Spectral Signal Denoising Method Based on Improved Wavelet Threshold Function

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Abstract: - In the quantitative analysis of spectral data, the noise often affects the accuracy of analytic results. In order to improve the analytic precision of the spectral data, the spectral signal collected with CCD need to denoise. According to the characteristics of spectral signal and combining with the advantages of the hard threshold function and the soft threshold function, we propose an improved threshold function denoising algorithm(ITFDA), which can not only overcome the discontinuity of hard threshold, but also overcome the constant error of soft threshold. The simulation experiments conducted with the improved threshold function are compared with the hard and soft threshold in terms of Signal to Noise Ratio(SNR) and Mean Square Error(MSE). The results show that the proposed denoising algorithm with improved threshold function can remove noise from the spectral signal in a certain extent, retain the important characteristics spectrum - peak of spectral signal, and reconstruct the spectral signal very well.

Key-Words: - Threshold function, Wavelet threshold denoising, Spectral signal, Threshold denoising algorithm

1 Introduction

With the rapid development of electronic technology and computer technology, the charge coupled device (CCD) has been widely applied to image sensing and noncontact measurement field. CCD has many characteristics of high sensitivity, low noise, fast reading, high dynamic range and a full spectral response. Using a linear CCD as the detection device of spectral signal, the full spectrum measurement and fast sampling can be achieved; and the measuring instrument can be realized to full spectrum and miniaturization. But the spectral signal collected by CCD may contain noise, which will affect the result accuracy of the spectral data quantitative analysis. In order to improve the analysis precision of spectral data, it is very necessary that the spectral signal collected by the linear array CDD is denoised.

The wavelet transform, which has a high frequency resolution in the low frequency region and has a high time resolution in the high frequency region, is a method of time frequency localization analysis. The wavelet transform is known as “mathematical microscope”, so it is used widely. There are main methods of wavelet denoising, such as modulus maxima denoising[1-2], correlation denosing[3-4], and threshold desnoising[5-6]. Because the wavelet threshold denoising algorithm is simple, it has got good application in image denoising. The hard threshold function and the soft threshold function proposed by Donoho[5] is usually chosen as threshold function in wavelet threshold desnoising algorithm. However, the signal reconstructed by the hard threshold function denoising algorithm(HTFDA) is easy to produce large variance and oscillation; the signal reconstructed by the soft threshold function denoising algorithm(STFDA) appears excessive smoothing, which makes the edge relative fuzzy. In order to improve the defects of the hard and soft threshold function, the various improved threshold function have been put forward to enhance the desnoising effect[7-10]. According to the characteristic of the CCD spectrum, an improved threshold function denoising algorithm(ITFDA) is proposed, which overcomes the shortcomings of the hard threshold function and the soft threshold function. So the good spectrum denoising effect is obtained.

2 The Basic Principle of Wavelet Threshold Denoising

The noisy signal is decomposed by wavelet, the signal energy is mainly distributed in the low resolution scale function, but the noise energy distribution is unchanged and still is distributed in all wavelet coefficients. So the wavelet coefficient of the useful signal is relatively large, and the wavelet coefficient of the noise is smaller. Based on this characteristic, Donoho [5-6] proposed a method of wavelet threshold denoising, in which threshold function was used and the appropriate threshold was selected. The large wavelet coefficients can be retained, and the smaller wavelet coefficients are removed. The purpose of removing noise from the noisy signal is achieved.

Assuming that a noisy signal is

$$y(i) = s(i) + n(i),\ i = 1, 2, \cdots, N-1 \qquad (1)$$

Where, $s(i)$ is the original signal and $n(i)$ is a Gaussian white noise $\left(0, \sigma^2\right)$.

Wavelet threshold denoising steps is shown in Fig.1.

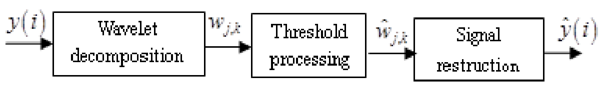

Fig.1. Wavelet threshold denoising procedure

In Fig.1, the noisy signal is decomposed according to the wavelet basis and the decomposition level J, and then the wavelet coefficients $w_{j,k}$ are obtained. The proper threshold value and the threshold function are used to process the high frequency wavelet coefficients, and then the estimated wavelet coefficient values $\hat{w}_{j,k}$ are gained. In signal reconstruction step, using inverse wavelet transform, the signal estimation value $\hat{y}(i)$, which is denoised signal, is achieved.

3 Improved Threshold Function

In the wavelet threshold denoising, the selection of threshold function is very important; it directly affects the signal denoising result.

3.1 Hard Threshold and Soft Threshold Function

The threshold function, which directly affects the signal denoising result, reflects the processing strategy of the wavelet coefficients. The hard threshold function and the soft threshold function are two popular threshold functions in wavelet threshold denoising. The hard threshold function can well reserve the detail characteristics of the signal, but it is discontinuity at the threshold ±δ. At the same time, the hard threshold function completely retains the wavelet coefficients larger than the threshold value without processing them, and cancels wavelet coefficients smaller than the threshold value. This easily makes the reconstruct signal produce large variance and oscillation. However, the soft threshold function is continuous at the threshold ±δ, which can avoid the denoised signal oscillation. But there is a constant deviation between the wavelet coefficients estimated value and actual signal wavelet coefficients, which results in reconstructed signal over smoothing and makes the edge relative fuzzy. These also affect the signal denoising results.

The hard threshold function

$$\hat{w}_{j,k} = \begin{cases} w_{j,k} & ,\left|w_{j,k}\right| \geq \delta \\ 0 & ,\left|w_{j,k}\right| < \delta \end{cases} \qquad (2)$$

The soft threshold function

$$\hat{w}_{j,k} = \begin{cases} \operatorname{sgn}\left(w_{j,k}\right)\left(\left|w_{j,k}\right| - \delta\right) & ,\left|w_{j,k}\right| \geq \delta \\ 0 & ,\left|w_{j,k}\right| < \delta \end{cases} \qquad (3)$$

where, $w_{j,k}$ is the noisy wavelet coefficient, $\hat{w}_{j,k}$ is the denoised wavelet coefficient, δ is the threshold value. The curves of the hard and soft threshold function are shown in Fig. 2.

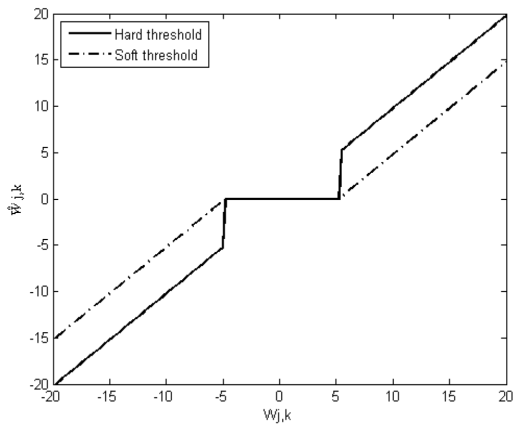

Fig.2. The hart threshold and soft threshold function

3.2 Improved Threshold Function

Aiming at the problem of the hard and soft threshold function, an improved threshold function is proposed in this paper, i.e.

$$\hat{w}_{j,k}=\begin{cases}\operatorname{sgn}\left(w_{j,k}\right)\left[\left|w_{j,k}\right|^{2}-\left(\delta e^{-\frac{\left|w_{j,k}\right|-\delta}{M}}\right)^{2}\right]^{\frac{1}{2}} & ,\left|w_{j,k}\right|\geq\delta \\ 0 & ,\left|w_{j,k}\right|<\delta\end{cases} \quad (4)$$

where, sgn(·) is the sign function, $w_{j,k}$ is the noisy wavelet coefficient, $\hat{w}_{j,k}$ is the denoised wavelet coefficient, δ is the threshold value, and *M*, which range is $M > 0$, is adjustable parameter. The improved threshold function changing along with the *M* changing is shown in Fig.3.

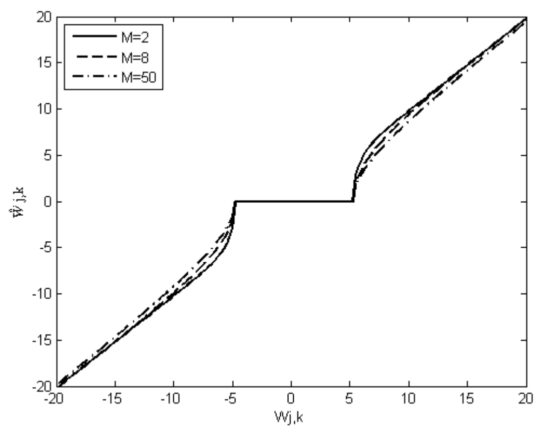

Fig.3. Improved threshold function with different *M* value

As shown in Fig.3, the improved threshold function approximates the hard threshold function when $M \to 0$. Along with the M value increasing, the improved threshold function embodies the characteristics of soft threshold function, the deviation. In addition, the improved threshold function has the following characteristics:

(1) Continuous at the threshold ±δ. When $w_{j,k} \to \pm\delta$, $e^{-\frac{\left|w_{j,k}\right|-\delta}{M}} \to 1$, then the desoised wavelet coefficient $\hat{w}_{j,k} \to 0$. This shows that the improved threshold function has the same nature with the soft threshold function in threshold ±δ: continuity.

(2) Taking $\hat{w}_{j,k} = w_{j,k}$ as the asymptote. When $w_{j,k} \to \infty$, $e^{-\frac{\left|w_{j,k}\right|-\delta}{M}} \to 0$, then the denoised wavelet coefficient $\hat{w}_{j,k} \to w_{j,k}$. This illustrates that the improved threshold function retains the property of the hard threshold function, that is, the estimating value tends to the actual value: progressive.

(3) High-order derivative. The improved threshold function is high-order derivative in the wavelet domain.

From the above analysis it can be drawn that the improved threshold function, which is proposed in this paper, is between the hard threshold function and the soft threshold function, not only overcomes the discontinuity of the hard threshold function, but also avoids the constant deviation phenomenon of the soft threshold function.

The comparison among of the hard threshold function, the soft threshold function and the improved threshold function is shown as Fig. 4

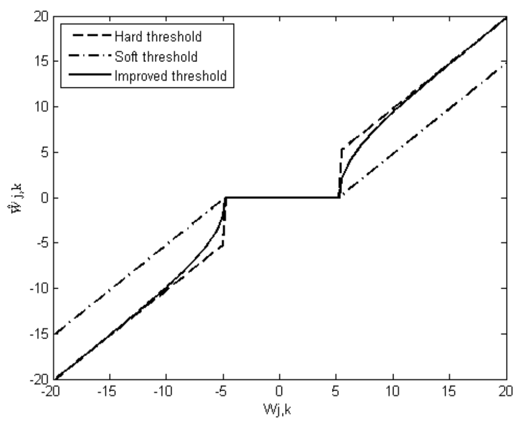

Fig.4. Comparison of the threshold functions

4 ITFDA for Spectral Signal

Given a discrete noisy spectral signal collected by the CCD

$$y(i)=s(i)+n(i),\quad i=1,2,\cdots,N-1 \quad (5)$$

where, *s*(*i*) is an original spectral signal and *n*(*i*) is a Gaussian white noise.

Using the improved threshold function given in formula (4), this paper presents an improved threshold function denoising algorithm(ITFDA) for spectral signal, which steps are as follows:

(1) Wavelet transforming $w_{j,k} = \mathrm{W}\left[y(i)\right]$. Selecting the wavelet basis, decomposing the noisy spectral signal *y*(*i*) level *j* , then getting every level wavelet coefficients $w_{j,k}$.

Because the noise is an additive noise, the wavelet coefficients after the wavelet transforming are composed of two parts, i.e. $w_{j,k} = u_{j,k} + v_{j,k}$. Where $u_{j,k}$ is the wavelet coefficient of the original spectral signal and $v_{j,k}$ is the wavelet coefficient of the noise.

(2) In every level, the high frequency wavelet coefficient is shrinked $\hat{w}_{j,k} = \mathrm{D}(w_{j,k}, \delta_j)$, in which the improved threshold function is used, i.e.

$$\hat{w}_{j,k} = \begin{cases} \operatorname{sgn}(w_{j,k})\left[|w_{j,k}|^2 - \left(\delta_j e^{-\frac{|w_{j,k}|-\delta_j}{M}}\right)^2\right]^{\frac{1}{2}} & ,|w_{j,k}| \geq \delta_j \\ 0 & ,|w_{j,k}| < \delta_j \end{cases} \tag{6}$$

where, $w_{j,k}$ is the noisy wavelet coefficient in level j, $\hat{w}_{j,k}$ is the denoised wavelet coefficient, δ_j is the thresholad in level j. the expression for δ_j is

$$\delta_j = \sigma_j \sqrt{2\ln(N_j)} \tag{7}$$

where, N_j is the length of the wavelet coefficient in level j, $\sigma_j = \frac{\operatorname{median}\{|w_{j,k}|\}}{0.6745}$ is noise standard deviation in level j.

Through threshold denoising, the denoised wavelet coefficient is obtained, which makes $\|\hat{w}_{j,k} - u_{j,k}\|$ as small as possible, so as to achieve the purpose of removing noise.

(3) The signal estimating value $\hat{y}(i) = \mathrm{W}^{-1}(\hat{w}_{j,k})$ obtained after the inverse wavelet transform. Using wavelet low frequency coefficients and high frequency coefficients that are processed with the improved threshold function, the denoised signal estimating value is obtained by IWT.

5 Experiment Simulations

The Signal to Noise Ratio (SNR) and Mean Square Error (MSE) generally are used to measure the denoising performance. The SNR is a measure of the degree of the noise impacting on the useful signal; its value is to be as small as possible. The definition of SNR

$$SNR = 10 \cdot \log\left[\frac{\sum_{i=0}^{N-1} x^2(i)}{\sum_{i=0}^{N-1}\left(\hat{x}(i) - x(i)\right)^2}\right] \tag{8}$$

Where, $x(i)$ is an original signal and $\hat{x}(i)$ is the denoised signal.

The Mean Square Error indicates the approaching degree of the reconstruction signal and the true signal. The smaller the value is, the closer between the reconstruction signal and the true signal are. The MSE is defined as below

$$MSE = \frac{1}{N}\sum_{i=0}^{N-1}\left(\hat{x}(i) - x(i)\right)^2 \tag{9}$$

5.1 The Threshold Denoising Experiment for General Signal

In order to validate the effect of the proposed ITFDA, two kinds of standard signals, which are Bumps and Blocks shown in Fig.5 (a) and Fig.7 (a), are selected as the experimental signals, and the noise is additive Gaussian white noise. The noisy signals are shown in Fig.5 (b) and Fig.7 (b). The hard threshold function, soft threshold function and the improved threshold function respectively are used to remove the noise from the noisy signal. Taking M=2, the wavelet basis db4 and decomposing level J=5, and the wavelet threshold value in every level is calculated from formula(7). The effects of denosing algorithm with various threshold function are shown in Fig.6 and Fig.8 when the noisy signal SNR=15dB.

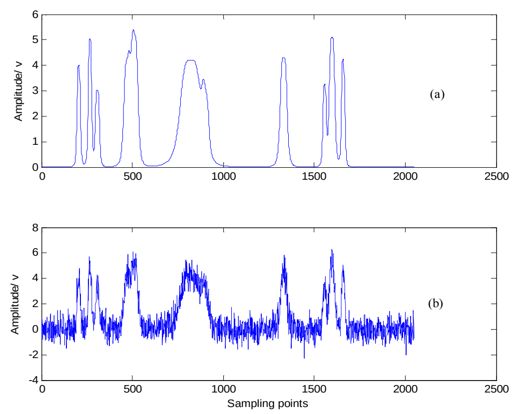

Fig.5. Bumps signal: (a) original signal, (b) noisy signal

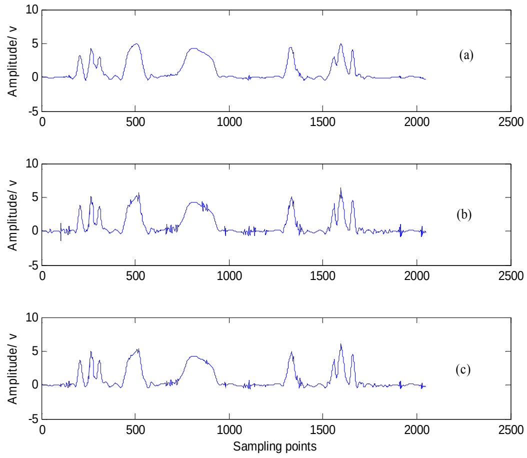

Fig.6. Denoising of Bumps signal: (a) denoised signal with STFDA, (b) denoised signal with HTFDA, (c) denoised signal with ITFDA

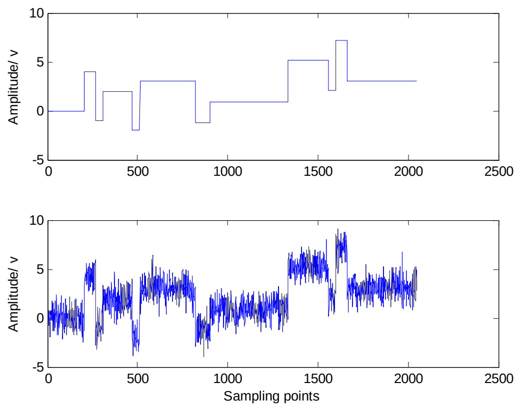

Fig.7. Blocks signal: (a) original signal, (b) noisy signal

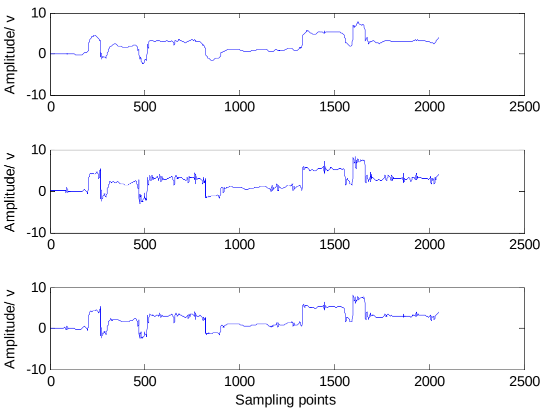

Fig.8. Denoising of Blocks signal: (a) denoised signal with STFDA, (b) denoised signal with HTFDA, (c) denoised signal with ITFDA

As shown in Fig.6 and Fig.8, the waveform processed with proposed ITFDA is not smoother than the waveform with STFDA, but smoother than HTFDA. At the same time, the waveform denoised with ITFDA has more signal details than STFDA. This is particularly important for the spectral signal.

Table 1 The SNR of denoised signal (dB)

Signal type	Noisy signal	STFDA	HTFDA	ITFDA
Bumps	10	16.9497	16.892	18.3386
	12	17.185	18.3598	19.4581
	15	17.8939	19.701	20.6023
	18	17.8747	21.4161	21.8942
Block	10	18.1528	17.4069	18.8743
	12	18.3707	18.544	19.37
	15	19.6366	21.5378	22.405
	18	20.5321	24.6972	24.7878

Table 2 The RSE of denoised signal

Signal type	Noisy signal	STFDA	HTFDA	ITFDA
Bumps	0.4478	0.2489	0.2174	0.1916
	0.3206	0.2294	0.1863	0.1679
	0.2283	0.2299	0.1529	0.1447
Block	0.748	0.3582	0.3511	0.3193
	0.525	0.3096	0.2488	0.2251
	0.3579	0.2793	0.1729	0.1711

Table 1 and table 2 show that the denoising effect with STFDA is better than the HTFDA when the SNR is low, and the case is opposite when the SNR is high. But whether on the high SNR or the low SNR condition, the denoising effect with ITFDA proposed in this paper is better than STFDA and HTFDA. Therefore, the improved threshold function denoising algorithm(ITFDA) is effective.

5.2 The Threshold Denoising Experiment for Spectral Signal

The experimental data is from the spectrum data acquisition system based on CCD, which is the Sony Corp production of ILX511 line array CCD, the number of pixels is 2048, the CCD device has a built-in timer, a clock driver and a hold circuit and sampling circuit, it is easy to operate[11]. The test signal is shown in Fig.9, in which Fig.9 (a) is the original spectral signal, Fig.9 (b) is a noisy spectral signal. Fig.10 is the denoised spectral signal obtained by using STFDA, HTFDA and ITFDA.

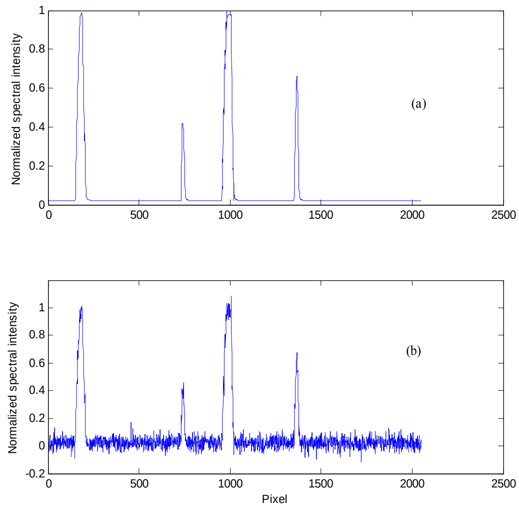

Fig.9. Spectral signal: (a) Original spectral signal (b) Noisy spectral signal

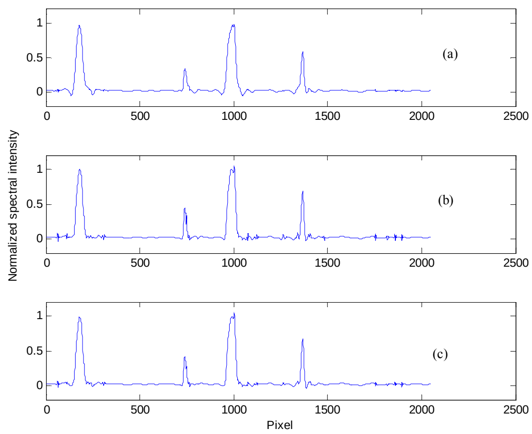

Fig.10. Denoised spectral signal: (a) Denoised spectral signal with STFDA (b) Denoised spectral signal with HTFDA (c) Denoised spectral signal with ITFDA

Fig.10 reveals that the denoised spectral signal waveforms obtained by using three denoising algorithms are similar to the original spectral signal waveform, this shows not only is the noise filtered, but also the peak characteristics of spectral signal is well preserved. The denoised spectrum waveform with STFDA is relatively smooth, but the second peak signal decreases. The denoised spectrum waveform with HTFDA has oscillation, and there is more burr where no spectrum. The denoised spectrum waveform with ITFDA not only retains the important peak features of the spectral signal, but also has smaller oscillation than with HTFDA. The spectral signal is reconstructed well

Table 3 is denoised spectral signal SNR and MSE when noisy spectral signal SNR=15dB, MSE=0.0012.

Table 3 Effect comparison of denoising method

Signal type	Noisy signal	STFDA	HTFDA	ITFDA
SNR (dB)	15	18.09	21.53	22.22
MSE	1.2×10^{-3}	5.61×10^{-4}	2.54×10^{-4}	2.17×10^{-4}

Table 3 shows an objective analysis and comparison of the effect of denoising method. In the two aspects of SNR and MSE, ITFDA is better than STFDA and HTFDA. Comparing with the STFDA, the denoised spectral signal, obtained by using ITFDA, SNR is improved 4.13dB and MSE is reduced by 61.3%.

6 Conclusion

For the sake of improving the analysis accuracy of spectral signal, this paper employs the improved threshold function denoising algorithm(ITFDA) and removes noise form the spectral signal. In order to validate the commonality of the ITFDA, simulation experiments are carried out on with the Bumps and Blocks signals. The denoised signal SNR and MES, which are obtained by using STFDA, HTFDA and ITFDA, are compared. The results show that the denoising effects of ITFDA are better than STFDA and HTFDA in different SNR conditions. Simulation experiment results show that the denoised spectral signal, which is obtained by using the ITFDA, the SNR increases 7.22dB, the MSE reduces nearly 82% with respect to noisy spectral signal. Therefore, it is effective method that the noisy spectral signal is denoised by using the ITFDA. But the spectral signal collected by linear array CCD may also contain the impulse noise, which impacts on the peak value and position of the spectral signal. This problem needs further research so as to improve the spectral signal denoising effect.

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