Slip Effects on MHD Non-Darcy Boundary layer Flow over a Stretching Sheet in a Porous Medium with radiation and Ohmic Dissipation

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Abstract: - In this study, MHD boundary layer slip flow of an incompressible fluid over a stretching sheet in Darcy-Forchheimer porous medium are investigated numerically. Analysis has been carried out in the presence of thermal radiation and ohmic dissipation. Velocity and thermal slips are considered instead of no-slip conditions at the boundary. The governing boundary layer equations along with the boundary conditions are transformed into a dimensionless form by a similarity transformation and the resulting coupled ordinary differential equations are then solved by shooting method. The effects of governing parameters on the flow and thermal fields are examined. The skin friction and wall temperature gradient with effects of slip parameter are reported graphically for various parametric conditions to show interesting aspect of the numerical solution.

Key-Words: - Boundary layer; heat transfer; stretching surface; slip parameters

1 Introduction

In recent years, the study of boundary layer flow behavior and heat characteristics has been investigated and extended by several authors [1-4] for different physical situation. The flow and heat transfer due to a stretching surface in viscous fluid is of great practical interest because it occurs in a number of engineering processes i.e. in the polymer industry when a polymer sheet is extruded continuously from a die, with a tacit assumption that the sheet is inextensible. During its manufacturing process a stretched sheet interacts with the ambient fluid both thermally and mechanically. The thermal interaction is governed by the surface flux. This quantity can either be prescribed or it is the output of a process in which the surface temperature distribution has been prescribed. The cooling of a large metallic plate in the bath (an electrolyte) is another problem belonging to this category.

However, in real situation one has encountered the boundary layer flow over the stretching sheet. For example, in a melt-spinning process, the extrudate is stretched into a filament or sheet while it is drawn from the die. Finally, this sheet solidifies while it passes through effectively controlled cooling system in order to acquire the top-grade property of the final product. The quality of final product depends on the rate of heat transfer at the stretching surface. The problem of flow due to a stretching sheet has been studied to many flow situations. Crane [1] was the first to examine the problem of steady two-dimensional boundary layer flow of an incompressible and viscous fluid caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point on the sheet. Pal and Mondal [2] have provided comprehensive discussion on hydromagnetic non-darcy flow and heat transfer over a stretching sheet in presence of thermal radiation and ohmic dissipation. Qasim et al. [3] examined the effects of slip conditions on stretching boundary layer heat transfer with thermal radiation.

Mukhopadhyay [4] numerically studied the mixed convection boundary layer flow along a stretching cylinder in porous medium. Combined effects of non-uniform heat source/sink and thermal radiation on heat transfer over an unsteady stretching permeable surface was presented by Pal [5]. Recently, Olenrewaju [6] analyzed the effects of internal heat generation on the hydromagnetic non-darcy flow and heat transfer over a stretching sheet in the presence of thermal radiation and ohmic dissipation. Pal and Modal [7] analyzed the effects of heat source/sink, variable viscosity and soret on non-Darcy MHD mixed convection boundary layer over a stretching sheet embedded in a fluid saturated porous media.

Therefore, the purpose of the present study is to study the slip effects on MHD Darcy-Forchheimer convective flow over a permeable stretching sheet in the presence of thermal radiation and Ohmic dissipation. It extends, in fact, the papers by Pal and Mondal [2] to the case of slip flow. Numerical results are compared with those of Pal and Mondal [2] for special cases, and are presented in tables.

2 Mathematical Formulation

We investigate the two-dimensional steady incompressible electrically conducting fluid flow over a continuous stretching sheet embedded in a porous medium. The flow region is exposed under uniform transverse magnetics fields $\vec{B} = (0, B_0, 0)$ and uniform electric field $\vec{E} = (0,0,-E_0)$. Since such imposition of electric and magnetic fields stabilizes the boundary layer flow . It is assumed that the flow is generated by stretching of an elastic boundary sheet from a slit by imposing two equal and opposite forces in such a way that velocity of the boundary sheet is of linear order of the flow direction. We know from Maxwell's equation that $\nabla.\vec{B} = 0$ and $\nabla.\vec{E} = 0$. When magnetic field is not so strong then electric field and magnetic field obey Ohm's Law $\vec{J} = \sigma(\vec{E} + \vec{q} \times \vec{B})$, where $\vec{J}$ is the Joule current, σ is the magnetic permeability and $\vec{q}$ is the fluid velocity. We assume that magnetic Reynolds number of the fluid is small so that induced magnetic field and Hall effect may be neglected.

We take into account of magnetic field effect as well as electric field in momentum and thermal boundary layer equations. Under the above stated physical situation, the governing boundary layer equations for momentum and energy under Boussinesq's approximation and the governing and viscous and Ohmic dissipations are

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma}{\rho}(E_0 B_0 - B_0^2 u) - \frac{\nu}{k}u - Fu^2 \quad (2)$$

$$\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma}{\rho C_p}(uB_0 - E_0)^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \quad (3)$$

where u and v are the velocity components in the x and y directions, respectively; ν is the kinematic viscosity; ρ is the density of the fluid; k is the permeability of the porous medium; q_r is the radiative heat flux in the y-direction; F is the empirical constant (Forchheimer number) in the second-order resistance and setting $F = 0$ in Eq. (2), the equation is then reduced to the Darcy's law, C_p is the specific heat at constant pressure and κ is the thermal conductivity. Thermal boundary layer Eq.(3) takes into account the Joule heating or Othmic dissipation due to the magnetic as well as electric fields The third and fourth terms on the right hand side of Eq. (2) stand for the first-order (Darcy) resistance and second-order porous inertia resistance, respectively. It is further assumed that the normal stress is of the same order of magnitude as that of the shear stress in addition to usual boundary layer approximations for deriving the momentum boundary layer Eq. (2) . Thermal boundary layer Eq. (3) takes into account the joule heating or Ohmic dissipation due to the magnetic as well as electric field and the internal heat generation. The appropriate boundary conditions are put into the following forms

$$u = U_w + N_0\frac{\partial u}{\partial y}, \quad v = 0,$$
$$T = T_w + K_0\frac{\partial T}{\partial y}, \quad \text{at } y = 0, \quad (4)$$

$$u \to 0, \; T \to T_\infty \text{ as } y \to \infty. \quad (5)$$

Here N_0 is the velocity slip factor and K_0 is the thermal slip factor . We now introduce the following similarity transformation and dimensionless stream function and temperature as follows

$$u = bxf'(\eta), v = -\sqrt{b\nu}f(\eta), \psi = x\sqrt{b\nu}\, f(\eta),$$
$$\eta = \sqrt{\frac{b}{\nu}}y, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (6)$$

Here, $f(\eta)$ is the dimensionless stream function and η is the similarity variable. Substituting Eq.(6) into (2) and (3), we obtain

$$f''' + ff'' - f'^2 + Ha^2(E_1 - f') - k_1 f' - F^* f'^2 = 0, \quad (7)$$

$$\frac{1+Nr}{Pr}\theta'' + (f\theta' - 2f'\theta) + E_c f''^2 + E_c Ha^2 (f' - E_1)^2 = 0, \quad (8)$$

where $k_1 = \nu/kb$ is the porous medium parameter, $Ha = B_0\sqrt{\sigma/\rho b}$ is Hartmann number, $E_1 = E_0/B_0 b_x$ is the local electric parameter, $F^* = Fx$ is the local inertia-coefficient, and $Re_x = U_x/\nu$, $Pr = \nu/\alpha$ is Prandtl number, $E_c = b^2 l^2 / A_0 C_p$ is Eckert number and $Nr = 16\sigma^* T_\alpha^3 / 3k\kappa$ is the thermal radiation parameter. The boundary conditions (4) and (5) take the form

$$f(0) = 0, \quad f'(0) = 1 + \delta f''(0), \quad \theta(0) = 1 + \beta\theta'(0) \quad (9)$$

$$f'(\eta) \to 0, \quad \theta(\eta) \to 0, \quad \eta \to \infty \quad (10)$$

where δ and β are the velocity slip and thermal slip parameters, respectively.

3 Results and Discussion

Equations (7) and (8) subjects to the boundary conditions (9) and (10) have been solved numerically by using the shooting method. The results are given to carry out a parametric study showing the influences of the non-dimensional parameters, namely the velocity slip parameter δ, thermal slip parameter parameter β, porous medium parameter k_1, the inertial parameter F^* and other physical quantities of interest. In order to validate the present results, we have compared them with those Pal and Mondal [2] for the no-slip case $\delta = \beta = 0$ as shown in Table 1. Table 2 gives the values of skin friction $f''(0)$ and wall temperature gradient $-\theta'(0)$ for several values of velocity slip parameter δ when $\beta = 0$ (no-slip) and $\beta = 0.1$. It is noticed that as the velocity slip parameter δ increases, this leads to significant changes in the values of skin friction $f''(0)$ and wall temperature gradient $-\theta'(0)$. The values of wall temperature gradient $-\theta'(0)$ for several values of thermal slip parameter parameter β when $\delta = 0$ (no-slip) and $\delta = 0.1$ are presented in Table 3. The values of $-\theta'(0)$ as shown in Table 3 decrease with the increasing values of velocity slip parameter δ. The effects of δ and β on the wall temperature gradient $-\theta'(0)$ is displayed in Fig. 1 and 2. It is observed that for a particular value of δ the wall temperature gradient $-\theta'(0)$ is decreased as the thermal slip parameter β is increased. The variations of wall temperature gradient $-\theta'(0)$ for several values of Hartmann number *Ha* are presented in Figs 3 and 4, respectively. As can be seen from Fig. 3, for a particular value of *Ha*, the values of wall temperature gradient $-\theta'(0)$ decreases as the velocity slip parameter δ increases. The same behavior is observed in Fig. 4, the wall temperature gradient $-\theta'(0)$ is decreased with increasing thermal slip parameter. Figs. 5 to 7 present some samples of velocity and temperature profiles for different values of δ and β. With the increasing values of δ and β, the fluid velocity increases monotonically. Due to slip condition at the plate the velocity of fluid adjacent to the plate has some positive value and accordingly the thickness of momentum boundary layer decreases. These figures also show that the boundary conditions (9) and (10) are satisfied.

4 Conclusion

The present work deals with the the non-Darcy fluid flow and heat transfer over a stretching sheet embedded in porous media with thermal radiation and ohmic dissipation as considered by Pal and Mondal [5]. We have extended the previous work by taking into consideration slip conditions at the boundary. The governing equations are transformed into ordinary differential equations using similarity transformation and are then solved numerically using the shooting method. The effects of the velocity slip and thermal slip parameters and some values of the physical parameters on the flow and heat transfer characteristics are studied. In general, the increase of velocity slip and thermal slip parameters reduces the momentum boundary layer thickness and also enhances the heat transfer from the plate.

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Table 1: Comparison of wall temperature gradient, $-\theta'(0)$ for *Ha*=0, λ=0 and various values of Pr.

Pr	Pal and Mondal [2]	Present
1.0	1.333333	1.333334
2.0	1.999996	1.999923
3.0	2.509715	2.509659
4.0	2.938782	2.938723
5.0	3.316479	3.316423
6.0	3.657769	3.657714
7.0	3.971509	3.971455
8.0	4.263457	4.263404
9.0	4.537609	4.537557
10.0	4.796871	4.796819

Table 2: Values of skin friction $-f''(0)$ and wall temperature gradient $-\theta'(0)$ for various values of β and δ when $Ha = 1.0$, $E_c = 1.0$, $E_1 = 1.0$, Pr = 3.0, $k_1 = 0.2$, $Nr = 0.1$ and $F^* = 0.1$.

		$\beta = 0.0$	$\beta = 0.1$
δ	$-f''(0)$	$-\theta'(0)$	$-\theta'(0)$
0.0	1.427441	0.692803	0.564245
0.1	1.206843	0.934193	0.770161
0.2	1.049681	1.070412	0.890987
0.3	0.931117	1.151235	0.965854
0.4	0.838018	1.200160	1.013620
0.5	0.762730	1.229567	1.044409
0.6	0.700440	1.246504	1.064090
0.7	0.647960	1.255184	1.076229
0.8	0.603082	1.258397	1.083200
0.9	0.564227	1.257781	1.086501
1.0	0.530232	1.254549	1.087181

Table 3: Values of wall temperature gradient $-\theta'(0)$ with $Ha = 0.1$, $E_c = 1.0$, $E_1 = 1.0$, Pr = 3.0, $k_1 = 0.2$, $Nr = 0.1$ and $F^* = 0.1$ with various velocity slip parameter δ against temperature slip β .

	$\delta = 0.0$	$\delta = 0.1$
β	$-\theta'(0)$	$-\theta'(0)$
0.0	0.692803	0.564205
0.1	0.564245	0.770161
0.2	0.475931	0.655097
0.3	0.411520	0.569966
0.4	0.362466	0.504417
0.5	0.323861	0.452389
0.6	0.292687	0.410091
0.7	0.266988	0.375026
0.8	0.245438	0.345485
0.9	0.227107	0.320258
1.0	0.211323	0.2984650

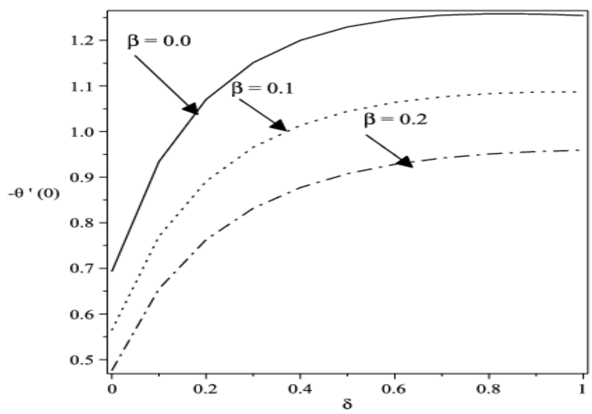

Fig.1 : Variation of wall temperature gradient $-\theta'(0)$ with δ when $Ha = 1.0$, $E_c = 1.0$, $E_1 = 1.0$, $k_1 = 0.2$ Pr = 3.0 and $F^* = 0.1$ for several values of β.

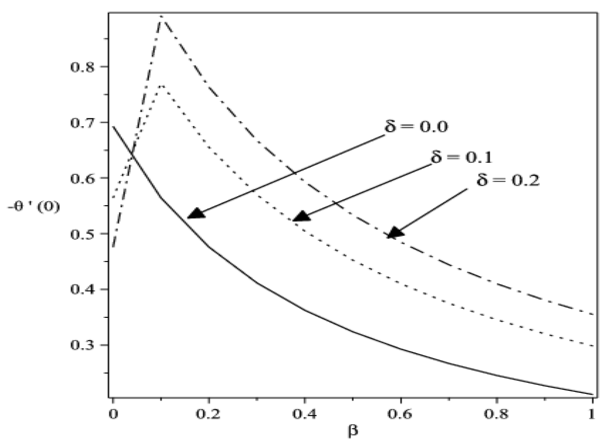

Fig.2 : Variation of wall temperature gradient $-\theta'(0)$ with β when $Ha = 1.0$, $E_c = 1.0$, $E_1 = 1.0$, $k_1 = 0.2$, Pr = 3.0 and $F^* = 0.1$ for several values of δ.

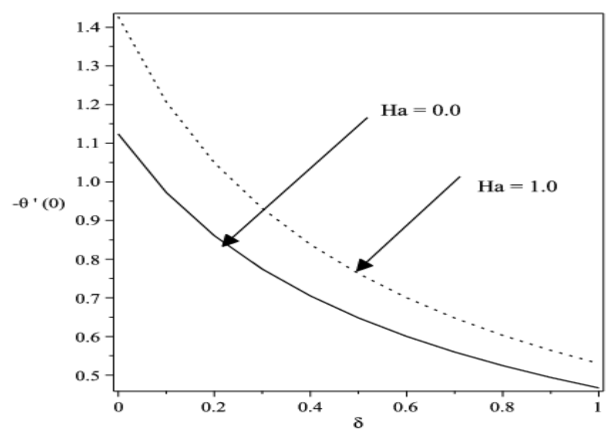

Fig.3: Variation of wall temperature gradient $-\theta'(0)$ with δ when $\beta = 0.1$, $E_c = 1.0$, $E_1 = 1.0$, Pr = 3.0 and $F^* = 0.1$ for several values of Ha.

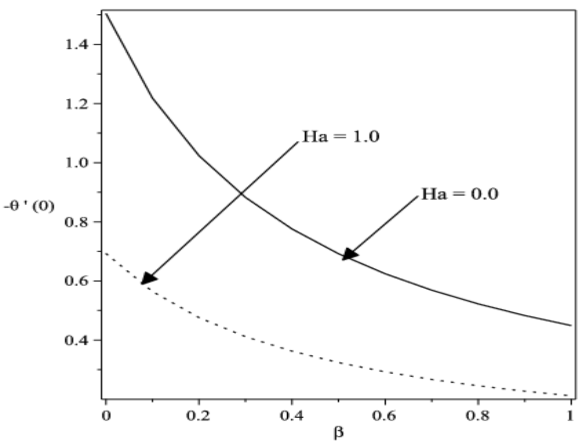

Fig.4: Variation of wall temperature gradient $-\theta'(0)$ with β when $\delta = 0.1$, $E_c = 1.0$, $E_1 = 1.0$, Pr = 3.0 and $F^* = 0.1$ for several values of Ha.

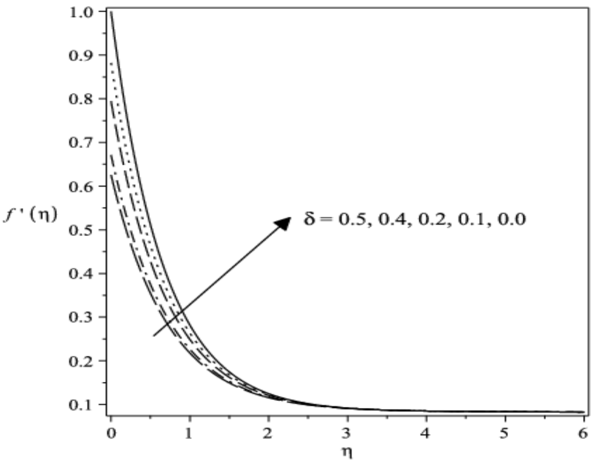

Fig.5: Velocity profiles $f'(\eta)$ for different values of δ when $\beta = 0.1$, Ha = 0.1, E_1 = 1.0, E_c = 1.0, k_1 = 0.1 , Nr = 0.1 , $F^* = 0.1$ and Pr = 3.0.

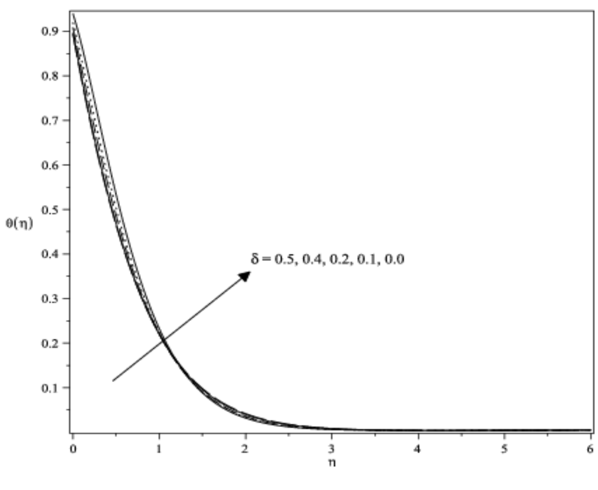

Fig.6: Temperature profiles $\theta(\eta)$ for different values of δ when $\beta = 0.1$, Ha = 0.1, E_1 = 1.0, E_c = 1.0, k_1 = 0.1, Nr = 0.1 , $F^* = 0.1$ and Pr = 3.0.

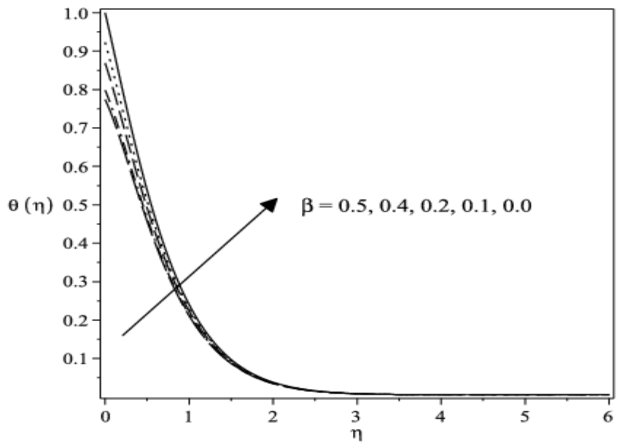

Fig. 6: Temperature profiles for different values of β when $\delta = 0.1$, Ha = 0.1, E_1 = 1.0, E_c = 1.0, k_1 = 0.1 , Nr = 0.1 , $F^* = 0.1$ and Pr = 3.0.

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