Nonstationary Blind Source Separation Using Slepian Spectral Estimator

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Abstract: Introduced by Priestley, evolutionary spectral theory generalizes the definition of spectrum for nonstationary signals while avoiding some of the shortcomings of bilinear time-frequency distributions. There have been different approaches to estimate Priesley's evolutionary spectrum such as evolutionary periodogram. In this paper, we present an estimator of evolutionary spectrum for blind separation of nonstationary signals. Our estimator uses a transform based on discrete prolate spheroidal sequences. Also known as Slepian sequences, DPSS are defined to be sequences with maximum spectral concentration for a given duration and bandwidth. Using the connection between discrete evolutionary transform and evolutionary periodogram, we derive the estimator for the evolutionary spectrum and demonstrate its performance for blind source separation of time-varying autoregressive moving average signals.

Key–Words: Evolutionary spectrum; Slepian sequences; blind source separation; nonstationary processes

1 Introduction

Most signals show some form of nonstationarity which can be described by time-dependent spectra [1]. Introduced by Priestley, evolutionary spectral theory generalizes the definition of spectrum for nonstationary signals while avoiding some of the shortcomings of bilinear time-frequency distributions such as cross-terms [2]. Processes composed of slowly varying amplitude modulated carriers, also called oscillatory processes, are considered to have an evolutionary spectrum [2]. There have been different approaches to estimate evolutionary spectrum (ES). For example, evolutionary periodogram (EP) [3, 4] can be used to estimate the ES by allowing nonstationary signals to be modeled as a sum of complex sinusoids with time-varying complex amplitudes [4]. In [5], the discrete evolutionary transform (DET) was proposed for the computation of a kernel and the corresponding ES. Similarly, in array signal processing, the signal received by each sensor of the array can be modeled as a sum of complex sinusoids with time-varying complex amplitudes [6]. As shown in [6], the time-varying amplitudes can be estimated using linear estimators obtained via minimum mean-squared error criteria. These estimates are then used for the estimation of time-varying cross-power distributions of the data across the array.

Spectral definition of nonstationary signals can be used in the Blind Source Separation (BSS) problem [7], as well. The BSS consists of several signals emitted from point sources placed in the far field and involve several sensors. In the simplest form, it can be defined as recovering n unknown sources from m observations (mixtures) of them [8]. In general, each sensor receives a linear mixture of source signals and BSS methods recover all individual sources from the mixture or at least separate a particular source. The BSS algorithms can be classified as the ones that are based on using statistical information available on source signals or those that are exploiting the difference in the time-frequency signatures of the sources to be separated [9, 10, 11]. An example for the over-deteminded case i.e., the number of observations are greater than the number of sources $n \leq m$, a method based on second-order statistics and joint-diagonalization of set of covariance matrices can be found in [12]. Other examples on spatial time-frequency distributions (TFDs) as a generalization of bilinear TFDs, in the case of nonstationary signals, are in [13, 14]. Although bilinear TFDs have good localization property, they display cross-terms and positivity of spectral estimates are not guaranteed [15].

The EP as an estimator of the Wold-Cramer ES, which is a special case of Priestley's ES, was used also for array processing in semi-homogeneous random fields [16]. In this paper, by expanding our work [17] on reconstruction of signals from nonuniform samples in the evolutionary spectral domain, we introduce a spectrum estimator for BSS problem. We will use the spatial evolutionary spectrum estimated by a transform based on discrete prolate spheroidal sequences (DPSS) [18] together with whitening technique to estimate the mixing matrix. Once the mixing matrix is estimated, we can separate the source signals. Also known as Slepian sequences, DPSS derive from the time-frequency concentration problem and are defined to be sequences with maximum spectral concentration for a given duration and bandwidth. The paper is organized as follows. In the next section, we review the evolutionary spectral theory and provide the fundamental equations of signal representation. In Section 3, we present the proposed estimator. We briefly review the BSS problem and related formulation in Section 4. In Section 5, we present experimental results. Conclusions follow.

2 Estimation of Evolutionary Spectrum

The evolutionary spectral theory describes the local power frequency distribution at each instant of time [2]. As a special case of Priestley's evolutionary spectrum (ES), the Wold-Cramér ES considers a nonstationary signal as the output of a linear time-varying system driven by a stationary white noise [20]. The evolutionary periodogram (EP) was presented for estimation of the Wold-Cramér ES [4].

In the following, we will review the Wold-Cramér ES starting with the representation of a discrete-time nonstationary process as the output of a casual, linear time-varying system with impulse response $h[n, m]$ as

$$x[n] = \sum_{m=-\infty}^{n} h[n, m]\varepsilon[m], \tag{1}$$

here discrete-time nonstationary process $x[n]$ is the output for the input $\{\varepsilon[m]\}$ which is a stationary, zero-mean, unit-variance, white noise process. The representation in (1) is known as the Wold-Cramér decomposition [19]. The white noise process, $\{\varepsilon[m]\}$ can be expressed as a sum of sinusoids with random amplitudes and phases as follows

$$\varepsilon[m] = \int_{-\pi}^{\pi} e^{j\omega m} dZ(\omega). \tag{2}$$

Accordingly, the nonstationary process $\{x[n]\}$ can be expressed as

$$x[n] = \int_{-\pi}^{\pi} H(n, \omega)e^{j\omega n} dZ(\omega), \tag{3}$$

where

$$H(n, \omega) = \sum_{m=-\infty}^{n} h[n, m]e^{-j\omega(n-m)}, \tag{4}$$

for $Z(\omega)$ being a process with orthogonal increments. The variance of $x[n]$

$$E\{|x[n]|^2\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |H(n, \omega)|^2 d\omega, \tag{5}$$

provides the power distribution of the nonstationary process $\{x[n]\}$ at each time n, as a function of the frequency parameter ω.

The Wold-Cramér ES is defined as $S(n, \omega) = |H(n, \omega)|^2$ and the cross-power ES for two processes $\{x[n]\}$ and $\{y[n]\}$ is given as $S_{xy}(n, \omega) = H_x(n, \omega)H_y^*(n, \omega)$. This definition was also proposed in [20] as a special case of Priestley's ES if one restricts the function $H(n, w)$ to the class of oscillatory functions that are slowly-varying in time. In [4], a similar condition was applied to model the component of $x[n]$ for a particular frequency of interest, ω_0 as

$$x_0[n] = H(n, \omega_0)e^{j\omega n} dZ(\omega_0), \tag{6}$$

such that

$$x[n] = x_0[n] + y_{\omega_0}[n] = A(n, \omega_0)e^{j\omega n} + y_{\omega_0}[n], \tag{7}$$

where $A(n, \omega_0)$ represents time-varying complex amplitude as $A(n, \omega_0) = H(n, \omega_0)dZ(\omega_0)$ and $y_{\omega_0}[n]$ being a zero-mean modeling error.

It can be derived that

$$E\{|A(n, \omega_0)|^2\} = S(n, \omega_0)\frac{d\omega_0}{2\pi}, \tag{8}$$

and using $x[n]$ and $A(n, \omega_0)$, we can estimate $S(n, \omega_0)$. Repeating this process for all frequencies ω, an estimate of the time-dependent spectral density $S(n, \omega)$ was obtained [4].

If we assume that $A(n, \omega_0)$ also varies with time, a representation as an expansion of orthonormal functions $\{\beta_i[n]\}$ over $0 \leq n \leq N-1$ is

$$A(n, \omega_0) = \sum_{i=0}^{M(\omega_0)-1} \beta_i^* a_i = \mathbf{b}[n]^H \mathbf{a}. \tag{9}$$

The vectors $\mathbf{a} = [a_0, ..., a_{M-1}]^T$ and $\mathbf{b}[n] = [\beta_0[n], ..., \beta_{M-1}[n]]^T$ represent a vector of random expansion coefficients and a vector of orthonormal functions at time n, respectively. The number of expansion functions $M \leq N$ depends on the frequency ω_0 and indicates the degree to which $A(n, \omega_0)$ varies with time. For small M, $A(n,\omega_0)$ is slowly varying and for large values of M, $A(n,\omega_0)$ is rapidly varying. Then, any time behavior of $A(n,\omega_0)$ can be approximated by changing M. However, the order of expansion must be kept at a minimum to improve frequency resolution [4]. The minimum mean squared error (MSE) estimate for $A(n,\omega_0)$ is

$$\hat{A}(n,\omega_0) = \sum_{i=0}^{M-1} \beta_i^*[n] \sum_{k=0}^{N-1} \beta_i[k]x[k]e^{-j\omega_0 k}, \quad (10)$$

and for all possible values of frequency, the time-varying spectral density is called the EP [4]. Therefore, the relation between the estimator and the time-varying spectral density can be found as

$$\begin{aligned} \hat{S} &= \frac{2\pi}{d\omega}|\hat{A}(n,\omega)|^2 \quad (11) \\ &= \frac{N}{M}|\sum_{i=0}^{M-1} \beta_i^*[n] \sum_{k=0}^{N-1} \beta_i[k]x[k]e^{-j\omega k}|^2. \end{aligned}$$

Rewriting (11)

$$\hat{S} = \frac{N}{M}|\sum_{k=0}^{N-1} v[n,k]x[k]e^{-j\omega k}|^2, \quad (12)$$

here $\hat{S}$ can be interpreted as the magnitude square of the Fourier transform of $x[k]$ windowed by a sequence $v[n,k]$ where $v[n,k] = \sum_{i=0}^{M-1} \beta_i^*[n]\beta_i[k]$. Using the model in (7) at frequency ω_0, the derivations above can be expanded for array processing as in [6]. For example, considering signals $\{x_l[n]\}$, $1 \le l \le L$, $0 \le n \le N-1$, where L is the number of sensors and N is the number of the data snapshots, $\{A_l(n,\omega_o)\}$ can be represented as an expansion of M orthogonal basis functions for the sensor data $x_l[n]$ as

$$A_l(n,\omega_o) = \sum_{i=0}^{M(\omega_0)-1} \beta_i^* a_i, \quad (13)$$

and $x_l[n]$ can be expressed over the observation interval in vector form

$$\mathbf{x}_l = \mathbf{F}(\omega_0)\mathbf{a}_l(\omega_0) + \mathbf{y}_l(\omega_0), \quad (14)$$

where $\mathbf{F}(\omega_0)$ is a matrix with entries $\mathbf{F}_{n+1,i+1} = \beta_i^*[n]e^{j\omega_0 n}$, [6]. Letting $\mathbf{a}[n] = \mathbf{b}[n]^H\mathbf{A}$ be a vector of amplitudes at time n, the estimates of the time-varying amplitudes are obtained as $\hat{\mathbf{a}}[n] = \mathbf{b}[n]^H\mathbf{F}^H\mathbf{x}$ via MSE estimator. Then, in array signal processing, the cross-power evolutionary spectral density estimator is

$$\hat{\mathbf{S}}_{xx}(n,\omega) = E\{\hat{\mathbf{a}}[n]^H\hat{\mathbf{a}}[n]\}, \quad (15)$$

which is also

$$\hat{\mathbf{S}}_{xx}(n,\omega) = (\mathbf{b}[n]^H\mathbf{F}^H) \otimes_l \mathbf{R} \otimes_r (\mathbf{F}\mathbf{b}[n]), \quad (16)$$

here $\otimes_l$ and $\otimes_r$ are the left and right block Kronecker product, respectively and $\mathbf{R} = E\{\mathbf{x}\mathbf{x}^H\}$ with E being the expectation operator. The cross-power between the data at sensors ℓ and m can be obtained as $\hat{\mathbf{S}}_{x_\ell x_m}(n,\omega)$ [6].

3 Derivation of Slepian Estimator for Evolutionary Spectrum

3.1 From Evolutionary Periodogram to Discrete Evolutionary Transform

In this section we will briefly review the discrete evolutionary transform. In [5], the discrete evolutionary transform (DET) was defined to represent a nonstationary signal and its spectrum. Using Gabor or Malvar representations with the Wold-Cramér representation, an evolutionary kernel can be obtained and the ES is the magnitude square of the evolutionary kernel [5]. The Wold-Cramér representation, similar to (1) can be written as

$$x[n] = \sum_{k=0}^{K-1} X(n,\omega_k)e^{j\omega_k n}, \quad (17)$$

where $\omega_k = 2\pi k/K$, $0 \le n \le N-1$ and $X(n,\omega_k)$ is called the evolutionary kernel [5]. In this case associating with the sinusoidal representation in (1)

$$X(n,\omega_k) = \sum_{\ell=0}^{N-1} x(\ell)W_k(n,\ell)e^{-j\omega_k \ell}, \quad (18)$$

is an inverse discrete transformation that provides the evolutionary kernel, $X(n,\omega_k)$ in terms of the signal. $W_k(n,\ell)$ is in general, a time and frequency dependent window [5]. Here the ES is defined as $S_E(n,\omega_k) = |X(n,\omega_k)|^2$. It becomes obvious that the DET is a generalization of the STFT and $S_E(n,\omega_k)$ is a generalization of the spectrogram. A similar representation for the kernel was obtained in [4] when developing the EP by expressing the time-varying window as a set of orthogonal functions.

3.2 From Evolutionary Transform to Slepian Evolutionary Periodogram

Stationary and nonstationary random processes can be represented by general orthogonal expansions as proposed by Priestley [19]. Discrete form of prolate spheroidal wave functions (PSWF) [18] can be used efficiently for signal representation [17] and called discrete prolate spheroidal sequences (DPSS). These sequences are also known as Slepian sequences. In general, a signal $x[n]$ can be represented in terms of an orthogonal basis $\{\phi_k[n]\}$ as,

$$x[n] = \sum_{k=0}^{K-1} d_k \phi_k[n], \quad 0 \le n \le N-1, \tag{19}$$
$$d_k = \sum_{n=0}^{N-1} x[n]\phi_k^*[n], \quad 0 \le k \le K-1.$$

We showed in [17] that $x[n]$can be written as follows:

$$x[n] = \sum_{k=0}^{K-1} \underbrace{\left[d_k \phi_k[n] e^{-j\omega_k n}\right]}_{X(n,\omega_k)} e^{j\omega_k n}, \tag{20}$$

where $\omega_k = 2\pi \frac{k}{N}$. We can obtain the evolutionary kernel $X(n, \omega_k)$ in terms of $x[n]$ by replacing the d_k coefficients with their definition in (19)

$$\begin{aligned} X(n,\omega_k) &= d_k \phi_k[n] e^{-j\omega_k n} \\ &= \sum_{m=0}^{N-1} x[m] W_k(n,m) e^{-j\omega_k m}, \end{aligned} \tag{21}$$

where $W_k(n, m) = \phi_k[n]\phi_k^*[m]e^{-j\omega_k(n-m)}$. In order to obtain the evolutionary kernel, specifically the window $W_k(n, m)$, we considered DPSS $\{\phi_k[n]\}$ as the bases of the representation in [17]. Accordingly, by taking the magnitude square as $|X(n, \omega_k)|^2$, we obtain the ES.

3.3 Evolutionary Slepian Estimator

The use of PSWF as data tapers for analysis of nonstationary and nonlinear time series is not new [21]. Indeed, PSWF have also been used in many other applications, one example is in communication theory [22] and their mathematical properties and computation are presented in [23]. Discrete form of the PSWF i.e., DPSS resulted from the work of Slepian about the problem of concentrating a signal jointly in temporal and spectral domains [18]. Given N and $0 < \Omega < 1/2$, the DPSS are a collection of N real valued, strictly bandlimited $|f| \le \Omega$ discrete time sequences $\phi_{N,\Omega} = \left[\phi_{N,\Omega}^{(1)}, \phi_{N,\Omega}^{(2)}, \cdots, \phi_{N,\Omega}^{(N)}\right]$ with their corresponding eigenvalues $1 > \lambda_{N,\Omega}^{(1)} > \lambda_{N,\Omega}^{(2)} \cdots \lambda_{N,\Omega}^{(N)} > 0$. The second Slepian sequence is orthogonal to the first Slepian sequence. The third Slepian sequence is orthogonal to both the first and second Slepian sequences. Continuing in this way, the Slepian sequences form an orthogonal set of bandlimited sequences.

There are $2N\Omega - 1$ Slepian sequences with energy concentration ratios approximately equal to one and for the rest, the concentration ratios begin to approach zero, (See Fig.1). For a given integer $K \le N$, we can get $N \times K$ matrix formed by taking the first K columns of $\phi_{N,\Omega}$. When $K \approx 2N\Omega$, it is a highly efficient basis that captures most of the signal energy. The details on how well signals can be represented using the DPSS can be found in [23, 22]. In the following,

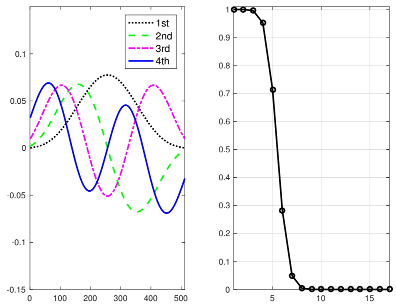

Figure 1: Left: First four Slepian sequences for chosen N=512 and NΩ=3.5; right: energy concentrations i.e., eigenvalues.

we derive an estimator for ES using Slepian sequences for solution of blind source separation (BSS) problem in the spectral domain. Let us start with the basic definition of DET and apply windowing as follows:

$$x[n] = \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} d_{m,k} \phi_k[n] h(n - mL), \tag{22}$$

if we modify (22) by multiplying with both $e^{-j\omega_k n}$ and $e^{j\omega_k n}$ (i.e., no effect introduced as $e^{-j\omega_k n} e^{j\omega_k n} = 1$), we obtain the transform from which we compute the evolutionary kernel $X(n, \omega_k)$ as

$$x[n] = \sum_{k=0}^{K-1} \underbrace{\left[\sum_{m=0}^{M-1} d_{m,k} \phi_k[n] h(n - mL) e^{-j\omega_k n}\right]}_{X(n,\omega_k)} e^{j\omega_k n}.$$

The coefficients $d_{m,k}$ can be calculated as follows

$$d_{m,k} = \sum_{\ell=0}^{N-1} x[\ell]\phi_k^*[\ell]\gamma^*(\ell - mL). \tag{23}$$

Rewriting $X(n, \omega_k)$ by replacing $d_{m,k}$ with (23), we obtain

$$X(n,\omega_k) = \sum_{\ell=0}^{N-1} \sum_{m=0}^{M-1} x(\ell)\phi_k^*[\ell]\phi_k[n]\varphi[n,\ell]e^{-j\omega_k n}, \tag{24}$$

where $\varphi[n,\ell] = h(n-mL)\gamma^*(\ell-mL)$ for $h(n-mL)$ and $\gamma^*(\ell-mL)$ being Gaussian functions. Arranging the terms by multiplying with $e^{-j\omega_k \ell}$ and $e^{j\omega_k \ell}$ again and rearranging

$$X(n,\omega_k) = \sum_{\ell=0}^{N-1} x(\ell) W(n,\ell) e^{-j\omega_k \ell}, \qquad (25)$$

gives us an expression similar to STFT for the evolutionary kernel $X(n,\omega_k)$ and also the time-frequency dependent window $W(n,\ell)$ where

$$W(n,\ell) = \sum_{m=0}^{M-1} \phi_k^*[\ell]\varphi[n,\ell] e^{j\omega_k \ell} \phi_k[n] e^{-j\omega_k n}. \quad (26)$$

Accordingly, we can obtain the proposed spectrum which we call the windowed evolutionary Slepian spectrum (WESS) by taking the magnitude square as $S = |X(n,\omega_k)|^2$ and illustrate in the simulations.

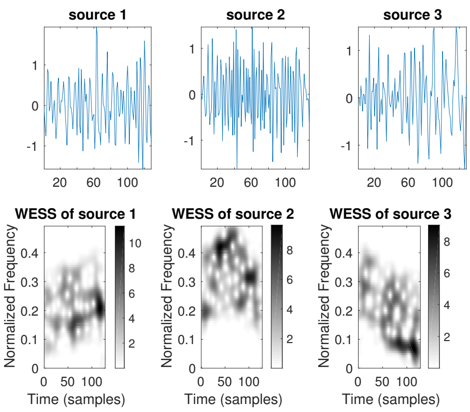

Figure 2: TVARMA sources and their corresponding windowed evolutionary Slepian spectra.

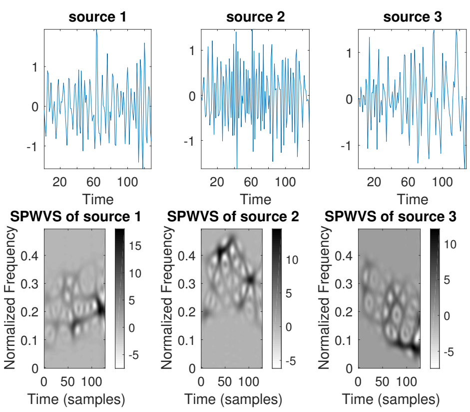

Figure 3: TVARMA sources and their corresponding SPWV spectra.

We will compare the Smoothed Pseudo Wigner-Ville (SPWV) distribution (for reduction of cross terms) [15] to the WESS in our experiments for the BSS problem, as we present in Section 5.

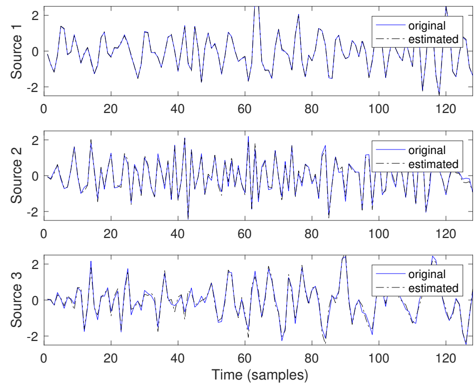

Figure 4: Separation of sources from noiseless observations and comparison with actual sources using windowed evolutionary Slepian transform.

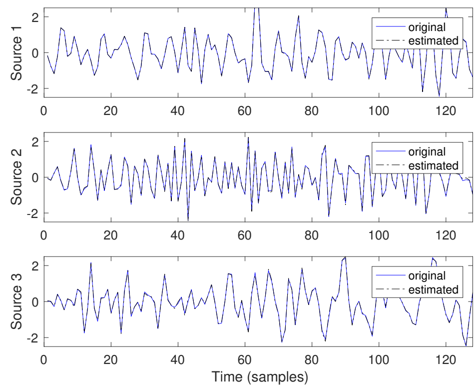

Figure 5: Separation of sources from noiseless observations and comparison with actual sources using SPWV distributions.

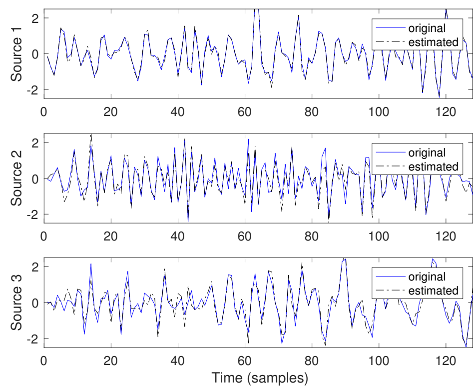

Figure 6: Separation of sources from noisy observations (SNR 20 dB) and comparison with actual sources using windowed evolutionary Slepian transform.

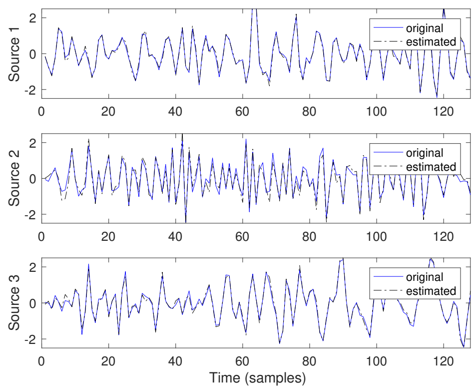

Figure 7: Separation of sources from noisy observations (SNR 20 dB) and comparison with actual sources using SPWV distributions.

4 Blind Source Separation Problem

4.1 Problem Formulation

Blind source separation (BSS) covers a wide range of applications in diverse fields such as digital communications, pattern recognition, biomedical engineering, and financial data analysis, among others. Separation of unknown signals that have been mixed in an unknown way has been a topic of great interest in the signal processing community, as well. In general, the available BSS methods use the following data model for each signal received at each sensor [12]:

$$\mathbf{x}[n] = \mathbf{C}\mathbf{s}[n] + \mu[n], \tag{27}$$

such that

- $\mathbf{x}[n] = [x_1[n], \ldots, x_p[n]]^T$ is a p vector of observations,
- $\mathbf{s}[n] = [s_1[n], \ldots, s_q[n]]^T$ is a q vector of unknown sources,
- $\mathbf{C}$ is a $p \times q$ mixing or array matrix,
- $\mu[n]$ is a zero-mean, σ^2 variance white noise vector.

The objective is to obtain an estimate $\hat{\mathbf{C}}$ of $\mathbf{C}$ and obtain sources as

$$\hat{\mathbf{s}}[n] = \hat{\mathbf{C}}\mathbf{x}[n] \approx \mathbf{G}\mathbf{s}[n] + \hat{\mathbf{C}}^{\#}\mu[n] \tag{28}$$

where $\#$ represents pseudoinverse and $\mathbf{G}$ is a matrix with only one nonzero entry per row and column [12]. In particular, the approaches using time-frequency signal representations for BSS involve the following steps [25]:

- Estimation of the spatial time-frequency spectra,
- Estimation of whitening matrix and noise variance,
- Joint-diagonalization of the noise compensated and whitened spatial time-frequency spectra matrices.

The details of these steps and full implementation of BSS can be found in [13, 25, 26].

4.2 Spatial Evolutionary Transform and BSS

In time-frequency approach for BSS, using the data model received at each sensor, the cross-power spectral estimate can be written as [6],

$$\hat{\mathbf{S}}_{xx}(n,\omega) = \mathbf{C}\hat{\mathbf{S}}_{ss}(n,\omega)\mathbf{C}^H + \sigma^2\mathbf{b}[n]^H\mathbf{b}[n]\mathbf{I}. \tag{29}$$

In this paper, in the equation above, $\hat{\mathbf{S}}_{xx}(n,\omega)$ is the evolutionary spatial Slepian estimate as (16). Representing $\mathbf{W}$ as the $p \times q$ whitening matrix and letting $\mathbf{U} = \mathbf{W}\mathbf{C}$, whitened and noise compensated matrices are

$$\begin{aligned} \tilde{\mathbf{S}}_{xx}(n,\omega) &= \mathbf{W}(\hat{\mathbf{S}}_{xx}(n,\omega) - \sigma^2\mathbf{I})\mathbf{W}^{\mathbf{H}} \\ &= \mathbf{U}\hat{\mathbf{S}}_{ss}(n,\omega)\mathbf{U}^H \end{aligned} \tag{30}$$

where $\mathbf{U}$ is unitary and diagonalizes the cross-power spectral estimate $\tilde{\mathbf{S}}_{xx}(n,\omega)$ for any (n,ω) [24, 25, 26]. The unitary matrix can be estimated from the eigenvectors of any $\tilde{\mathbf{S}}_{xx}(n,\omega)$ with distinct eigenvalues and the mixing matrix is obtained using $\mathbf{C} = \mathbf{W}^{\#}\mathbf{U}$. The source signals are then estimated as in (28) [25].

5 Experimental Results

In our experiments, we test the applicability of Slepian transform as a valid spectrum estimator of nonstationary signals for BSS problem in particular for a time varying autoregressive moving average (TVARMA) process. Using the time-frequency representation based BSS algorithm in [25], for simulation of an overdetermined case, we use three sources and four observations. The observations are considered as noise free and noisy (20 dB SNR). We compare our method with the smoothed pseudo Wigner Ville Spectrum (SPWVS) of the sources (See Fig. 2 and Fig. 3). SPWV representation was preferred for increased readability (i.e., for the reduction in the number of cross-terms that happen in the Wigner Ville spectrum) [15]. The separation of three sources and estimation from noise free observations using the proposed method is presented in Fig. 4 and estimation using the SPWV representation is presented in Fig. 5 with the same BSS algorithm given in [25]. As we see from these figures, the results are very similar; however SPWV representation showing slightly more precise estimation performance. Then we test the estimation of sources using noisy observations for SNR 20 dB. Again, the results are very similar for the proposed method and the SPWV representation, with the SPWV representation showing slightly more precise estimation (See Fig. 6 and Fig. 7).

6 Conclusion

In this paper, we defined a spectral estimation method using Slepian sequences similar to the evolutionary periodogram and showed that Slepian transform can be used for blind source separation problem for nonstationary signals. The proposed method perform similar to the smoothed pseudo Wigner Ville representation for noiseless and noisy cases (SNR 20 dB). We will consider robust blind source separation algorithms for performance improvement of noisy observations as our future work. Additionally, we will analyze a wide variety of processes with increased number of sources and observations for more detailed observations of nonstationary processes in BSS problem.

References:

[1] Boashash, B.: 'Time-frequency signal analysis,' Advances In Spectrum Analysis And Array Processing, vol. 1, S. Haykin, Ed. NJ: Englewood Cliffs Prentice-Hall, (1991).

[2] Priestley, M. B.: 'Power spectral analysis of nonstationary random processes,' J. Sound Vib, vol. 6, pp. 86–97, (1967).

[3] Kayhan, A., S., El-Jaroudi, A., and Chaparro, L. F.: 'Wold-Cramer evolutionary spectral estimators', Proc. IEEE-SP Int. Sympos. Time-Frequency Time-Scale Analysis, Victoria, Canada, pp. 115–118, (1992).

[4] Kayhan, A. S., El-Jaroudi, A., and Chaparro, L. F.: 'Evolutionary periodogram for nonstationary Signals,' IEEE Trans. on Signal Processing, vol. 42, pp. 1527–1536, (1994).

[5] Suleesathira, R., Chaparro, L. F. and Akan, A.: 'Discrete evolutionary transform for time-frequency analysis,' Proc. Asilomar Conference on Sig., Sys. and Comp., pp. 812–816, (1998).

[6] Kayhan, A. S., and Moeness, G. A.: 'Spatial evolutionary spectrum for DOA estimation and blind signal estimation,' IEEE Trans. on Signal Processing, vol. 48, pp. 791–797, (2000).

[7] Hosseini, S., and Deville, Y.: 'Blind separation of nonstationary sources by spectral decorrelation,' In: Puntonet, C.G., Prieto, A.G. (eds.) ICA 2004. LNCS, vol. 3195, pp. 279–286. Springer, Heidelberg (2004).

[8] Johnson, D. H. and Dudgeon, D. E.: 'Array Signal Processing: Concepts and Techniques,' Englewood Cliffs, NJ: Prentice-Hall, (1993).

[9] Guo, J., Zeng, X., and She, Z.: ' Blind source separation based on high-resolution time-frequency distributions,' In: Computers and Electrical Engineering, vol. 38, Issue 1, pp. 175–184, (2012).

[10] Guo, Q., Ruan, G., and Liao, Y.: 'A time-frequency domain underdetermined blind source separation algorithm for MIMO radar signals,' Symmetry, vol. 9, (2017).

[11] Pal, M., Roy, R., Basu, J. and Bepari, M.: 'Blind source separation: A review and analysis,' Asian Spoken Language Research and Evaluation Conference, pp. 1–5, (2013).

[12] Belouchrani, A., Meraim, K., Cardoso, J. F., and Moulines, E.: 'A blind source separation techniques using second-order statistics,' IEEE Trans. Signal Processing, vol. 45, pp. 434–444, (1997).

[13] Belouchrani, A. and Amin, A.: 'Blind source separation based on time-frequency signal representation,' IEEE Trans. Signal Processing, vol. 46, pp. 2888–2897, (1998).

[14] Sekihara, K, Nagarajan, S., Poeppel, D., and Miyashita, Y.: 'Time-frequency MEG-MUSIC algorithm,' IEEE Trans. Med. Imag., vol. 18, pp. 92–97, (1999).

[15] Cohen, L.: 'Time-frequency analysis,' Englewood Cliffs, NJ: Prentice-Hall, (1995).

[16] Bohme, J. F.: 'Array processing in semi-homogeneous random fields,' in Proc. Septieme Colloque Sur le Traitment di Signal est Ses Applications, pp. 104/1–104/4, (1979).

[17] Oh, J., Senay, S. and Chaparro, L. F.: 'Signal reconstruction from nonuniformly spaced samples using evolutionary Slepian transform-based POCS,' EURASIP Journal on Advances in Signal Processing - Special issue on applications of time-frequency signal processing in wireless communications and bioengineering, (2010).

[18] Slepian, D.: 'Prolate spheroidal wave functions, Fourier Analysis and Uncertainty IV: extensions to many dimensions; generalized prolate spheroidal functions,' Bell Syst. Techn. Journal, vol. 43, pp. 3009–3057, (1962).

[19] Priestley, M. B.: 'Spectral Analysis and Time Series,' Academic Press, (1981).

[20] Melard G. and Schutter, A. H.: 'Contributions to evolutionary spectral theory,' Journal of Time Series Analysis, vol. 10, pp. 41–63, (1989).

[21] Moghtaderi, A., Takahara, G. and Thomson, D. J.: 'Evolutionary spectrum estimation for uniformly modulated processes with improved frequency resolution,' IEEE Workshop on Statistical Signal Processing, (2009).

[22] Moore, I. C. and Cada, M.: 'Prolate spheroidal wave functions: An introduction to the Slepian series and its properties,' Applied and Computational Harmonic Analysis, vol. 16, pp. 208–230, (2004).

[23] Walter, G. G. and Shen, X.: 'Sampling with prolate spheroidal functions,' Am. Math. Soc. Abstract, pp. 41–227, (2003).

[24] Tong, L., Liu. R, Soon, V. and Huang, Y.: ' Indeterminacy and identifiability of blind identification,' IEEE Trans. on Circuits and Systems, vol. 38, pp. 499–509, (1991).

[25] Fevotte, C. and Doncarli, C.: 'Two contributions to blind source separation using time-frequency distributions,' IEEE Sig. Pro. Letters, vol. 11, pp. 386–389, (2004).

[26] Cardoso, J. F. and Souloumiac, A.: 'Blind beamforming for non Gaussian signals,' Proc. Inst. Elect. Eng. vol. 140, pp. 362–370, (1993).