Error Reconstruction Functions of Non-Optimal algorithms based on Atomic Functions

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Abstract—A non-optimal algorithm based on atomic function is investigated for the Sampling – Reconstruction Procedure (SRP) of Gaussian process realizations. The reconstruction error function of this non-optimal algorithm is compared with the reconstruction error functions obtained in the optimal algorithm, represented by the conditional mean rule, and in another non-optimal algorithm based on Balakrishnan's theorem. Results show that the application of atomic function has disadvantages reflected in a bigger magnitude of the error reconstruction.

Keywords—Atomic functions, Conditional mean rule, Error reconstruction function, Non-optimal algorithm.

1 Introduction

There have been several investigations to try to establish a statistical description that defines the reconstruction of the realizations of a random process through the set of their samples. This problem is called by the Sampling – Reconstruction Procedure (SRP). Perhaps the most known work devoted to this problem was done by A. Balakrishnan in his theorem (BT) [1]. He mentioned that any realization of a stationary random process $x(t)$ with a power spectrum $S(\omega)$ restricted by the boundary frequency ω_b can be reconstructed by:

$$\hat{x}(t) = \lim_{N \to \infty} \sum_{j=-N}^{j=N} x(T_j) \frac{\sin \omega_b (t - \Delta T_j)}{\omega_b (t - \Delta T_j)} \qquad (1)$$

here $x(T_j)$ is a sample in the time T_j, ΔT is the sampling interval determined by $\Delta T = \pi / \omega_b$ and $2N + 1$ is the number of samples taking into account in the reconstruction algorithm. As one can see, the base function $\phi_j(t)$ for any sample $x(T_j)$ is the *sinc* function

$$\varphi_j(t) = \frac{\sin \omega_b (t - \Delta T_j)}{\omega_b (t - \Delta T_j)} = \sin c(\cdot). \qquad (2)$$

A.Balakrishnan [2] wrote that "regardless of statistics, whether Gaussian or not, the best mean-square estimate of $x(t)$ from $x(T_j)$ is linear" and it is determined by (1). This point of view is practically not refuted in the literature. Furthermore, the reconstruction error is equal to zero for all types of the processes with a restricted power spectrum.

It is important to mention some restrictions and disadvantages of BT [3]: 1) This theorem does not take into account the most important characteristic of each stochastic process – its probability density function (pdf). 2) The conditions of BT are not realizable because the sampled process is singular and the number of samples is equal to infinity. 3) Following BT, the sampled process must be stationary. 4) Among many statistical characteristics of a sampled process, BT limits its focus on only the numerical value: the boundary frequency ω_b of the power spectrum $S(\omega)$. 5) Following BT, other very important statistic characteristics, like pdf, the covariance function $K(t_1, t_2)$ or the type of the power spectrum $S(\omega)$ $(\omega < \omega_b)$, do not influence outcome, whether in the reconstruction function or in the error reconstruction function. 6) BT declares that realizations of all types of random stationary processes can be optimally reconstructed by the unique linear algorithm with the base function-*sinc*. 7) No matter what types of random process is considered, with BT the reconstruction error function is equal to zero and the reconstruction is based on the same function. 8) The formula (2) determines the impulse response of the linear non-realizable filter and the summing procedure (1) demands infinite delay.

The application of the Conditional Mean Rule (CMR) in the statistical SRP description of random process realizations provides a possibility to make some important conclusions:
1) Every random process must have its own reconstruction algorithm and its own reconstruction error function. 2) BT is valid for Gaussian processes only. 3) The linear reconstruction algorithm is valid for Gaussian processes only. 4) If the Gaussian process has a limited spectrum and the number of samples is finite, the *sinc* function is not the base optimal function. 5) In SRP description of Gaussian realizations, the reconstruction error function does not depend on the values of samples, but depends only in respect to axis of time. 6) BT is a particular case of the CMR algorithm for Gaussian processes with a finite spectrum.

Following Balakrishnan's theorem the reconstruction error is equal to zero. This result is valid, when the number of samples N is infinite, though this condition is not realizable. But considering a limited number of samples, the reconstruction error is different to zero, and for the Gaussian processes it can be found by the formula:

$$\hat{\sigma}^2 = 1 - 2\sum_{j=1}^{N} R(t - T_j)\phi(t - T_j)$$
$$+ \sum_{j=1}^{N}\sum_{i=1}^{N} R(T_j - T_i)\phi(t - T_j)\phi(t - T_i), \tag{3}$$

where $R(\cdot)$ is the normalized covariance function.

In recent years, in order to overcome these drawbacks, one methodology has been studied extensively. It is based on the *conditional mean rule*. On the basis of this method one can analyze the SRP of random processes with different types of probability density functions, taking into account the following aspects: the process can be stationary or non-stationary; the number of samples is arbitrary and limited; the intervals between neighbor samples can be arbitrary or periodical; etc. The rule provides the reconstruction procedure of any realization of a random process with the minimum error possible. In this case the main statistical properties of the given process are taking into account. It gives a possibility to obtain for any random process its own *optimal reconstruction algorithm* and the *optimal reconstruction error algorithm* [2]-[10].

Besides Balakrishnan's theorem, there are some others reconstruction methodologies which are used other set of orthogonal functions. For example, there is the Khurgin-Yakovlev's theorem [11]. The main idea of this methodology is connected with using the set of derivatives of a given function instead of the set of samples of the same function. Generally one can use all derivatives of an arbitrary order M. The method is valid for functions with a restricted spectrum on the boundary frequency ω_b. The statistical expression of Khurgin-Yakovlev's theorem is:

$$g(t) = \sum_{i=0}^{M-1} \sum_{j=-\infty}^{\infty} g^{(i)}(Mj\Delta T)(t - Mj\Delta T)^i \frac{\left[\sin c(q)\right]^i}{i!}$$

(4)

Where $g^{(i)}(Mj\Delta T)$ is the derivative of the order i of the function $g(t)$ with $\Delta \le 0.5\omega_b$, $q = \pi(t - Mj\Delta)/N\Delta$,

$$\sin c(q) = \frac{\sin(q)}{(q)}.$$

It is quite possible to use this algorithm for the SRP of realizations of random processes as well. But its effectiveness is not good because it is necessary to send all derivative values in a separate channel. The method using a set of samples of a given function is more productive because the reconstruction error is smaller (see [4, 9] and citation there).

There are some generalizations of Sampling Theorem (1) on the basis of so called atomic functions [12 – 13, 15 - 17]. There are algorithms of Strang - Fix and of Levitan. In the last variant one can introduce some trigonometric series in order to use in the description of SRP of various functions [13]. Below we concentrate our attention on the direct SRP algorithm based on *atomic functions*. Because there are not any English publications about atomic functions, we give some initial information about such functions here.

2 About Atomic Functions

The theory of these functions was described by V. L. Rvachiov by first time in [12]. The application of this theory in SRP problems was suggested for *deterministic* functions [13, 16].

The atomic functions are characterized by one principal property: the analytical expression for the derivative of any order of a function is the same analytical expression of the original function, though the amplitudes, durations and signs of the produced functions by the derivatives could be different. The atomic functions $h_\beta(x)(\beta > 2)$ are the solutions of the differential equation [12, 13]:

$$y'(x) = \frac{\beta^2}{2}\left[y(\beta x + 1) - y(\beta x - 1)\right].$$

(5)

The Fourier transformation for the function $h_\beta(x)$ is:

$$F_\beta(\rho) = \prod_{j=1}^{\infty} \sin c\left(\frac{\rho}{\beta^j}\right).$$

(6)

This function tends to zero at points $2\pi j$, $(j \ne 0)$. In Fig. 1 are presented the functions $h_\beta(x)$ for different values β. While Fig. 2 shows the normalized first order derivative for each type of function expressed in Fig. 1.

In Fig. 3 are the non-normalized derivatives of the function $h_2(x)$. In these graphs, the form of the curves of the different order derivatives is the same, but the amplitudes, the durations and the signs vary. In [12, 13] are investigated more types of atomic functions in detail.

Any random realization has many ways to be reconstructed. It means one can use an alternative methodology to the optimal methodology according the properties of each realization. This parallel technique can have some others statistical parameters than the optimal technique uses to make the reconstruction. Because of this, such reconstruction is not adequate, so it is called *non-optimal reconstruction algorithm*. Balakrishnan's

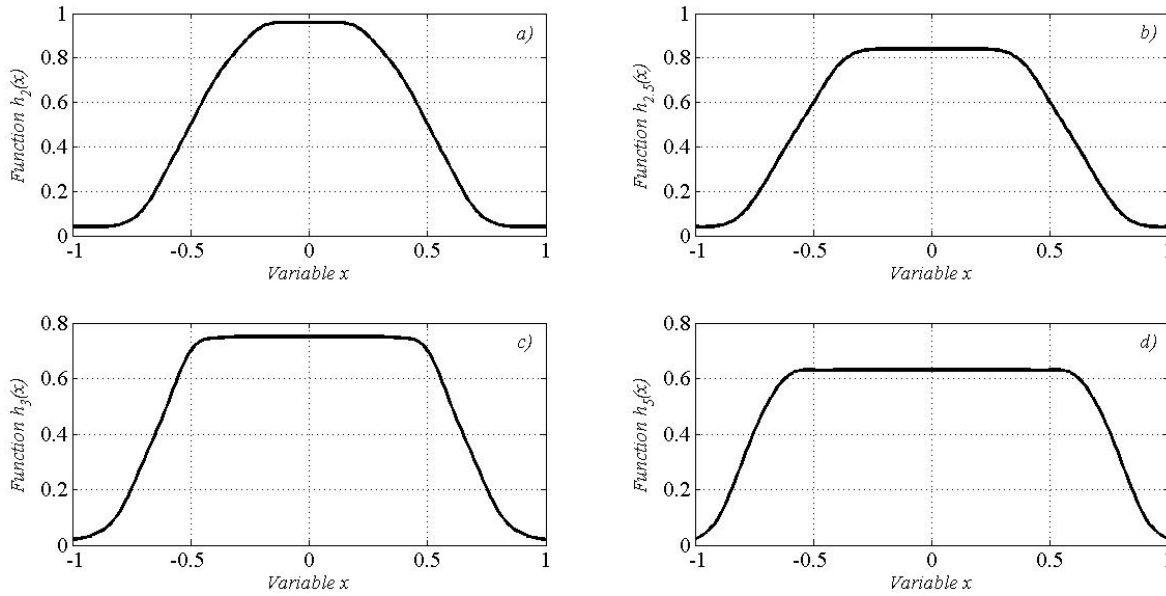theorem with the limited number of samples *N* is

Fig. 1. Function $h_\beta(x)$ for: a) $\beta = 2$, b) $\beta = 2.5$, c) $\beta = 3$ and $\beta = 5$.
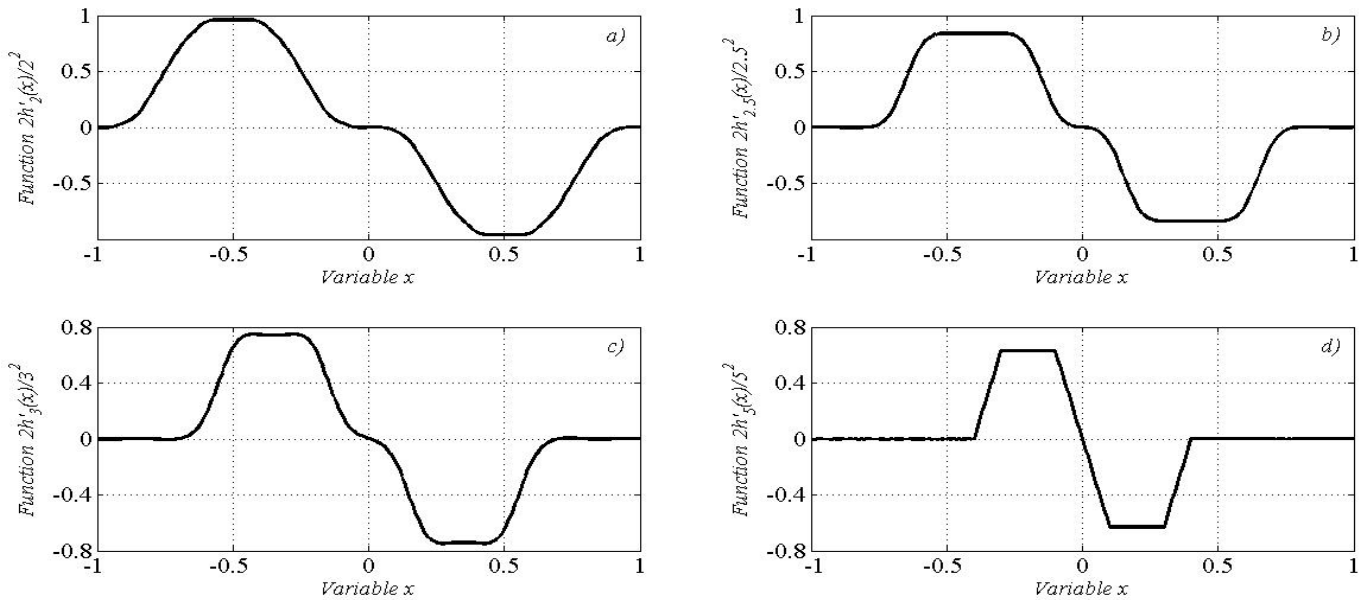

Fig. 2. Normalized derivative of the function $h_\beta(x)$ for: a) $\beta = 2$, b) $\beta = 2.5$, c) $\beta = 3$ and $\beta = 5$.

Also, the conditional mean rule algorithm can be considered as a non-optimal algorithm if it does not take into account the appropriate parameters. The main reason to apply such algorithms is having a simpler methodology.

The present paper is devoted to the investigation of a new non-optimal algorithm based on atomic functions. Besides this, it is necessary to compare the reconstruction SRPquality of Gaussian process realizations.

Therre are three the most interesting variants: the optimal algorithm based on the conditional mean rule, Balakrishnan's algorithm, and the non-optimal algorithm based on atomic functions. The number of samples is finite in all cases.

The investigation is centered on the statistical SRP description

3 The Optimal Reconstruction Algorithm

The optimal algorithm is based on the conditional mean rule. It allows obtaining the reconstruction of realizations of a random process by the knowledge of their complete statistical description.

The main idea of this methodology has been proposed in [2] (see also [4, 9, 10]). Firstly, let us consider a random process $x(t)$ characterized by its multidimensional probability functions $w_m[x(t_1), x(t_2),..., x(t_m)]$. One realization of this process is sampled in time instant $T = \{T_1, T_2,..., T_N\}$. Therefore, there is a set of samples $X, T = x(T_1), x(T_2),..., x(T_N)$, where the number of samples N and their times of occurrence T_i are arbitrary. It means that the probability density function, all initial and central moment functions are conditional now.

The conditional mean function $\tilde{m}(t) = \langle x(t) | X, T \rangle$ is used as reconstruction function. The quality of the reconstruction is evaluated by the conditional variance function $\tilde{\sigma}^2(t) = \langle [x(t) - \tilde{m}(t)]^2 | X, T \rangle$ or reconstruction error function. Both characteristics $\tilde{m}(t)$ and $\tilde{\sigma}^2(t)$ can be found on the basis of the conditional multidimensional pdf $w_{N+1}(x(t) | X, T)$ of the given process. It is clear that one can't know exactly the sampled realization, but with this rule it is possible to reconstruct the realization. The rule also provides the minimum estimation reconstruction error for realizations with an arbitrary pdf.

The investigation is centered on the statistical Sampling – Reconstruction Procedure description of Gaussian process.

Considering that the realization sampled is Gaussian, their conditional characteristics are [14]:

$$\tilde{m}(t) = m(t) + \sum_{i=1}^{N} \sum_{j=1}^{N} K(t, T_i) a_{ij} [x(T_j) - m(T_j)], \quad (7)$$

$$\tilde{\sigma}^2(t) = \sigma^2(t) - \sum_{i=1}^{N} \sum_{j=1}^{N} K(t, T_i) a_{ij} K(T_j, t), \quad (8)$$

where $m(t)$ and $\sigma^2(t)$ are respectively the mathematical expectation and the variance of the initial process $x(t)$; $K(\cdot)$ is the covariance function, and a_{ij} represents the elements of the inverse covariance function. Let assume that the realization is stationary. Let us assume: $m(t) = 0$ and $\sigma^2(t) = 1$. From (7) and (8), one can see: the reconstruction function is the linear function of samples and the reconstruction error function does not depend on samples..
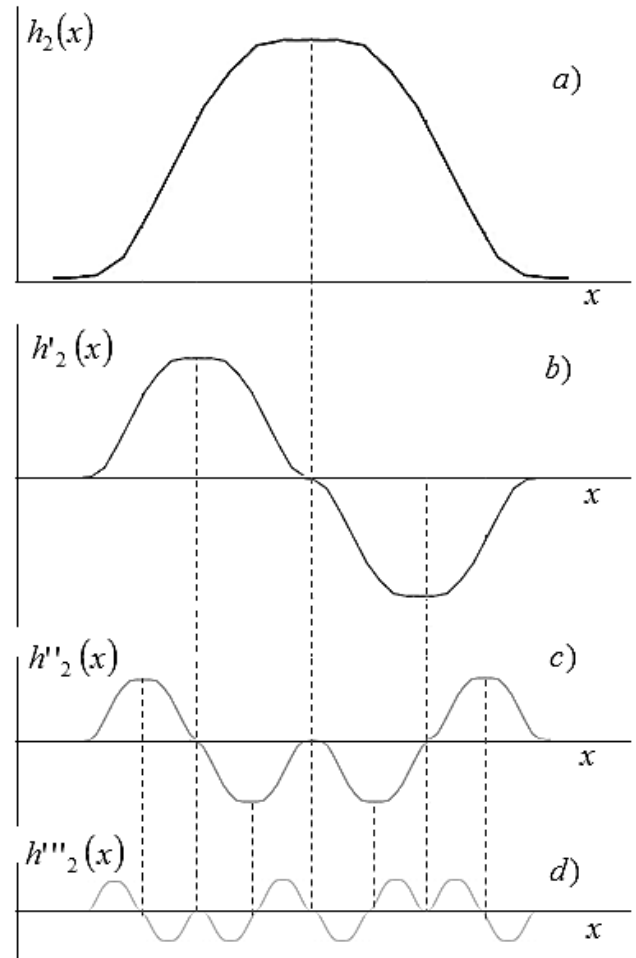

Fig. 3. Non-normalized derivatives of the function $h_2(x)$.

4 The Non-optimal Reconstruction Algorithm

Now we shall discuss the non-optimal algorithm based on atomic functions. For making the reconstruction of any realization of a given process with its restricted power spectrum, it is possible to use the Fourier transformation (6) of

the atomic functions [12, 13, 15 - 17]. The reason is that the zeros of (6) are located periodically. Besides this, the functions described by (6) tend to zero in the infinite faster than other functions [13]. Considering this methodology, the reconstruction function $\hat{m}(t)$ for any random processes with a limited power spectrum on the basis of the samples $x(\Delta T_i)$ is:

$$\hat{m}(t) = \sum_{i=-\infty}^{\infty} x(\Delta T_i) F_\beta \left[\frac{\pi}{\Delta T_i}(t - \Delta T_i) \right], \qquad (9)$$

where $F_\beta(\rho)$ is given by (6), and [13]:

$$\beta > 2; \Delta T \le \frac{\pi}{\omega_b} \frac{\beta - 2}{\beta - 1}; \Delta T \le \frac{\pi}{\omega_b}. \quad (10)$$

Then the equation (9) changes to:

$$\hat{m}(t) = \sum_{i=-\infty}^{\infty} x(\Delta T_i) \prod_{j=1}^{\infty} \sin c \left(\frac{\beta \pi}{\Delta T_i \beta^j}(t - \Delta T_i) \right). \qquad (11)$$

Expression (11) can be interpreted as the disintegration of the function $x(t)$ on the basis of the displacement-narrowing of the imaginary Fourier functions of the atomic function $h_2(\cdot)$. For its application it can be restricted by a finite amount on the right side, being indicated as [13]:

$$\hat{m}(t) = \sum_{i=-\infty}^{\infty} x(\Delta T_i) \prod_{j=1}^{N} \sin c \left(\frac{\beta \pi}{\Delta T_i \beta^j}(t - \Delta T_i) \right), \qquad (12)$$

when:

$$\beta \left(1 - \beta^{-N}\right) > 2; \Delta T = \frac{\pi}{\omega_b} \frac{\beta \left(1 + \beta^{-N}\right) - 2}{\beta - 1}. \quad (13)$$

The minimum values are determined by the solution of the equation $\beta\left(1 + \beta^{-N}\right) = 2$. When $N = 1$, the WKS series is obtained; and if $N = \infty$, the series (13) is transformed into (12).

To calculate the reconstruction error, let us introduce the base function $\varphi_j(t)$:

$$\varphi_j(t) = \prod_{j=1}^{N} \sin c \left(\frac{\beta \pi}{\Delta T_j \beta^j}(t - \Delta T_j) \right). \qquad (14)$$

Then, assuming a finite number of samples N, the reconstruction error function is [18]:

$$\hat{\sigma}^2(t) = 1 - 2 \sum_{j=1}^{N} R(t - \Delta T_j) \varphi(t - \Delta T_j) + \sum_{j=1}^{N} \sum_{i=1}^{N} R(\Delta T_j - \Delta T_i) \varphi(t - \Delta T_j) \varphi(t - \Delta T_i) \qquad (15)$$

Here there is the same situation like in (7), (8): the reconstruction function depends on the values of the samples and the error reconstruction function does not depend on them.

5 Comparisons Between Both Reconstruction Algorithms

Let us chose a realization from a Gaussian Markovian process. It is formed on the output of an one-stage integrated *RC* circuit driven by Gaussian white noise $n(t)$. The normalized covariance function $R(\tau)$ and the power spectrum density $S(\omega)$ are:

$$R(\tau) = \exp(-\alpha|\tau|), \qquad (16)$$

$$S(\omega) = \frac{2\alpha}{\alpha^2 + \omega^2}. \qquad (17)$$

As the covariance time τ_c is unitary, then $\alpha = 1$.

The Gaussian Markovian process is not differentiable because the variance of its derivative is equal to infinity. Restricting the spectrum (17) in the frequency ω_b, the properties of the given process change radically. The resulting process is non-Markovian and infinitely differentiable. So, the covariance function now is:

$$R(\tau) = \left(\int_{-\omega_b}^{\omega_b} \frac{d\omega}{\beta^2 + \omega^2} \right)^{-1} \int_{-\omega_b}^{\omega_b} \frac{e^{j\omega\tau} d\omega}{\beta^2 + \omega^2} . \qquad (18)$$

The first term in (18) ensures the normalization for the power spectrum of the obtained process with any value of the boundary frequency ω_b. Fig 4 shows the normalized covariance function $R(\tau)$ for different values of ω_b. When ω_b is equal to infinity, the curve is similar to the curve obtained with one-stage RC filter.
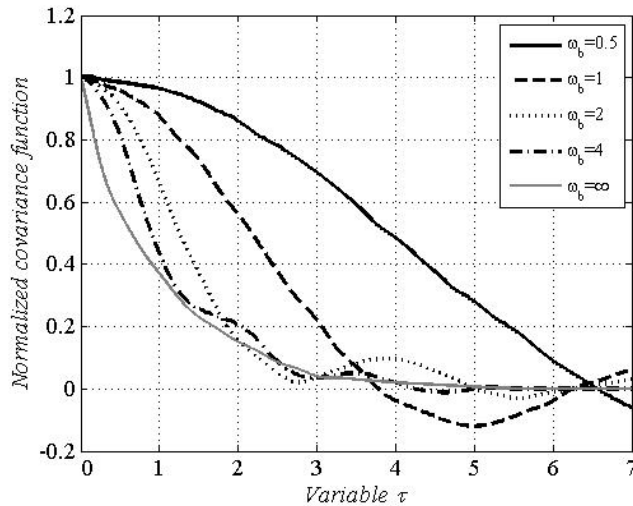

Fig. 4. Normalized covariance function of a Gaussian process with different values of ω_b.

In order to have a complete comparison of the efficiency of the optimal algorithm and the non-optimal algorithms, let us introduce another type of non-optimal algorithm. This algorithm is based on Balakrishnan's theorem [1] when the number of samples N is finite. The reconstruction function and the reconstruction error function are represented by (1), (3) respectively.

Firstly, it is necessary to know the type of base functions for each mentioned algorithms: optimal algorithm (conditional mean rule), non-optimal algorithm of the case A (Balakrishnan's theorem with a finite set of samples), and non-optimal algorithm of the case B (atomic functions). Fig. 5 shows the base functions. All curves have a value of zero at the sampling points, except at the initial point because it corresponds to the multiplication of the sample

with the base function. The forms of the curves are different. In the non-optimal algorithm, the curve of the case A tends to zero more slowly as a consequence of its large peak values. The curve of the case B tends to zero fastest. (This curve was calculated with $\beta = 2.1$ because it is a near value to the limit value which is $\beta = 2$.) This means that the influence between the samples is minimal in this case. There is the same situation if the coefficient $\beta = 4$. When the value β increases, then the amplitude of the oscillations increase as well. However, all base functions of the case B tend to zero faster than the optimal algorithm and the case A of the non-optimal algorithm.
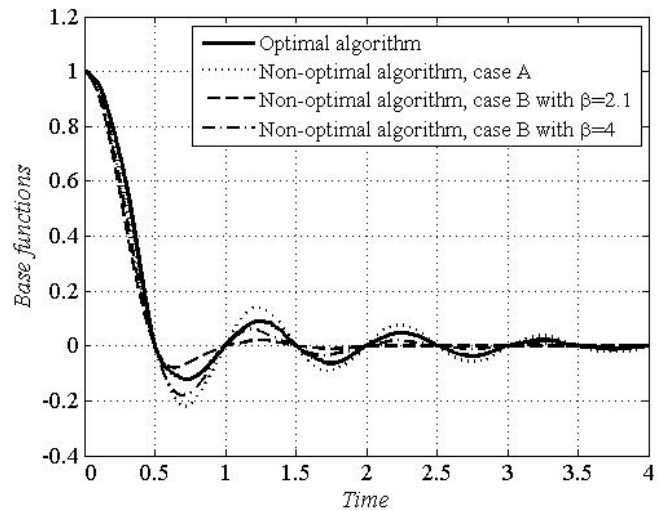

Fig. 5. Comparison of base functions.

Fig. 6 illustrates the error reconstruction function for the case B of the non-optimal algorithm (atomic functions) for various values N and $\beta = 2.1$. It is important to note that the unit of measurement on the x-axis is the boundary frequency ω_b, not the time. Obviously, when the number of samples increases the reconstruction error decreases. The maximum error is obtained at the half of the sampling interval $t = \Delta T / 2$.
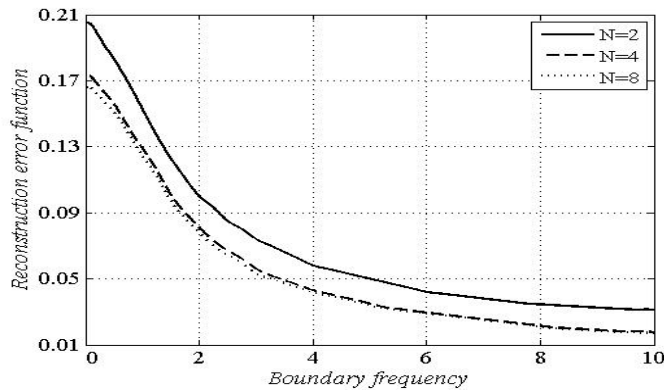
Fig. 6. Reconstruction error functions using atomic functions with N and $\beta = 2.1$.

Now we shall compare the effectiveness of the algorithm with atomic functions the reconstruction errors of the optimal algorithm and the non-optimal algorithm of the case *A*. In Fig. 7 the reconstruction error functions are presented for all above mentioned methodologies with $N = 2$. In Fig. 8 the curves are calculated with $N = 4$. In Fig. 9 the number of samples is equal to $N = 8$.

In the non-optimal algorithm (the case *B)*, when values of the boundary frequency are $\omega_b < 2$, the magnitude of the error increases with respect to the error of the optimal algorithm if the number of samples N grows. When the boundary frequency is $\omega_b > 2$ the difference with the optimal error decreases gradually (with the exception of the situation when $N = 2$ and $\beta = 4$). The curves in the non-optimal algorithm (the case *A*) have a similar behavior like the curves of the optimal algorithm while the number of samples grows.

When $N = 2$, the error in case *A* is the biggest. When $N = 4$, the error in case *A* is smaller than the error in case *B* with $\beta = 2.1$, but bigger than the error in case *B* with $\beta = 4$. When $N = 8$, the error in case *A* is the smallest in all non-optimal cases. The physical interpretation of this effect is related with the special property of atomic functions. They tend to zero very quickly, so the influence between the neighboring samples for making the reconstruction operation is small. So, with $N = 2$ the influence is minimal, and the error grows considerably. With more samples the influence rises, and the error decreases.
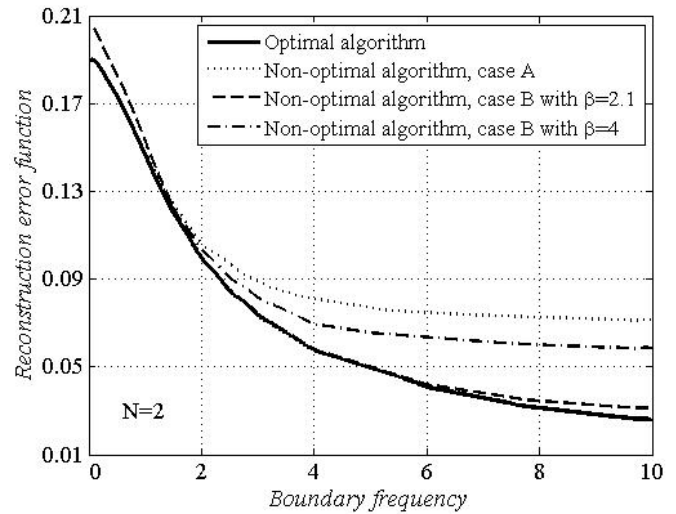

Fig. 7. Comparison of the reconstruction error function with $N = 2$.
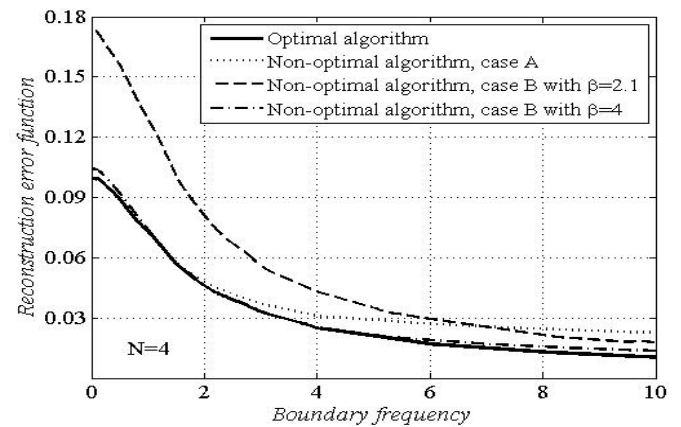

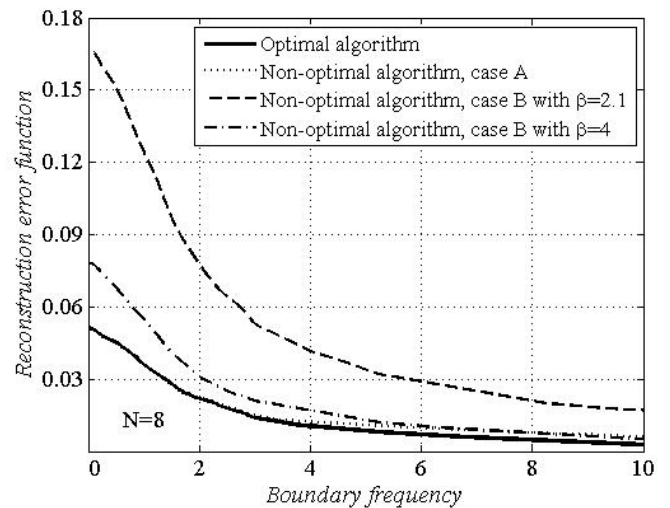Fig. 8. Comparison of the reconstruction error function with $N = 4$.

Fig. 9. Comparison of the reconstruction error function with $N = 8$

Obviously, the magnitude of the error in the optimal algorithm is smaller than the non-optimal algorithms. But this is a natural effect due to the structure of its functions.

6 Conclusions

Three different reconstruction algorithms are analyzed to describe the sampling-reconstruction procedure of realizations of a Gaussian process with a finite power spectrum. Both principal characteristics (reconstruction functions and reconstruction error functions) are calculated. In general, the smallest reconstruction error is obtained by the algorithm with the conditional mean rule, also called optimal algorithm. Then it is the non-optimal algorithm based on the Balakrishnan's theorem. And the biggest reconstruction error is obtained with the non-optimal algorithm based on atomic functions. The non-optimal function does not depend on the covariance function for making the reconstruction operation. They depend on the boundary frequency only. The algorithm on the base of the conditional mean rule takes into account the covariance function of the sampled process, and for this reason the optimal algorithm has the best characteristics of the reconstruction procedures.

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