Effects of the individual terms of the lunar potential in the motion of satellites around the moon

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Abstract: The present work study analyzes the influence of non homogeneity of the lunar gravitational field in the orbit of an artificial satellite. The model is based in spherical harmonics, according to the development presented by Konopliv. This model allows us to consider spherical harmonics until degree and order 100. Simulations are performed for the analysis of the following facts: the contribution of each term of the potential; the variation of the perturbation due to the inclination of the orbit; the actuation of the control system; the influence in transfer and correction maneuvers made to minimize the perturbative effects in the artificial satellite orbit. The transfer and correction maneuvers of lunar satellites are simulated using continuous thrust and trajectory control in closed loop.

Key Words: Astrodynamics, Lunar potential, Orbital Motion, Orbital Maneuvers, Satellite, Control System

1. Introduction

The behavior of an artificial satellite in lunar orbit is disturbed mainly by the non homogeneity of the gravitational field of the Moon, which tends to cause variations in the orbital elements of the artificial satellite.

Usually, lunar missions remain in activity for a long period of time, such as the Indian lunar mission Chandrayaan-1, which found evidence of water in the lunar soil, had a life-time of two years. The mission GRAIL, from NASA, which provided data for the preparation of the complete and detailed map of the distribution of the gravity field of the Moon, had a year-long duration.

Considering the long period of activity of the space mission, it is necessary to perform orbital maintenance maneuvers to keep the satellite in a predetermined orbit. To study of the perturbations due to the non-sphericity of the lunar gravitational field, the model LP100K (Konopliv [6]) was inserted into the simulator trajectory STRS, showed in Rocco [8]. This model allows the study of the influence of the degree and order of the gravitational harmonic in the orbit of an artificial lunar satellite. This model has maximum degree and order of 100. Studies and analyzes of such influence, as well as simulations considering the disturbance due to the lunar gravitational potential of the orbit of an artificial satellite, can be found in Gonçalves [5].

The simulator allows the orbital control to be done with the objective of minimizing the maximum deviations and errors using a continuous propulsion system and in a closed loop control. A more detailed description of the orbital trajectory control can be found in Rocco[9].

2. Lunar Gravitacional Potencial

The gravitational potential of the moon is expressed by the coefficients of the normalized spherical harmonics, given by Equation (1) [6], [7]:

$$U(r,\lambda,\phi) = \frac{\mu}{r} + \\ + \frac{\mu}{r}\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n (\bar{C}_{nm}\cos m\lambda + \\ + \bar{S}_{nm}\,\mathrm{sen}\,m\lambda)\,\bar{P}_{nm}\,(\mathrm{sen}\,\phi) \qquad (1)$$

where n is the degree, m is the order, μ is the gravitational constant and r is the lunar equatorial radius. $\overline{P}_{nm}$ are the fully normalized associated Legendre polynomials; a_e is the reference radius of the Moon, ϕ is the latitude, and λ is the longitude.

3. The Model LP100K

The lunar gravitational field was determined by using data from different lunar missions already performed.

Although many models of the lunar gravitational field exist in the literature, our study is limited to the results obtained from the implementation of the model LP100K (Konopliv, [6]) in the trajectory simulator STRS, Rocco [8].

Data for development of the model LP100K were obtained by one of the most important mission, the Lunar Prospector (1998-1999), which is the third mission from NASA's exploration program called Discovery, which provided the first measurement of the gravitational field of the Moon. Since at the time of the data modeling using the LP mission there were no direct observations of the far side of the Moon, the information about ita gravity comes from the long-term effects observed in the satellite orbit.

The model LP100K presented by Konopliv is a representation of the spherical harmonics due to the gravity planetary based in the gravitational potential of the celestial body, given by Equation (1).

The output calculated by the model provides the components x, y and z for the gravity acceleration at each instant of time along the orbit of an artificial satellite. It is possible to consider the spherical harmonics up to degree and order 100. By a comparison between the acceleration of the gravity from a central field and the gravitational acceleration provided by the Konopliv model, it is obtained the velocity increase that the disturber term applies to the satellite, enabling, by means of the inverse problem, to obtain the Keplerian elements that characterize the orbit of the artificial satellite to perform an analysis of the orbital motion.

4. Simulations and Results

Since the objective of this work is to analyze the influence of the gravitational potential in a lunar orbit of an artificial satellite, three studies were performed, as described below.

First we perform a study seeking to analyze the influence of each term of the gravitational potential in the orbit of an artificial satellite. This task was done in an indirect way, obtaining the values of the velocity increments implemented by the N terms of the potential during one orbit of the satellite, and then repeating this calculation for N +1 terms. Making the difference between the velocity increments, the contribution of the addition of one more term in the potential is obtained, and so this idea can be repeated until we can evaluate all the terms.

This study is then made according to the variation of the inclination, in order to show the influence of this orbital element in the orbital perturbation felt by the spacecraft.

The use of the situation where the variation of the orbital elements is used in order to minimize the fuel consumption for orbital maintenance is usual in practice. However, the objective here is to measure the effects of adding each term of the potential and not finding the strategy that minimizes the fuel consumption. A similar idea was done in Prado [10] for a spacecraft that travels around the Earth and it is perturbed by the Moon and the Sun.

After completing the analysis of the influence of the terms of the harmonics of the gravitational potential, in particular its influence for different values of the inclination, four orbital transfer maneuvers are performed.

We analyzed the cases in which only correction maneuvers are performed and the case where transfers and correction maneuvers are performed. For both cases, there were maneuvers to transfer the satellite from a high lunar orbit to a low lunar orbit and maneuvers to transfer the satellite from a low lunar orbit lunar to a high lunar orbit.

In all four simulations, the situations where the control system works only with the purpose of performing orbital transfers and the situations where it operates with the double objective of making orbital transfers and station-keeping are compared. We analyzed the velocity increment applied to the satellite to compensate the disturbing force, the evolution of the orbital elements and the fuel consumption.

4.1 Velocity increment

We begin our study by analyzing the importance of each term of the lunar gravitational potential.

This task is repeated 100 times, each time adding the next term of the gravitational potential, which means that for each term of the lunar gravitational potential, a simulation is made during the period of one orbit. For all simulations the control was not used, since the objective is to analyze the effects of the disturbance due to the gravitational potential in the orbit of the satellite.

Thus, the total variation of velocity applied to the satellite by the disturbing terms is obtained (the integral over the time of the acceleration caused), assuming that the motion is governed only by the potential of the Moon according to the Konopliv's model. This measurement gives the effect of adding a new element to the Moon's gravitational potential, and not the contribution of each term individually. After these simulations are done, the subtracting of the contribution of the previous terms shows how the addition of a new term changes the orbital characteristics.

Figure 1 shows the total change in the velocity due to the non-homogeneity of the distribution of the lunar mass using the dynamics that takes into account N +1 terms of the gravitational potential less the same variation resulting from the gravitational potential considering N terms.

The initial conditions of the orbit are: semi-major axis: 1900000 m; eccentricity: 0.001, inclination: 45°; right ascension of the ascending node: 20°; argument of the periapsis: 100°; mean anomaly: 1°.

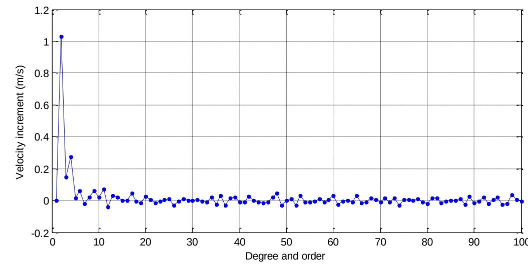

Figure 1- Difference between the velocity variation applied to the spacecraft with N +1 and considering only N terms of lunar potential.

Note the expected domain of the first term of the disturbing potential, as observed in Carvalho [1], [2], [3], [4]. However, it is also possible to see that the results of the next terms are not monotonically decreasing, which means that a term can be of a higher order and still provide a significant contribution to the increase of the velocity of the satellite, as studied by Gonçalves [5]. A similar study can be found in Ramanan [11].

Negative terms appears in Figure 1, meaning that the addition of this term in the lunar gravitational potential decreases the perturbation suffered by the satellite, so reducing the velocity variation required for the maintenance of a Keplerian orbit. In other words, this term helps to control the artificial satellite, because its effect is opposite to the resultant of the previous terms, then in favor of controlling the system.

4.2 Inclination

As seen in Section 4.1, each term of the lunar gravitational potential causes a different influence on the orbit of an artificial satellite.

Our objective now is to analyze the effect of the inclination of the lunar artificial satellite in the orbit perturbation. A study of an artificial satellite perturbed by the gravitational potential for the case of critical inclination is carried out in Carvalho [3], [4].

To perform this task, the values of velocity increment on a lunar artificial satellite due to the perturbing potential were obtained, based in the model presented by Konopliv. To avoid an excessive quantity of results, the simulations were performed considering terms up to degree and order 2, 10, 50 and 100. The results are shown in Figure 2.

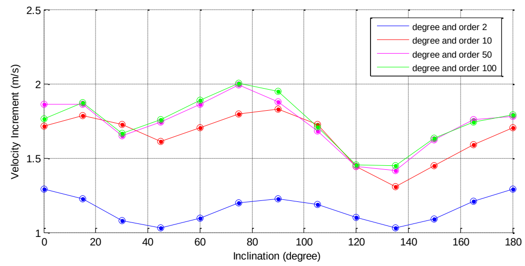

Figure 2 - Velocity variation provided by the disturbance as a function of the inclination of the orbit of the satellite and the number of terms considered for the lunar potential.

Orbits were analyzed with the following values of inclination: 0 º, 15 º, 30 º, 45 º, 60 º, 75 º, 90 º, 105 º, 120 º, 135 º, 150 º, 165 º and 180 º, all with the following initial conditions: semi- major axis: 1900000 m, eccentricity: 0.001; right ascension of the ascending node: 20 º argument of the periapsis: 100 º; mean anomaly: 1 º.

We can notice a significant difference when we consider only degree and order 2 and when terms are considered up to degree and order 10, 50 and 100. It is also noticed that there are values of inclination where the disturbance generated by the lunar gravitational potential is larger. Analyzing the case with more terms (degree and order 100), we conclude that the inclination of 75º has a higher value for the velocity increment, and the inclination of 135º has the lowest value for the velocity increment, with the difference between them of approximately 0.55 m/s, which is very significant in the scale of values obtained, about 25%. It means that the inclination plays an important role in the perturbations received by the satellite.

Note also that the simulations performed with 50 and 100 potential terms are very similar to each other, thus emphasizing the small contribution of the terms between 50 and 100, as expected. Even considering only 10 terms the results are already similar to the cases with 50 and 100 terms, thus demonstrating the strong domination of the first 10 terms. But there are significant differences between considering only the first term and the others until the number 10, so a Keplerian model is not very accurate to study the motion of a satellite around the Moon.

Thus, we can see in Figure 3, the velocity increment for the satellite due to the gravitational potential for the first 10 terms of the spherical harmonics.

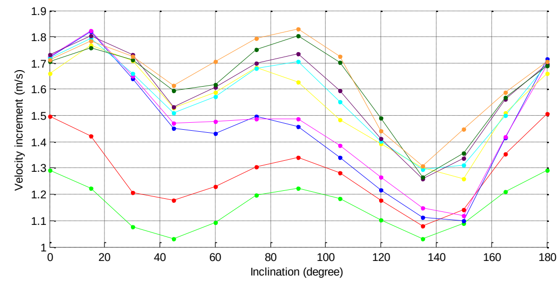

Figure 3 - Velocity variation provided by the disturbance as a function of the inclination of the orbit of the satellite and the number of terms considered for the lunar potential for the first 10 terms.

In the Figure 3, the green light line represents the study realized for degree and order 2, the red line degree and order 3, the blue line degree and order 4, magenta line degree and order 5, yellow line degree and order 6, cyan line degree and order 7, purple line degree and order 8, dark green degree and order 9, and orange line degree and order 10.

4.3 Station-Keeping maneuvers

The following comparative study aims to analyze the influence of the gravitational potential in the orbit of an artificial satellite. To perform this task, four simulations have been made whose initial conditions and characteristics are presented in Table 1.

<table>
<tr><td>Simulation 1</td><td rowspan="2">Semi-major axis: 2600000 m, eccentricity: 0.001, inclination: 45º, ascending node: 20º, argument of the periapsis: 100º, mean anomaly: 1º</td><td>Transfer maneuver</td></tr>
<tr><td>Simulation 2</td><td>Transfer maneuver and station-keeping</td></tr>
<tr><td>Simulation 3</td><td rowspan="2">Semi-major axis: 1900000 m, eccentricity: 0.001, inclination: 45º, ascending node: 20º, argument of the periapsis: 100º, mean anomaly: 1º</td><td>Transfer maneuver</td></tr>
<tr><td>Simulation 4</td><td>Transfer maneuver and station-keeping</td></tr>
</table>

Table 1. Simulations realized

Simulations 1 and 2 perform an orbital transfer of an orbit of altitude around 815 km for an orbit with altitude 100 km, and simulations 3 and 4 perform an orbital transfer from an altitude of 115 km to an orbit with altitude 830 km. Figures 4 and 5 show the behavior of the semi-major axis for the four simulations.

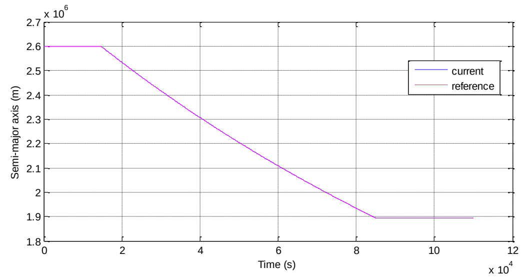

Figure 4 - Behavior of the semi-major axis during simulations 1 and 2.

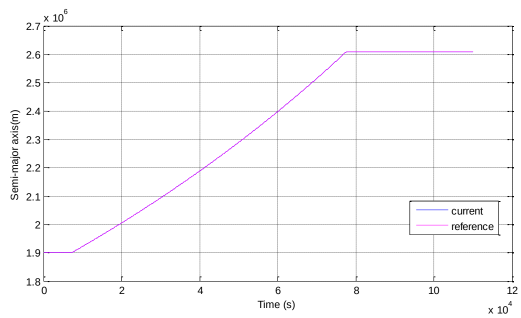

Figure 5 - Behavior of the semi-major axis during simulations 3 and 4.

Figures 4 and 5 show that the objective of the simulations was successfully achieved, since the semi-major axis decreased for simulation 1 and increased for simulation 2. In both cases the thruster is turned on to accomplish the transfer orbit after the satellite performs a complete orbit around the Moon and it is turned off when the satellite reaches the altitude of 100 km for simulation 1 and 815 km of altitude for simulation 2.

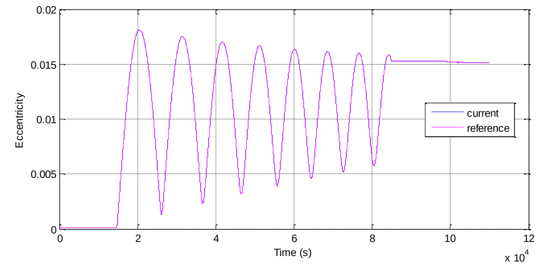

Figure 6 - Behavior of the eccentricity during simulations 1, 2, 3 and 4

In Figure 6 we can see that the orbit remained with low eccentricity, with small variations due to the gravitational potential and the use of thrusters to perform the maneuvers.

Figure

Figures 7 to 10 show the deviation in the semi-major axis and Figures 11 to 14 show the deviations from the eccentricity in the four simulations.

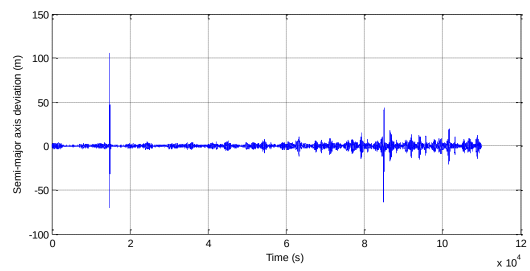

Figure 7 - Semi-major axis deviation during simulation 1.

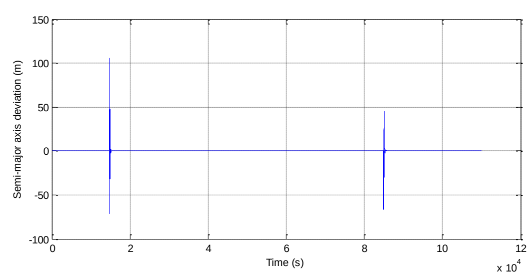

Figure 8 - Semi-major axis deviation during simulation 2.

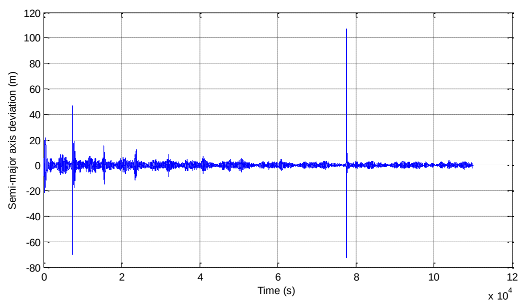

Figure 9 -Semi-major axis deviation during simulation 3.

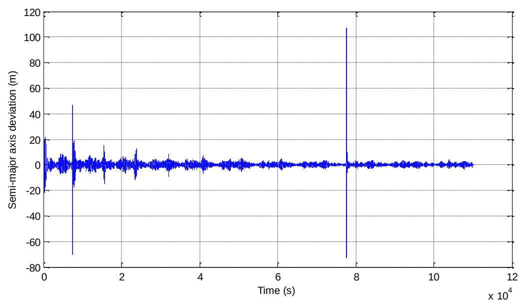

Figure 10 - Semi-major axis deviation during simulation 4.

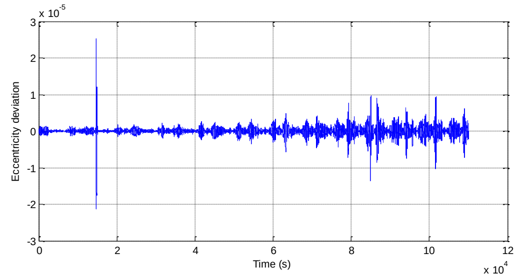

Figure 11 - Eccentricity deviation during simulation 1.

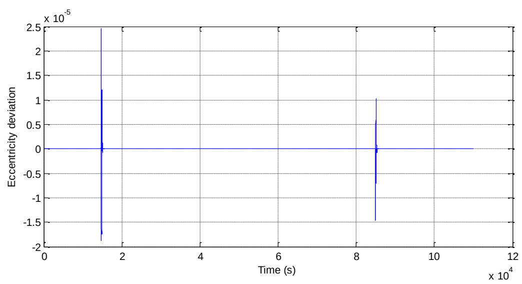

Figure 12 - Eccentricity deviations during simulation 2.

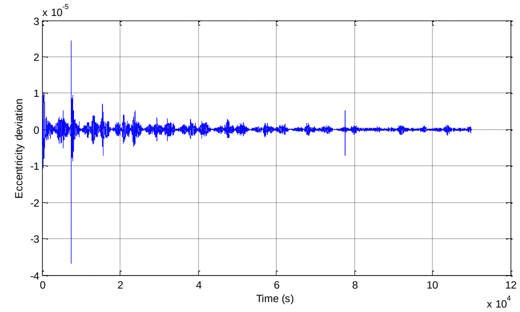

Figure 13 - Eccentricity deviations during simulation 3.

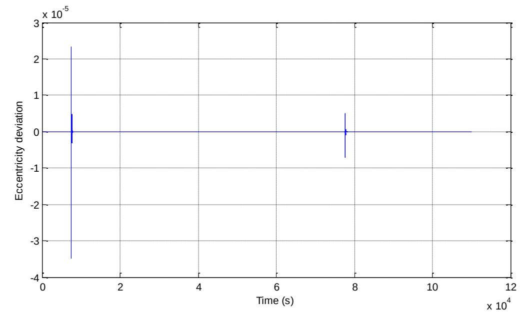

Figure 14 - Eccentricity deviations during simulation 4.

In Figures 8, 10, 12 and 14 we note that the use of the thrusters to perform the correction maneuvers keeps the actual trajectory of the satellite together with the reference trajectory, as expected. The absence of the use of the thrusters for station-keeping generates deviations throughout the simulation, as can be seen in Figures 7, 9, 11 and 13.

The results of the two cases studied and shown in Figures 7 to 14 show a deviation peak at times when the thruster is turned on and off. These spikes tend to stabilize after a sequence of oscillations that occur due to the transitional control system, as can be seen in Figure 15.

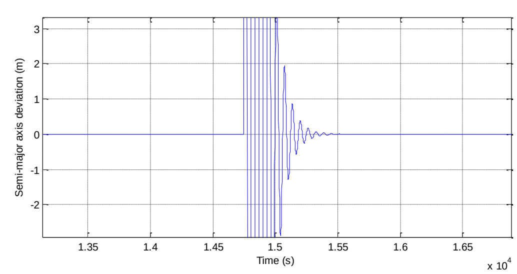

Figure 15 - Transitory control system.

These peaks seen in Figures 7 and 14 mean that there is a difference between the actual trajectory and the reference trajectory, as can be seen in Figure 16, which is a magnification of Figure 5. This difference tends to be corrected by the control system, which operates with the goal of canceling the differences between the trajectories caused by the thrusters.

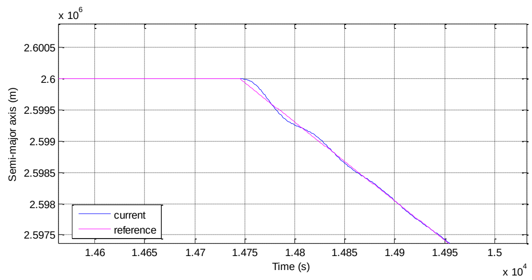

Figure 16 - Magnification of Figure 5.

Figures 17 and 20 show the altitude of the satellite, Figures 18 and 21 show the disturbance due to the lunar gravitational potential and Figures 19 and 22 the resultant force due to the lunar gravitational potential.

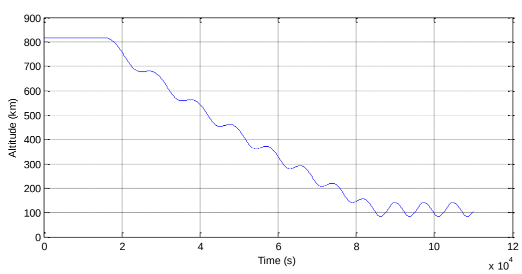

Figure 17 - Altitude of the satellite during simulations 1 and 2.

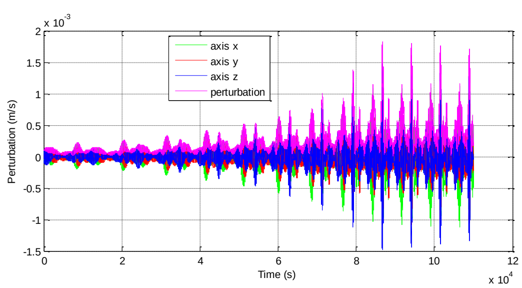

Figure 18 - Perturbations due to the lunar gravitational potential during simulations 1 and 2.

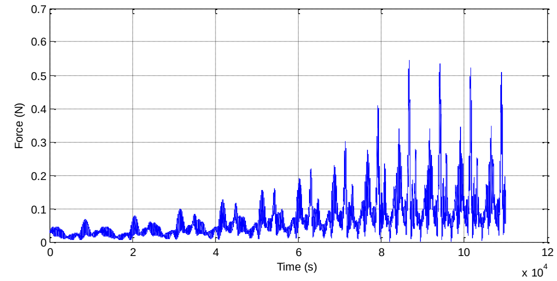

Figure 19 - Perturbations due to the lunar gravitational potential during simulations 1 and 2.

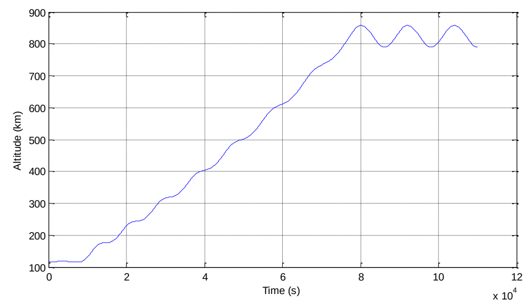

Figure 20 - Altitude of the satellite during simulations 3 and 4.

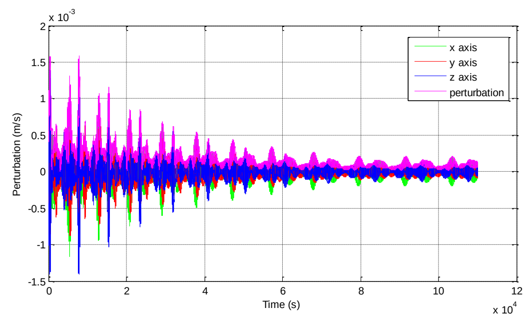

Figure 21 - Perturbations due to the lunar gravitational potential during simulations 3 and 4.

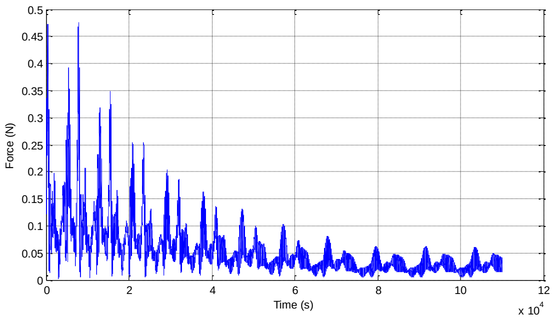

Figure 22 - Perturbations due to the lunar gravitational potential during simulations 3 and 4.

According to the objectives of the simulated missions, we can see in Figure 10 the variations of the altitude of the satellite during the time of the simulations. The satellite had a variation in altitude of approximately 715 km, and it was hovering around the altitude of 100 km for simulations 1 and 2 and 830 km of altitude for simulations 3 and 4, since these were the mission objectives.

From Figures 19 and 22 we see the force due to the perturbation of the gravitational lunar potential satellite, that is, the force that the disturbance makes on the vehicle, which causes the displacement of the satellite from the keplerian trajectory. We can see that the behavior is quite not constant, causing difficulties to the performance of the control system.

It is also observed that the intensity of resultant perturbation on the satellite increases as the satellite approaches the lunar surface and that it decreases when the satellite moves away from the lunar surface. It is also visible that it stabilizes after some time.

In Figures 23 and 24 we can observe the thrust applied during the simulations. We see that the thruster applies force in all the three axes. Therefore, it is possible that the control operates separately on each axis.

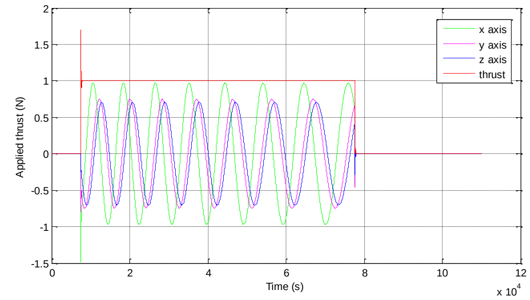

Figure 23 - Thrust applied to the satellite during the simulation without orbital correction.

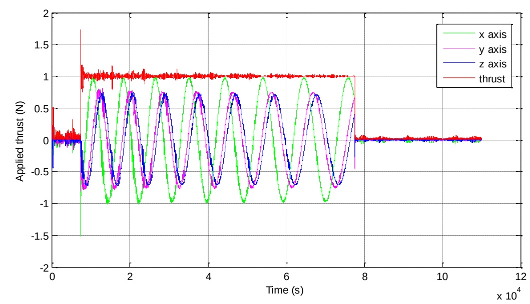

Figure 24 - Thrust applied to the satellite during the simulation with orbital correction.

It is noted in Figures 23 and 24 that, when the maneuver is completed, in accordance with Figures 17 and 20, the altitude reaches the specified value it tend to stabilize. At this time the thruster shuts down. It is also possible to see, in Figures 18 and 21, the inconstancy of the gravitational potential, which requires an intense performance of the control system.

The results of the simulations show that there are differences between the situations when only orbital correction maneuvers and when corrections and transfer maneuvers are performed. For the simulations 1 and 2, the fuel consumption is around 2.8685 kg for the case when the orbital corrections are not performed and consumption of 2.9273 kg when the orbital correction maneuvers are also performed. It means that there is a difference of 0.0588 kg of propellant. For the simulations 3 and 4, the consumption is 2,8641 kg for the first case and 2,8953 kg for the second case, so there is a difference of 0,0312 kg of propellant.

This small difference occurs because, in addition to fuel consumed for the transfer maneuver, there is still a fuel consumption used to minimize the effects of perturbation on the trajectory of the artificial satellite.

The Figures 25 and 26 show de velocity increment in the simulations 1 and 2, where we can justify the increased fuel consumption when performed correction maneuvers. It is possible to see in the Figure 26 that the velocity increment continues to increase when the transfer manuever terminates.

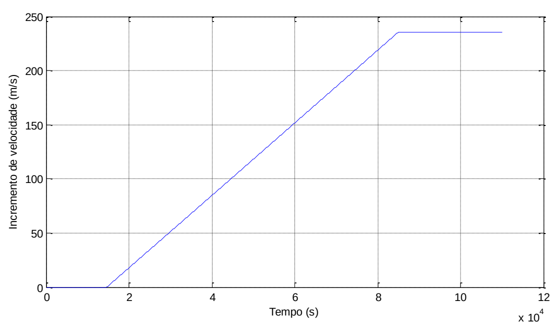

Figure 25 - Thrust applied to the satellite during the simulation without orbital correction.

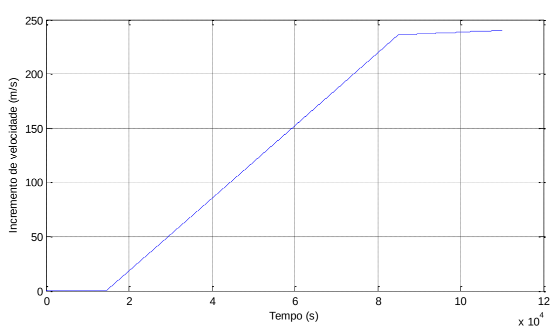

Figure 26 - Thrust applied to the satellite during the simulation with orbital correction.

4. Conclusions

The results of this study are consistent with the model developed by Konopliv (2001), which shows the correlation between the lunar gravitational acceleration and the topography of the Moon, showing the fluctuation of the gravity acceleration due to the non-uniform distribution of mass of the Moon.

The study performed here analyzed the effects of each term of the gravitational potential in the orbit of an artificial lunar satellite. The results showed that the first term (degree and order 2) dominates the motion, but the contributions of the other terms are relevant, especially the first 10 terms. Terms higher than 10 are responsible for smaller contributions.

Negative values appear in this study, showing that some terms act in the opposite direction with respect to the resultant of the previous terms, so helping to maintain the artificial satellite orbit near a keplerian orbit.

The study of different values for the inclination showed that the variation of the inclination may change the perturbation suffered by the artificial satellite significantly, so those possibilities must have been taking into account when planning a mission.

Those studies show that the disturbance in an artificial satellite due to non uniform distribution of the mass of the Moon is significantly inconstant, requiring an intense performance of the control system to attenuate the deviations caused in trajectory.

We can observe that the deviations in the state variables were always small, that is, the control system was able to reduce the error in the state variables by the action of the thrusters.

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