Strong and Weak Interactions in Stochastic Systems of Nonlinear Mechanics

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Abstract: - Sensitivity Analysis is crucial both in the modelling phase and in the interpretation of model results. In order to reduce computational costs in structural global sensitivity analysis, it has been suggested to utilize approximate response functions for sensitivity assessment. In the present paper, the approximation of nonlinear finite element model is presented by the polynomial function which conserves all higher order interactions among input quantities and the output of the model. The application of response functions for global sensitivity assessment is demonstrated on the analysis of output of the nonlinear simulation model of structural mechanics.

Key-Words: - Sensitivity analysis, simulation, computer experiment, stability, buckling, imperfections, steel

1 Introduction

The solution of numerous research problems lies in creating specialized models to encourage the decision making. With regard to the complexity of calculation models, it is not possible for an analyst to understand the reaction of a model system to changes in model inputs by intuition. The diversity of solved problems based on scientific modeling led, in recent years, to development of various methods of sensitivity analysis.

The reviews of sensitivity methods in interdisciplinary contexts were published in [1-6]. The majority of sensitivity analyses of simulation models (also called computer codes), treating the simulation model as a black box; i.e., the sensitivity analyses use only the Input/Output (I/O) data of the simulation model (not the internal variables and functions) [7].

In general, the models of structural mechanics are based on the finite element method (FEM). The complexity of nonlinear FEM models is usually large, and obtaining the outputs on contemporary computers can take minutes, even hours. If the input quantities are random, so the stochastic calculation model is in our mind. Although the demand of such a model for computing power can be high, the number of input random quantities can be low. In such a case, it is highly effective to approximate the response of the model by a polynomial. Global sensitivity analysis [7] usually requires the use of polynomial of higher order than structural reliability assessment based on the estimation of probabilities of failure [8]. The sensitivity analysis of static resistance of a slender imperfect member under compression the buckling of which is partly limited by bracing stiffness, is elaborated in the presented paper. The paper links up with recently published sensitivity [9] and reliability [10] analyses, focused on stability problems of slender steel structures. All the calculations presented in this paper were carried out by using the software created by the author of this paper.

2 Stability system with interactions

In the paper, the influence of initial imperfections on the resistance of the strut which is supported, in the half of its length, by the tensile stiffness K. The stiffness partly prevents the buckling. The strut length is L=6m, cross section is HEA 200. The support with stiffness K influences the buckling length L_{cr} of compress strut, which changes from $0.5 \cdot L$ to L. The stiffness in the half of beam models the link of the beam with neighbouring structure.

2.1 Initial imperfections

The initial beam curvature was chosen in the shape of one half-vawe of the function sinus with amplitude the mean value of which is zero, and standard deviation, L/1307=6m/1307=0.00459m. The standard deviation fulfills the assumption that 95 % realizations of random imperfections are within the interval $\pm L \cdot 0.15$ % = $\pm 6 \cdot 0.15 = \pm 9$ mm; this is a frequently considered criterion in reliability analyses.

Table 1: Input random imperfections.

Characteristic	Mean value	St. deviation
Cross-sectional height h	190 mm	0.0 mm
Cross-sectional width b	200 mm	1.9736 mm
Web thickness t_1	6.5 mm	0.0 mm
Flange thickness t_2	10.0 mm	0.45859 mm
Yield strength f_y	297.3 MPa	16.8 mm
Young's modulus E	210 GPa	10 GPa
Amplitude e_0	0	4.59 mm

Deviations from nominal values of the dimensions of hot-rolled steel cross sections were published in [11]. In the presented study, the profile HEA 200 was considered, which is current structural element for columns. Standard deviations of cross-sectional height and web thickness were considered by the zero value, because the influence of their variability on the resistance is low [12]. Statistical characteristics of yield strength and of Young's modulus were considered analogously as in the studies, see, e.g., [13], [14]. All inputs v Table 1 have Gauss probability density functions.

2.2 Sensitivity approach

Sensitivity analysis is crucial both in the modelling phase and in the interpretation of model results. From a general, scientific perspective, sensitivity analysis is the set of methods that allow us to understand key insights of scientific codes: "The judicious application of sensitivity analysis techniques appears to be the key ingredient needed to draw out the maximum capabilities of mathematical modelling" [15].

2.2.1 Correlation coefficients

If stochastic systems with interactions are in our mind, the evaluation of sensitivity analysis in the form of correlation coefficients is inappropriate in general. At the same time, it is the same whether the correlation is evaluated by using the Pearson correlation coefficient or the Spearman's rank correlation coefficient or the Kendall's tau coefficient. The correlation is the method of the first application, where there is the need to identify the dependence between the input and output of additive calculation models the output of which is monotonously dependent on each input quantity [2].

2.2.2 Sobol' decomposition theory

Theoretical background of the Sobol decomposition, and its practical significance for many branches of research are published in [2]. The basic formula describing the influence of random quantity Y_i on output quantity Z is usually written in the form (1).

$$S_i = \frac{V(E(Z|Y_i))}{V(Z)} \tag{1}$$

From the Sobol decomposition, the formula (2) can be further written which evaluates the interaction between the pairs of input quantities Y_i and Y_j on the model output Z. The interaction effect is important for understanding the calculation models in which it is not possible to substitute the effect of all the output quantities by the sum of effects of quantities Y_i on the model output Z. The second order sensitivity index is:

$$S_{ij} = \frac{V\left(E\left(Z|Y_i, Y_j\right)\right)}{V(Z)} - S_i - S_j$$
⁽²⁾

The influence of the groups of three on the output *Z* is written analogously by S_{ijk} etc. The sum of all the indices which the Sobol decompotion consists of, is expressed by relation (3). The Sobol decomposition theory does not require the inputs with Gauss probability density functions. When substituting the number of input quantities for the variable *M*, the total number of sensitivity indices in relation (3) is 2^{M} -1.

$$\sum_{i} S_{i} + \sum_{i} \sum_{j>i} S_{ij} + \sum_{i} \sum_{j>i} \sum_{k>j} S_{ijk} + \dots + S_{123\dots M} = 1 \quad (3)$$

2.3 Computational model

The strut was modelled by means of the method of beam finite elements. The resistance was calculated by geometrically nonlinear solution [14]. The magnitude of load step was decreased according to the approach to resistance. The resistance was calculated with accuracy 0.1 %; it manifested itself as satisfactory in many studies, e.g. [16]. Details concerning geometrically nonlinear solution and matrices of tangential stiffness are published in [12].

2.4 Polynomial approximation

A large number of simulation runs is needed to evaluate the sensitivity analysis. It requires that the calculation of resistances proceeds on computer rapidly. The rapid response can be reached by approximation of resistance by polynomial. The approximation was elaborated for five input random quantities. The resistance was approximated by the polynomial which contained linear and quadratic terms, and the absolute term c_0 (4).

$$Z = c_{0} + \sum_{a=1}^{2} \sum_{i=1}^{5} c_{\alpha} Y_{i}^{a} + \sum_{a=1}^{2} \sum_{b=1}^{5} \sum_{i=1}^{5} c_{\alpha} \cdot Y_{i}^{a} \cdot Y_{j}^{b}$$

$$+ \sum_{a=1}^{2} \sum_{b=1}^{2} \sum_{c=1}^{2} \sum_{i=1}^{5} \sum_{j=i+1}^{5} \sum_{k=j+1}^{5} c_{\alpha} \cdot Y_{i}^{a} \cdot Y_{j}^{b} \cdot Y_{k}^{c} \qquad (4)$$

$$+ \sum_{a=1}^{2} \sum_{b=1}^{2} \sum_{c=1}^{2} \sum_{d=1}^{2} \sum_{i=1}^{5} \sum_{j=i+1}^{5} \sum_{k=j+1}^{5} \sum_{l=k+1}^{5} c_{\alpha} \cdot Y_{i}^{a} \cdot Y_{j}^{b} \cdot Y_{k}^{c} \cdot Y_{l}^{d}$$

$$+ \sum_{a=1}^{2} \sum_{b=1}^{2} \sum_{c=1}^{2} \sum_{d=1}^{2} \sum_{e=1}^{2} c_{\alpha} \cdot Y_{1}^{a} \cdot Y_{2}^{b} \cdot Y_{3}^{c} \cdot Y_{4}^{d} \cdot Y_{5}^{e}$$

where c_{α} are two hundred and forty polynomial coefficients, which are calculated by the method of the least squares. For calculation of the polynomial regression, four hundred simulation runs of Latin Hypercube Sampling method (LHS) were applied [17], [18]. The first hundred simulation runs were considered according to Table 1. Other three hundred simulation runs were considered according to Table 2. All the quantities in Table 2 have rectangular probability density functions. Standard deviations of random quantities presented in Table 2 were artificially magnified so that the domain of definition of the polynomial (4) was satisfactory for evaluation of the sensitivity analysis by the LHS method. For the construction of polynomial, and subsequently for the evaluation of sensitivity analysis, random realizations e_0 were considered by absolute value.

Table 2: Random characteristics for approximation

Characteristic	Min. value	Max. value
Sect. height h	190 mm	0.0 mm
Sect. width b	190.344mm	209.6551 mm
Web thickness t_1	6.5 mm	0 mm
Flange thick. t_2	7.75665mm	12.24335MPa
Yield strength f_y	215.11259 MPa	379.48741GPa
Young's m. E	161.07893 GPa	258.9211 GPa
Amplitude e_0	-22.46337 mm	22.46337 mm

2.5 Sensitivity analysis results

Software to generate random realizations LHS and to calculate the Sobol sensitivity indices in (3) was created by the author of the present paper. For the evaluation of $E(Z|Y_i)$ in (1), ten thousand LHS runs, and for the evaluation of $V(E(Z|Y_i))$, other ten

thousand LHS runs were used. Indices S_i are in Fig.1. The variance V(Z) was evaluated with the use of one hundred thousand LHS runs. Analogously, all twenty six sensitivity indices of higher orders were evaluated, see Fig.2. Sensitivity indices were evaluated in dependence on *K* with step 0.1. The polynomial (4) approximates the nonlinear FEM solution with high accuracy, and enables the evaluation of all the sensitivity indices in (3).



Fig.1: First order sensitivity indices



Fig.2: Crucial higher order sensitivity indices

3 Conclusion

The approximation of stochastic calculation model by polynomial with quadratic members was presented. The polynomial reduces computational costs in structural global sensitivity analysis. The global sensitivity analysis presented identifies all stochastic interactions in the calculation model which was approximated by the polynomial. The application of global sensitivity analysis was presented on the example of the analysis of ultimate limit state of slender member under compression.

Analogous studies can be performed for nonlinear dynamic models, see, e.g. [19], [20]. The presented evaluation of global sensitivity analysis is appropriate for the study of response of models of nonlinear structural mechanics in the cases when the number of input random quantities is small.

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