influence on the velocity profile. Melt can be single-phase.

Here similarly to the above considered yields

$$r_* = 6\sqrt{\frac{1}{5Pe}\left(\frac{1-T_0}{\Delta_x} - x\right)}, \qquad (33)$$

or in a linear by x/Pe approach:

$$r_* = 6\sqrt{\frac{1-T_0}{5\Delta_x Pe}}\left[1 - \frac{\Delta_x x}{2(1-T_0)}\right],$$

where from seen that $r_*(x)$, in contrast to the considered cases, depends on melt temperature on the channel axis (does not depend of $\bar{\kappa}_2$). On position of front crystallization all melt influences, not only the boundary area as in the previous case.

Let consider consecutively a few of the above models. The unperturbed state of system is described by correlations (20) or boundary task (22)-(26). Then for small-amplitude perturbations of the equilibrium parameters in a linear approach

$$\rho_1 c_1\left[\frac{\partial \tau_j}{\partial t} + (2-j)\left(w_0\frac{\partial \tau_j}{\partial x} + \vec{v}_1 \nabla T_1\right)\right] =$$
$$\kappa_j\left(\frac{\partial^2 \tau_j}{\partial x^2} + \frac{1}{r}\frac{\partial \tau_j}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \tau_j}{\partial \varphi^2} + \frac{\partial^2 \tau_j}{\partial r^2}\right), \qquad (34)$$
$$\frac{\partial \vec{v}_1}{\partial t} + w_0\frac{\partial \vec{v}_1}{\partial x} = -\frac{1}{\rho_1}\nabla p_1, \quad div\vec{v}_1 = 0,$$

where *j*=1,2, $\vec{v}_1\{u_1, v_1, w_1\}(r) w_0 \exp i(kx + m\varphi - \omega t), p_1$- perturbations of melt velocity and pressure.

The boundary conditions are: on the axis:

$$r = 0, \quad u_1 = 0, \quad \tau_1 = 0. \qquad (35)$$

On the boundary of phase transition $r = R_0\left[1 + \zeta \exp i(kx + m\varphi - \omega t)\right]$ with accuracy to the linear terms by perturbations the mass and heat flux conditions are written as previously. On the external surface of a channel the impedance boundary condition is stated.

3.2.2 Mathematical model of perturbed system

With account of the above and (12), (13) the mathematical model of the perturbed system is:

$$div\vec{v}_1 = 0, \quad \frac{\partial \vec{v}_1}{\partial Fo} + Pe\frac{\partial \vec{v}_1}{\partial x} = -\frac{1}{\bar{\rho}_1}\nabla p_1, \qquad (36)$$
$$\frac{\partial \tau_1}{\partial Fo} + \vec{v}_1 \nabla T_1 + Pe\frac{\partial \tau_1}{\partial x} = \frac{div(\bar{\kappa}_1 \nabla \tau_1)}{\bar{\rho}_1 \bar{c}_1}, \quad \frac{\partial \tau_2}{\partial Fo} = \frac{div(\bar{\kappa}_2 \nabla \tau_2)}{\bar{\rho}_2 \bar{c}_2},$$

with a difference that system (36) contains the term with multiplier *Pe* describing the action of convective component of the unperturbed melt flow in a channel. The boundary conditions are similar.

For solution of the boundary task obtained, the equations for amplitudes of perturbations are got as

$$u_1 = \frac{1}{\bar{\rho}_1(\lambda - ikPe)}\frac{dp_1}{dr}, \quad \frac{du_1}{dr} + \frac{u_1}{r} + i\left(m\frac{v_1}{r} + kw_1\right) = 0,$$
$$v_1 = \frac{mp_1}{\bar{\rho}_1(\omega - kPe)r}, \quad w_1 = \frac{kp_1}{\bar{\rho}_1(\omega - kPe)}, \qquad (37)$$

$$\frac{d^2\theta_2}{dr^2} + \frac{1}{r}\frac{d\theta_2}{dr} - \left(\delta_2^2 + \frac{m^2}{r^2}\right)\theta_2 = 0, \ \frac{d^2\theta_1}{dr^2} + \frac{1}{r}\frac{d\theta_1}{dr} - \left(\delta_1^2 + \frac{m^2}{r^2}\right)\theta_1 =$$

$$=\frac{1}{\bar{a}_1^2}\left[\frac{\partial p_1/\partial r}{\lambda-ikPe}\frac{\partial T_1}{\partial r}+p_1\left(\frac{m/r^2}{\omega-mPe}\frac{\partial T_1}{\partial\varphi}+\frac{k}{\omega-kPe}\frac{\partial T_1}{\partial x}\right)\right],$$

where $\delta_1^2=k^2-(\lambda+ikPe)/\bar{a}_1^2$, $\delta_2^2=k^2-\lambda/\bar{a}_2^2$. Here $\bar{\kappa}_n=const$, therefore heat diffusivity coefficients $\bar{a}_i$ are constant too. For the amplitudes of velocity and pressure perturbations the same designations, as for the perturbations themselves, are kept.

As observed from (36), at high melt flow velocities $(Pe\gg 1)$ when convective heat transfer prevails over molecular heat transfer, the amplitudes of the perturbed velocities are small, except the waves with $\omega=kPe$. In the latter case strong instability of system can take place. These waves at any Pe are resonant and lead to considerable fluctuations of melt temperature. Distribution of temperature in a solid phase doesn't depend on Pe but depends on k,m and frequency ω.

Boundary conditions for (37) with account (35):

$$r=0,\quad u_1=\theta_1=0; \tag{38}$$

$$r=1,\quad u_1=(\rho_{21}-1)\lambda\zeta,\quad \theta_j=-\zeta(\partial T_j/\partial r)_{r=1},$$

$$\bar{\kappa}_2\frac{d\theta_2}{dr}-\frac{d\theta_1}{dr}=\zeta\frac{Pe}{\bar{a}_1^2}\left(\frac{\partial T_1}{\partial x}\right)_{r=1}-\lambda R_\lambda\zeta; \tag{39}$$

$$r=s,\ d\theta_2/dr=-Bi_{k,m}\theta_2. \tag{40}$$

Using (40) and accounting the correlation

$$\sin(\alpha_{2nk}s)=\frac{J_0(\alpha_{2nk})Y_0(\alpha_{2nk}s)-J_0(\alpha_{2nk}s)Y_0(\alpha_{2nk})}{J_1(\alpha_{2nk})Y_0(\alpha_{2nk})-J_0(\alpha_{2nk})Y_1(\alpha_{2nk})},$$

results in

$$(\partial T_1/\partial r)_{r=1}=\sum_{j=1}^{2}\sum_{n=1}^{4}\sum_{k=1}^{\infty}\alpha_{1k}J_1(\alpha_{1k})d_{jk}\exp(q_{3-j}\alpha_{2n}x), \tag{41}$$

$$(\partial T_2/\partial r)_{r=1}=\sum_{j=1}^{2}\sum_{n=1}^{4}\sum_{k=1}^{\infty}\frac{\alpha_{2nk}b_{jnk}}{\sin(\alpha_{2nk}s)}\exp\left[(-1)^{j+1}\alpha_{jnk}x\right]\cdot$$

3.2.3 Calculation of the Eigen values for the task

Parameter R_λ as it was noted, for crystalline solids is big owing to what there is an opportunity to define Eigen values from the last boundary condition (39) solving (37)-(40), taking into account (41) by means of asymptotic decomposition of required functions in a series by λ. From (37), using $\theta_j=\theta_j^0+\lambda\theta_j^1+\lambda^2\theta_j^2+...,$ $\vec{v}_j=\vec{v}_j^0+\lambda\vec{v}_j^1+\lambda^2\vec{v}_j^2+...,$ $p_1=p_1^0+\lambda p_1^1+\lambda^2p_1^2+...,$ yields

$$u_1^0=-\frac{i}{k}\frac{dw_1^0}{dr},\qquad \frac{d^2\theta_2^0}{dr^2}+\frac{1}{r}\frac{d\theta_2^0}{dr}-\left(k^2+\frac{m^2}{r^2}\right)\theta_2^0=0,$$

$$\frac{d^2w_1^0}{dr^2}+\frac{u_1}{r}\frac{dw_1^0}{dr}-\left[\frac{m^2(\omega-kPe)}{(\omega-mPe)r^2}+k^2\right]w_1^0=0, \tag{42}$$

$$\frac{d^2\theta_1^0}{dr^2}+\frac{1}{r}\frac{d\theta_1^0}{dr}-\left(\delta_{10}^2+\frac{m^2}{r^2}\right)\theta_1^0=\frac{1}{\bar{a}_1^2}\left(u_1^0\frac{\partial T_1}{\partial r}+w_1^0\frac{\partial T_1}{\partial x}\right),$$

where $\delta_{10}^2=k^2-ikPe/\bar{a}_1^2$. Obviously even in zero approach the solutions exist only when the melt temperature gradients by r and x in equilibrium state are functions only of r. General solution of the differential equation array (DEA) (42):

$$u_1^0=-i\left[c_1I_q'(kr)+c_2K_q'(kr)\right],\ w_1^0=c_1I_q(kr)+c_2K_q(kr),$$

$$\theta_1^0=I_m(\delta_{10}r)\left[c_3-\int_0^r B_1^0(r)K_m(\delta_{10}r)dr\right]+ \tag{43}$$

$$+K_m(\delta_{10}r)\left[c_4+\int_0^r B_1^0(r)I_m(\delta_{10}r)dr\right],\ \theta_2^0=c_5I_m(kr)+c_6K_m(kr),$$

where dash means derivative by independent variable in a brackets, c_j $(j=\overline{1,6})$- constants computed from boundary conditions. Here:

$$B_1^0=\frac{1}{\bar{a}_1^2\delta_{10}A_1^0}\sum_{j=1}^{2}\sum_{n=1}^{4}\sum_{k=1}^{\infty}\left\{\alpha_{2n}q_{3-j}J_0(\alpha_{1k}r)\left[c_1I_q(kr)+c_2K_q(kr)\right]+\right.$$

$$\left.-i\alpha_{1k}J_0'(\alpha_{1k}r)\left[c_1I_q'(kr)+c_2K_q'(kr)\right]\right\}d_{jk}\exp(q_{3-j}\alpha_{2n}x),$$

$$A_1^0=I_m(\delta_{10}r)K_m'(\delta_{10}r)-I_m'(\delta_{10}r)K_m(\delta_{10}r), \tag{44}$$

$q=m\sqrt{\dfrac{\omega-kPe}{\omega-mPe}}$. From the equations obtained is seen that in a resonance $\omega=kPe$ is $q=0$, and by $k=m$ results $q=m$. By $\omega=mPe$ due to properties of I_q follows $w_1^0\approx 0$ by all values kr, except $kr=\infty$.

$B_1^0(r)$ can be only slowly changing function x, then for example at $Pe\gg 1$ it is necessary to keep in (44) only two values of α_2: $\alpha_2=\pm 0{,}5\left(Pe-\sqrt{Pe^2+4\alpha_1^2}\right)$, having equated zero coefficients at two other values of α_2 ($T_{1x}^0=0$ in expression (41)).

Boundary conditions (38)-(40) in zero approach:

$$r=0,\quad u_1^0=\theta_1^0=0; \tag{45}$$

$$r=1,\quad u_1^0=0, \tag{46}$$

$$\theta_2^0=\chi_2^0\zeta=\zeta\sum_{j=1}^{2}\sum_{k=1}^{\infty}\sum_{n=1}^{4}\frac{\alpha_{2nk}b_{jnk}}{\sin(\alpha_{2nk}s)}\exp\left[(-1)^{j+1}\alpha_{2nk}x\right],$$

$$\theta_1^0=\chi_1^0\zeta=-\zeta\sum_{j=1}^{2}\sum_{k=1}^{\infty}\sum_{n=1}^{4}\alpha_{1k}d_{jk}J_1(\alpha_{1k})\exp(q_{3-j}\alpha_{2n}x),$$

$$\bar{\kappa}_2\frac{d\theta_2^0}{dr}-\frac{d\theta_1^0}{dr}=\zeta\frac{Pe}{\bar{a}_1^2}\left(\frac{\partial T_1}{\partial x}\right)_{r=1}-\lambda R_\lambda\zeta;$$

$$r=s,\quad d\theta_2^0/dr=-Bi_{k,m}\theta_2^0 \tag{47}$$

Analysing the equations and boundary conditions one must note that physically substantiated condition $u_1(0)=0$ $(u_1^0=0)$ does not request zero values of two other velocity components at the symmetry axis of the channel.

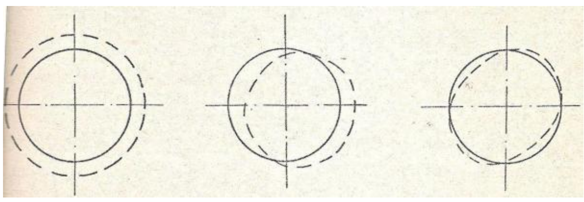
Substituting solution (43) into boundary conditions (45), (46), we get $u_1^0\equiv 0,\ w_1^0\equiv 0,$ so that in zero approach the melt velocity perturbations are absent, while for other constants is got:

$$c_3=\frac{\chi_1^0\zeta}{I_m(\delta_{10})},\quad c_5=-\frac{K_m'(ks)k+Bi_{k,m}K_m(ks)}{A_4k+A_3Bi_{k,m}}\chi_2^0\zeta,$$

$$c_6=-\frac{I_m'(ks)k+Bi_{k,m}I_m(ks)}{A_4k+A_3Bi_{k,m}}\chi_2^0\zeta,\quad c_4=0,\qquad (48)$$

By $m=0$ (axisymmetrical perturbations) yields $\theta_1^0\equiv 0$, what contradicts to boundary condition (46). This requires more detail analysis of the boundary condition (45) for θ_1. Let analyze it accounting the situations shown in Fig.3. For symmetric modes, apparently, physically reasonable is an absence of the perturbed heat flux on a channel axis, especially for axisymmetric perturbations ($m=0$) as mutually opposite equal in size fluxes are mutually compensated. For perturbations with $m\neq 0$ both boundary conditions are insufficiently physically substantiated, therefore approximately it is possible to state a condition of absence of perturbations on axis owing to their small amplitude and remoteness of an axis from boundary of phase transition, on which also perturbations are small:

$$r=0,\quad m=0,\ d\theta_1/dr=0.\qquad (49)$$



$m=0$ $m=1$ $m=2$

Fig.3 Symmetric and antisymmetric modes of perturbations by circular coordinate φ

Considering asymptotic estimates of the modified Bessel and Hankel functions at the arguments approaching to zero, we get $I_m'(0)=0$, $K_0'(0)=\infty$, owing to what with account (42) solution is obtained in the same form but temperature perturbation of symmetric and antisymmetric modes on an axis, generally speaking, isn't zero: $\theta_1^0(0)=\chi_1^0\xi/I_0(\delta_{10})$. From this follows that temperature perturbation on the boundary of phase transition in relation to the corresponding value on axis of the channel makes $\Delta_\theta=I_0(\delta_{10})$. At $Pe\ll k$ it turns out $\delta_{10}\approx k,$ therefore yields:

$$k\ll 1,\quad \Delta_\theta\approx 1;\qquad k\gg 1,\quad \Delta_\theta\approx\sum_{n=0}^{\infty}(k/2)^{2n}/(n!)^2,$$

where from seen that in case of long-wave perturbations by slow melt flow the temperature oscillations on the boundary of phase transition and on the axis are approximately equal, while short-wave perturbations rapidly fade by power low on removal from the front of crystallization.

By $Pe\gg k$, approximate correlation may be used

$$\delta_{10}=\sqrt{kPe}\left[\cos(\pi/4+\pi l)-i\sin(\pi/4+\pi l)\right],\ l=0,1,2,3,...,$$

where from seen that the waves on the boundary and on axis are stronger and may be substantial even by $k\ll 1,$ if $kPe\gg 1$.

4 Conclusion

At the insignificant forced convection in case of long-wave perturbations of boundaries of phase transition, the temperature oscillations in all area can be of the same order, and short-wave perturbations of boundary always fade near boundary (the higher is Pe, the more strongly they fade). The models obtained are useful for analysis of the instability and stabilization of front crystallization in cylindrical channel.

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