MIMO Radar Orthogonal Polyphase Code waveforms Design Based on Sequential Quadratic Programming

JUN LI
Department of Electronic Engineering
University of Electronic Science and Technology of China
No.2006, Xiyuan Ave, West Hi-Tech Zone, Chengdu, Sichuan
China
lijun_sc@qq.com

NA LIU
Department of Electronic Engineering
University of Electronic Science and Technology of China
No.2006, Xiyuan Ave, West Hi-Tech Zone, Chengdu, Sichuan
China
441940230@qq.com

Abstract: Orthogonal polyphase code is one of the most important waveforms for MIMO radar. In this paper, the sequential quadratic programming (SQP) method is used to design orthogonal polyphase code. The object is to obtain the waveforms not only with low autocorrelation sidelobe levels and low cross-correlation levels, but also with low integrated output (the summation of the outputs of all matched-filters in MIMO radar) sidelobe level, since it is the direct factor that affects the radar target detection. Phase quantization is applied to facilitate the waveform generating in practical radar system. Simulation results verify the validity and practicability of the method, and the effects of the optimization weighting coefficients and the number of quantization bits are discussed.

Key-Words: - Orthogonal polyphase code, sequential quadratic programming (SQP), auto-correlation sidelobe, cross-correlation, integrated output, phase quantization, waveform design, MIMO radar

1 Introduction

Waveform designing is an important basic work in MIMO radar research[1]-[4]. Orthogonal polyphase code is one of the most important forms of MIMO radar waveform. Intelligence methods such as simulated annealing and genetic algorithm are used to design orthogonal polyphase codes[5]-[7]. In order to further lower the autocorrelation sidelobe levels and cross-correlation levels, many optimization methods, such as Fletcher-Reeves algorithm[8] and sequential quadratic programming (SQP)[9], are adopted in waveform designing. The codes designed by these optimization methods usually have continuous phases.

Most existing methods focused on the autocorrelation sidelobe levels and cross-correlation levels of polyphase codes, and the obtained codes did have the superior autocorrelation and cross-correlation function properties. But they didn't necessarily have low integrated output sidelobe level. The integrated output signal is considered as summation of matched-filter outputs of all orthogonal signal channels in a MIMO radar receiver. Obviously, the integrated output sidelobe level is the direct factor that affects the target detection performance in practical radar systems. So it should be considered in MIMO radar waveform designing.

In this paper, based on SQP, we proposed a method to design orthogonal polyphase codes for MIMO radar. In the method, the integrated output sidelobe level, autocorrelation sidelobe levels and cross-correlation levels are optimized simultaneously, and phase code waveforms with continuous phase can be obtained at first. Then, by means of phase quantization, we can get the discrete phase codes, which are more available for actual radar system. The number of quantization bits can be decided by digital waveform generator devices (such as Direct Digital Synthesizer) of the radar transmitter.

Simulation results in the latter part of the paper shows the good performance of the method, compared to the methods based on simulated annealing algorithm[5] and genetic algorithm[6]. The effects of the optimization weighting coefficients and the impact of the number of quantization bits are illustrated at the end of the paper.

2 Signal Model and Design Method

Consider a phase code set with code length N and set size L as expressed by

$$\left\{s_l(n)=e^{j\phi_l(n)}, n=1,2,...,N\right\}, l=1,2,...,L \quad (1)$$

where $\phi_l(n)$ is the phase of sub-pulse n of sequence l in the set. If the number of the distinct phase is K, the phase for one sequence can only be selected from the following admissible values:

$$\phi_l(n)\in\left\{0,\frac{1}{K}2\pi,\frac{2}{K}2\pi...,\frac{K-1}{K}2\pi\right\} \quad (2)$$

The autocorrelation function $A(\phi_l,k)$, cross-correlation function $C(\phi_p,\phi_q,k)$ and integrated output function $S(k)$ can be defined as:

$$A(\phi_l,k)=\begin{cases}\dfrac{1}{N}\displaystyle\sum_{n=1}^{N-k}e^{j[\phi_l(n)-\phi_l(n+k)]} & 0\le k<N\\ \dfrac{1}{N}\displaystyle\sum_{n=-k+1}^{N}e^{j[\phi_l(n)-\phi_l(n+k)]} & -N<k<0\end{cases}$$

$$l=1,2,...,L \quad (3)$$

$$C(\phi_p,\phi_q,k)=\begin{cases}\dfrac{1}{N}\displaystyle\sum_{n=1}^{N-k}e^{j[\phi_q(n)-\phi_p(n+k)]} & 0\le k<N\\ \dfrac{1}{N}\displaystyle\sum_{n=-k+1}^{N}e^{j[\phi_q(n)-\phi_p(n+k)]} & -N<k<0\end{cases}$$

$$p,q=1,2,...,L, \text{ and } p\ne q \quad (4)$$

$$S(k)=\sum_{l=1}^{L}A(\phi_l,k)+\sum_{p=1}^{L}\sum_{q=1}^{L}C(\phi_p,\phi_q,k) \quad k=1,2,...,N-1 \quad (5)$$

Note that $S(k)$ expresses the sidelobe level of integrated output, which is the real output of signal processing for a radar target echo (Supposing there is no error between the direction of target and of beamformer). Phase code waveforms with low autocorrelation sidelobe and cross-correlation don't mean that the summation of the waveforms has low sidelobe level. The high sidelobe of integrated output may lead to high false alarm rate and deteriorated target detection performance. So, $S(k)$ should be optimized simultaneously while optimize $A(\phi_l,k)$ and $C(\phi_p,\phi_q,k)$.

To design MIMO radar orthogonal polyphase code waveforms, the object function of optimization can be expressed as

$$\min_{\Phi}\max\{\max_{\substack{k=-N,\cdots N,k\ne0\\ l=1,\cdots,L}}\left|A(\phi_l,k)\right|,$$

$$\lambda_1\max_{\substack{k=-N,\cdots N\\ p,q=1,\cdots,l}}\left|C(\phi_p,\phi_q,k)\right|,\ \lambda_2\max_{k=1,2,\cdots N}\left|S(k)\right|\} \quad (6)$$

Where $\mathbf{\Phi}\in R^{LN\times1}$ is the phase vector which contains LN free variables, which range of value is $[0,2\pi)$. $\max_{\substack{k=-N,\cdots N,k\ne0\\ l=1,\cdots,L}}\left|A(\phi_l,k)\right|$ is the autocorrelation peak sidelobe level (APSL), $\max_{\substack{k=-N,\cdots N\\ p,q=1,\cdots,l}}\left|C(\phi_p,\phi_q,k)\right|$ is the cross-correlation peak level (CPL), and $\max_{k=1,2,\cdots N}\left|S(k)\right|$ is the integrated output peak sidelobe level (IPSL). λ_1, λ_2 denote the optimization weighting coefficients of cross-correlation and integrated output respectively.

In order to solve the optimization problem of formula (6), an auxiliary variable z is introduced, and the object function is converted to inequality constraints. Namely, let autocorrelation sidelobe, cross-correlation and integrated output sidelobe be less than or equal to auxiliary variable z, and then minimize variable z. So formula (6) can be converted to the nonlinear programming problem with constraints as follow:

$$\begin{aligned}&\min_{\mathbf{\Phi}\in R^{LN\times1},z\in R} z\\ &s.t.\ \left|A(\phi_l,k)\right|\le z, \quad k=1,2,...,N-1; l=1,2,...,L\\ &\lambda_1\left|C(\phi_p,\phi_q,k)\right|\le z \quad \begin{array}{l}k=-N+1,-N+2,...,N-1;\\ p,q=1,2,...,L;\ p\ne q\end{array}\\ &\lambda_2\left|S(k)\right|\le z \quad k=1,2,...,N-1\end{aligned} \quad (7)$$

Problem (7) can be solved by using sequential quadratic programming (SQP) algorithm[10]. SQP is an iterative algorithm to solve nonlinear optimization problems. When the objective function and the constraints are quadratic continuous differentiable, SQP can be adopted. So, in formula (7), every variable of phase vector $\mathbf{\Phi}$ must have continuous value.

There is a package of SQP algorithm in MATLAB. By using function *fmincon* in the MATLAB optimization toolkit, we can solve problem (7) conveniently. Here, as an initial condition, every element of $\mathbf{\Phi}$ is set uniformly distributed on $[0,2\pi)$ and z is set to be 1.

After solving problem (7) by SQP algorithm, phase code sequences with continuous values are obtained. In order to being used in practical radar system, the continuous phases should be converted to discrete phases by quantization. If the number of quantization bits of the phase code permitted by the radar transmitter is B, then the number of phase will be $K=2^B$, and the discretized phase can be expressed by:

$$\hat{\phi}_i=\lfloor\phi_i\cdot K/2\pi\rfloor\frac{2\pi}{K} \quad i=1\sim LN \quad (8)$$

where ϕ_i denotes the element with continuous value in $\mathbf{\Phi}_{opt}$, and $\lfloor\cdot\rfloor$ denotes round down operation.

3 Design Results and Discussions

In this section, the design results of orthogonal polyphase code sequences with $L=4$, $N=40$ and $K=128$ are presented. The performance of code sequences designed by SQP method are compared with that by intelligence methods[5]-[6]. The loss of the phase quantization and the effects of the optimization weighting coefficients are discussed.

3.1 Designed Waveforms and Their Properties

We set $\lambda_1 = \lambda_2 = 1$. Table 1 gives the values of the polyphase code sequences designed by the method of this paper.

Table 1 Values of the polyphase code sequences ($\times 2\pi$)

Code1	0.8750 0.8750 0.0781 0.6719 0.3044
	0.0000 0.3047 0.5078 0.8984 0.0000
	0.0859 0.9375 0.7344 0.4766 0.0000
	0.0000 0.4609 0.7109 0.7813 0.9453
	0.8203 0.0000 0.8672 0.9141 0.5313
	0.5313 0.7422 0.5469 0.4375 0.3203
	0.8906 0.1328 0.4219 0.0000 0.0859
	0.6094 0.6094 0.3672 0.9609 0.1406
Code2	0.4375 0.2969 0.5234 0.4375 0.0156
	0.3594 0.4140 0.6797 0.5234 0.6484
	0.1953 0.6641 0.8672 0.3281 0.0000
	0.7422 0.9375 0.4766 0.3828 0.2031
	0.8203 0.1797 0.2031 0.8203 0.6484
	0.0000 0.5000 0.0078 0.1719 0.1641
	0.8438 0.2422 0.9531 0.2969 0.9297
	0.1797 0.2812 0.2109 0.7188 0.5078
Code3	0.1953 0.8516 0.5391 0.6563 0.8203
	0.1797 0.7109 0.7031 0.5156 0.6563
	0.3125 0.0078 0.5000 0.9609 0.0000
	0.0625 0.3984 0.4922 0.0000 0.4297
	0.1250 0.9688 0.4609 0.6719 0.2031
	0.6016 0.1797 0.7422 0.7813 0.6094
	0.5391 0.1719 0.3828 0.8828 0.5234
	0.7422 0.6016 0.0000 0.0078 0.4063
Code4	0.2500 0.0000 0.0469 0.8594 0.5781
	0.7031 0.2500 0.5859 0.4688 0.6953
	0.7422 0.9531 0.0000 0.1484 0.1328
	0.2891 0.0000 0.3672 0.6484 0.0625
	0.8828 0.7188 0.3672 0.6406 0.5078
	0.1719 0.3594 0.1719 0.2656 0.3359
	0.4453 0.0781 0.8906 0.9297 0.4219
	0.5234 0.2188 0.9063 0.3828 0.0000

Fig.1, Fig.2 and Fig.3 show the autocorrelation curves, cross-correlation curves and integrated output curve respectively.

Table 2 compares the APSL and CPL of the code sequences designed by the methods in this paper and in literature [5][6]. It can be seen that the code sequences designed by SQP method have much lower APSL and CPL than that by the intelligence methods[5][6].

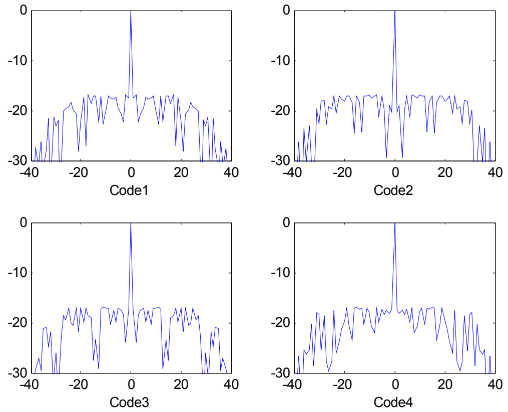

Fig.1 Autocorrelation curves of the sequences

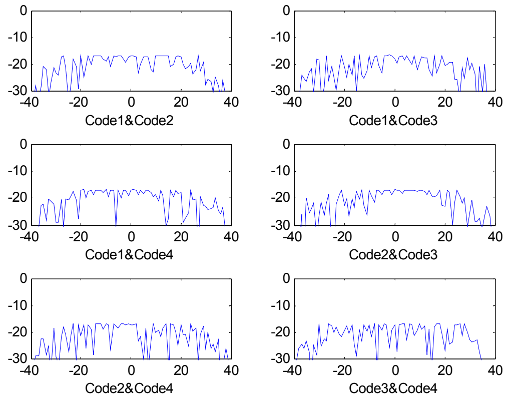

Fig.2 Cross-correlation curves of the sequences

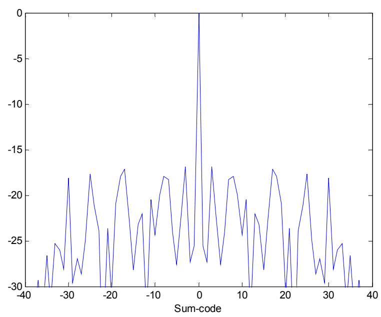

Fig.3 Integrated output curve of the sequences

Table 2 Properties (APSL and CPL) of designed codes

	APSL/dB	CPL/dB	IPSL/dB
This paper	-16.9	-16.9	-16.9
Literature[5]	-13.5	-14.8	-
Literature[6]	-16.0	-13.5	-

As shown in Table 3, if we ignore the constraint of integrated output sidelobe in this paper method (namely $\lambda_2 = 0$), the IPSL of the code sequences will increase remarkably while APSL and CPL decrease a little. That is to say, IPSL can be lowered remarkably by sacrificing a little performance of APSL and CPL.

Table 3 Properties of designed codes with/without the constraint of IPSL

	APSL/dB	CPL/dB	IPSL/dB
$\lambda_1=1,\lambda_2=1$	-16.9	-16.9	-16.9
$\lambda_1=1,\lambda_2=0$	-17.1	-17.1	-14.2

3.2 Effects of phase quantization

There is no doubt that phase quantization will lead to performance loss of designed phase code sequences. The less of the number of quantization bits, the bigger loss will be led to. Fig.4 shows the APSL, CPL and IPSL change curves versus the number of quantization bits B, where the condition is same as before, namely $\lambda_1=1$, $\lambda_2=1$, $L=4$, $N=40$. The number of phase is $K=2^B$, $B=1,2,3,\cdots,7$. It is indicated that the orthogonal polyphase codes design method based on SQP is not applicable for the occasion of small B.

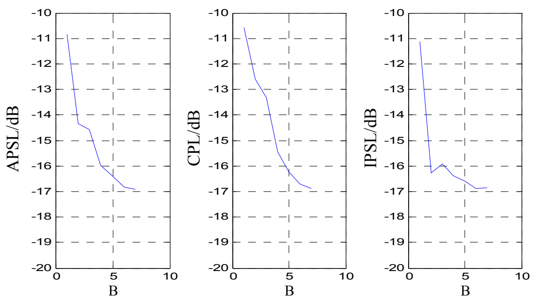

Fig.4 APSL, CPL and IPSL versus the number of quantization bits B

3.3 Effects of the optimization weighting coefficients

The optimization weighting coefficients λ_1 and λ_2 can be used to control the relative magnitudes of APSL, CPL and IPSL, although they are usually set as $\lambda_1=\lambda_2=1$. For example, APSL could decrease greatly by reducing λ_1, while sacrificing a little performance of CPL. IPSL could decrease greatly by increasing λ_2, while APSL and CPL get a little worse.

Fig. 5 shows the change curves of APSL, CPL and IPSL versus λ_2, where $\lambda_1=1$, $L=4$, $N=40$, $K=128$. It is indicated that with λ_2 increasing, IPSL decreases rapidly while APSL and CPL change slowly. So, we can obtain much lower IPSL with sacrificing a little performance of APSL and CPL by increasing λ_2.

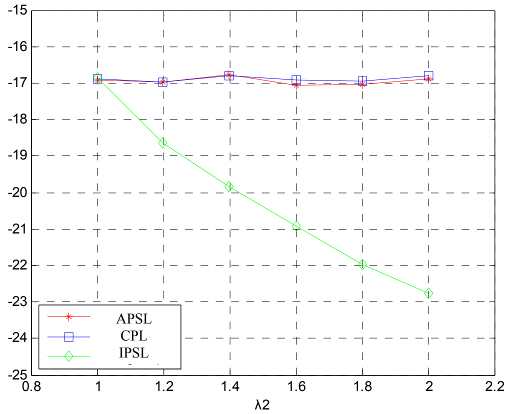

Fig.5 APSL, CPL and IPSL versus λ_2

4 Conclusion

This paper proposed an orthogonal polyphase code waveforms design method based on sequential quadratic programming. Under the constraints of autocorrelation sidelobe levels, cross-correlation levels and integrated output sidelobe level, phase code sequences with continuous phase were generated at first. In order to be available for a practical radar system, the phases were quantized according to the number of quantization bits permitted by the transmitter. When the number of quantization bits is relatively large, the designed orthogonal polyphase codes possess good properties of autocorrelation and cross-correlation, and also have high mainlobe to peak sidelobe ratio for the integrated output signal, which is the direct factor affecting target detection performance of a MIMO radar. APSL, CPL and IPSL of the orthogonal polyphase codes can be adjusted properly by controlling the optimization weighting coefficients.

Acknowledgment

This paper is supported by the National Natural Science Foundation of China (61201280) and the Fund of China Scholarship Council.

References:

[1] D. J. Rabideau. Adaptive MIMO radar waveforms[C]. in Proc. IEEE Radar Conference, Rome, 2008, 1-6

[2] C. Y. Chen, P. P. Vaidyanathan. MIMO radar waveform optimization with prior information of the extended target and clutter[J]. IEEE Transactions on Signal Processing, 2009, 57(9): 3533-3544

[3] K. W. Forsythe, D. W. Bliss. MIMO radar waveform constraints for GMTI[J]. IEEE Journal of Selected Topics in Signal Processing, 2010, 4(1): 21-32
[4] Chen Yifan et al. Adaptive Distributed MIMO Radar Waveform Optimization Based on Mutual Information. IEEE Trans. on Aerospace and Electronics Systems. 2013, 49(2): 1374-1385.
[5] H. Deng. Polyphase code design for orthogonal netted radar systems[J]. IEEE Transactions on Signal Processing, 2004, 52(11): 3126-3135
[6] Bo Liu, Zishu He, JianKuiZeng, Benyong Liu. Polyphase Orthogonal Code Design for MIMO Radar Systems. CIE '06. International Conference on Radar, Oct. 2006, 1:113-116
[7] Sun Ying, Zishu He, Hongming Liu, Li Jun, Shangwei Gao. Binary orthogonal code design for MIMO radar systems[J]. IEEE 2010 interational symposium on intelligent signal processing and communication
[8] Kevin J G. Generation of uniform amplitude complex code sets with low correlation sidelobes[J]. IEEE Trans. on Signal Processing, 1992,40(2):343-351
[9] Liangbing Hu，Hongwei Liu，Shunjun Wu. Orthogonal waveform design for MIMO radar via constrained nonlinear programming. Systems Engineering and Electronics， 2011,Vol. 33, No. 1:64-68
[10] Nocedal,J. And S.J.Wright. Numerical Optimization, Second Edition. Springer Series in Operations Research, Springer Verlag,2006