It can be reminded that the frequency of the rotor current in the actual rotor winding flowing into the converter is ω_{sl} not the stator frequency ω_s. In order to integrate the converter equivalent impedance into the steady-state model with ω_s, Z_{eq} should be divided by slip s. The equivalent impedance referred to the stator side is then given by [11]

$$\frac{\overline{Z}eq}{s} = \frac{R_{eq}}{s} + j\frac{\omega_{sl}L_{eq}}{s} = \frac{R_{eq}}{s} + j\omega_s L_{eq} \quad (17)$$

Where $\omega_{sl} = s.\ \omega_s$.

At unity power factor, the air-gap power(P_{ag}) of the generator can be calculated by:

$$P_{ag} \approx 3(V_s - I_s R_s)I_s \quad (18)$$

From the induction machine theory, P_{ag} is given by:

$$P_{ag} = \frac{\omega_s T_m}{P} \quad (19)$$

Equating equation (19) and (18) yields

$$3(V_s - I_s R_s)I_s = \frac{\omega_s T_m}{P} \quad (20)$$

$$R_s I_s^2 - V_s I_s + \frac{\omega_s T_m}{3P} = 0 \quad (21)$$

Applying quadratic expression on equation (21)

$$I_s = \frac{V_s \pm \sqrt{V_s^2 - \frac{4R_s T_m \omega_s}{3P}}}{2R_s} \quad (22)$$

The voltage across the magnetizing branch:

$$\overline{V}m = \overline{V}s + \bar{I}s(R_s + j\omega_s L_{ls}) \quad (23)$$

At unity pf, the stator voltage & current are given by

$$\overline{V}s = V_s < 0^0; \bar{I}s = I_s < 180^0 \quad (24)$$

The stator voltage and current are 180° out of phase, indicating that the DFIG is in the generating mode and the stator power factor (PF_S) is unity.

The magnetizing current can be determined by

$$\bar{I}m = \frac{\overline{V}m}{j\omega_s L_m} \quad (25)$$

The rotor current is

$$\bar{I}r = \bar{I}s - \bar{I}m \quad (26)$$

The rotor voltage can be calculated by

$$\frac{\overline{V}r}{s} = \overline{V}m - \bar{I}r(\frac{R_r}{s} + j\omega_s L_{lr}) \quad (27)$$

$$\overline{V}r = s.\overline{V}m - \bar{I}r(R_r + js.\omega_s L_{lr}) \quad (28)$$

The rotor voltage & current can relate to R_{eq} & X_{eq} as given by (see Figure 4)

$$\frac{\overline{V}r/s}{\bar{I}r} = \frac{R_{eq}}{s} + j\frac{X_{eq}}{s} \quad (29)$$

$$R_{eq} + jX_{eq} = \frac{\overline{V}r}{\bar{I}r} \quad (30)$$

In other way, from Figure 4 & assuming the direction of rotor current flow reversed,

$$\overline{V}r = \bar{I}r\left(R_r + js\omega_s L_{\sigma r}\right) + \left(\bar{I}s + \bar{I}r\right)js\omega_s L_m \quad (31)$$

By collecting terms in $j\omega_s$ and rearranging

$$\overline{V}r = \bar{I}rR_r + js\omega_s\left[L_{\sigma r}\bar{I}r + \left(\bar{I}s + \bar{I}r\right)L_m\right] \quad (32)$$

The equations for stator and rotor flux linkages [8]:

$$\overline{\psi_s} = L_m\bar{I}r + L_{\sigma s}\overline{I_s} \quad (33)$$

$$\overline{\psi_r} = L_m\bar{I}s + L_{\sigma r}\overline{I_r} \quad (34)$$

When the flux linkage expression is recognized in the voltage equations:

$$\overline{V_s} = \overline{I_s}R_s + j\omega_s\overline{\psi_s} \quad (35)$$

$$\overline{V_r} = \overline{I_r}R_r + js\omega_s\overline{\psi_r} \quad (36)$$

Once the values of R_{eq} & X_{eq} are determined from the above expressions, the analysis of power conversion of DFIG wind energy system, where P_m, P_r, stator and rotor winding losses, $P_{cu,s}$ and $P_{cu,r}$ & efficiency to be determined can be carried out.

To facilitate the steady-state analysis of the DFIG wind energy system, the equivalent circuit of Figure 4 above can be rearranged as shown in Figure 5, in which P_m, P_r, $P_{cu,s}$ and $P_{cu,r}$ can be easily calculated by a general equation of : ***$P = 3I^2R$.***

Figure 6 shows the power flow diagram of the DFIG operating under the super and sub synchronous modes of operation. Neglecting the rotational losses, P_{rot} of the turbine, the power transferred or dissipated in the generator can be calculated by [12]:

$$\left.\begin{aligned} &P_m = 3I_r^2(R_r + R_{eq})(1-s)/s \\ &P_s = P_m/(1-s); P_{ag} = P_s - 3I_s^2 R_s \\ &P_r = 3I_r^2 R_{eq} =; P_{cu,r} = 3I_r^2 R_r \\ &P_s = 3V_s I_s \cos\varphi_s\ ; P_{cu,s} = 3I_s^2 R_s \end{aligned}\right\rangle \quad (37)$$

The power delivered to the grid, P_g, the sum of the stator & rotor power is given by:

$$\left|P_g\right| = \begin{cases} |P_s| + |P_r| \to \text{Super synchronous mode} \\ |P_s| - |P_r| \to \text{Sub synchronous mode} \end{cases} \quad (38)$$

In the super synchronous operating mode as shown in Figure 6a, R_{eq} of the RSC has a positive value and hence P_r is positive. This implies that R_{eq} consumes power similar to the winding resistances, R_r and R_s. In reality, P_r is not dissipated in R_{eq}, but transferred from the rotor to the grid through the converters. In the sub-synchronous mode, R_{eq} has a negative value and P_r is also negative. This indicates that the rotor circuit receives power from the grid through the converters as shown in Figure 6b [11].

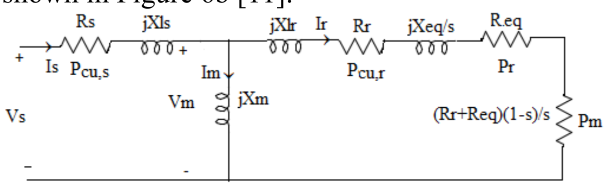

Figure 5. Steady-state equivalent circuit of DFIG for system performance evaluation

Table 1: List of simple and useful expressions [10]

$P_s+P_r \approx P_m$	$P_r \approx -sP_s$	$P_m \approx (1-s)P_s$	$\lvert V_r\rvert \approx \lvert sV_s\rvert$
$P_m \approx T_{em}*\omega_m/P$	$P_r \approx T_{em}*\omega_r/P$	$P_s \approx T_{em}*\omega_s/P$	

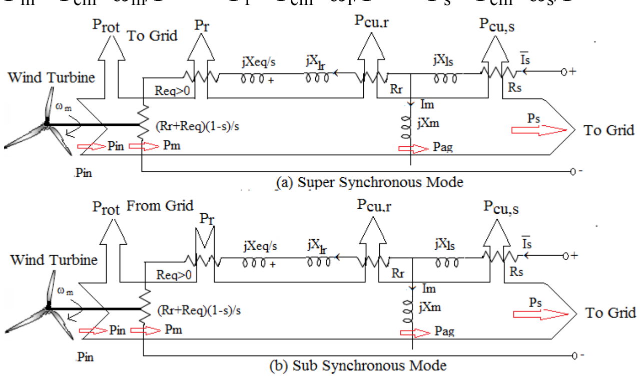

Fig. 6 DFIG Power flow with RSC by R_{eq} & X_{eq}.

3.3.RSC Equivalent Impedance & its Power Conversion at Super Synchronous Mode

Having all the necessary parameters of the DFIG 1.56 MW, 690 V, 50 Hz, 1800 rpm wind energy system as obtained from the name plate data of manufacturer, factory test report and mathematical analysis, investigation on the relationship between the rotor voltage, rotor current, the equivalent impedance of the RSC and power conversion system can be carried out at three operating points. In [13], it is mentioned that the stator leakage reactance (X_{ls}) and rotor leakage reactance X_{lr} (X_{2s}) are typically about 5% of the mutual reactance (X_m). R_s and R_r are typically about 0.5% of the X_m. In this regard L_{ls}, L_{lr} and X_m are calculated and similarly the remaining parameters are determined. In the first analysis, the generator speed at super synchronous mode is used and similar analyses are carried out on other speeds of operation.

From Equation 22, the stator current is calculated by

$$I_s = \frac{V_s \pm \sqrt{V_s^2 - \frac{4R_s T_m \omega_s}{3P}}}{2R_s}$$

I_s value with plus (+) sign is omitted as it results with huge current which is not reasonable and I_s is minus in generating mode of operation and is used. Accordingly, T_m is derived as from equation 22.

$$T_m = \frac{3P[V_s^2 - (2R_s I_s - V_s)^2]}{4R_s \omega_s} \quad (39)$$

In [7], the rated stator current, I_s is given as 1110A. Thus, the rated mechanical Torque, T_m can be calculated as:

$$T_m = \frac{3P[V_s^2 - (2R_s I_s - V_s)^2]}{4R_s \omega_s} = 9.5922 \text{ kNm}$$

Using the equivalent circuit of Figure 5,

$$\overline{V}m = \overline{V}s + \overline{I}s(R_s + j\omega_s L_{ls}) = 411.19 < 9.71^0 \text{ V}$$

This is the referred voltage. To obtain the actual voltage from transformation ratio, a (u) =0.42 [8]:

$$\overline{\overline{V_m}} = \frac{1}{a}\overline{V_m} = \frac{1}{0.42}(411.19 < 9.71^0) = 979 < 9.71^0 \text{ V}$$

This voltage can be the DFIG rotor voltage or RSC voltage. Rated continuous voltage of the RSC is 975V [3] resulted with small deviation as of calculated one.

$$\overline{I}_m = \frac{\overline{V}_m}{j\omega_s L_m} = 329.19 < -80.29^0 \text{A} = 55.522 - j324.5\text{A}$$

$I_r = I_s - I_m = -1165.522 + j324.5\text{A} = 1209.52 < -15.56^0\text{A}$

This is the referred rotor current. The actual rotor current from transformation ratio, a (u) =0.42:

$$\overline{I}_r = a\overline{I}_r = 508.14\angle -15.56° A$$

$$\overline{V}_r = s\overline{V}_m - \overline{I}_r(R_r + js\omega_s L_{lr}) = 79 < 23.92^0 = 79 < -156^0 \text{ V}$$

Or $\overline{V}r = \overline{I}rR_r + js\omega_s\overline{\psi}_r$ Where

$$\overline{\psi}_r = L_m I_s + L_r I_r = 1.442 < -71.43^0 \text{ wb}$$

Then, $\overline{V}r = \overline{I}rR_r + js\omega_s\overline{\psi}_r = 79.77 < 23.97\text{V}$

Where, $\omega_s = 2\pi * f_s = 2\pi * 50 = 314.16$ rad/s and

$s = \omega_s - \omega_r/\omega_s = -0.2$

The voltages are almost the same in the two ways.

Actual rotor voltage:

$$\overline{V'r} = \frac{\overline{Vr}}{a} = \frac{79.77\angle -156.03°}{0.42} = 189.93\angle -156.03°V$$

Or using the equation

$$|V_r| \approx |sV_s|,$$

V_r = 0.2*690/√3=79.7 V which is nearly the same as the above one. The equivalent impedance for the RSC at rated speed is given by

$$\overline{Z_{eq}} = \frac{\overline{V_r}}{\overline{\overline{I_r}}} = \frac{79 < 23.92^0}{1209.852 < -15.56^0} = 0.0504 + j0.0412\Omega$$

From which R_{eq}=0.0504 Ω X_{eq}=0.0415 Ω

Similarly, the DFIG at unity power is:

$$P_m = 3I_r^2(R_r + R_{eq})\frac{1-s}{s} = -1619.7kW \approx -1.6MW$$

Alternatively, the P_m can also be calculated by

$$P_m = T_m.\omega_m = -8592.2*1800*2*\pi/60 = -1.6MW$$

P_m in the two methods is the same. In addition, the nameplate rated mechanical power from the turbine is 1.56MW (≈ -1.6MW) [7] which is again nearly the same with calculated result. The rotor power is:

$$P_r = 3I_r^2R_{eq} = 3*1209.852^2*0.0504 = 221.318\,kW$$

The rotor and stator winding losses are

$$P_{cu,r} = 3I_r^2R_r = 3*1209.852^2*0.011074 = 48.63\,kW$$

$$P_{cu,s} = 3I_s^2R_r = 3*1110^2*0.006243 = 23.1\,kW$$

$$P_s = 3V_sI_s\cos\varphi_s = -1326.578\,kW$$

Where the stator power factor angle φ_s is 180^0. The total power delivered to the grid is then,

$$|P_g| = |P_s| + |P_r| = 1326.578 + 221.318 = 1547.896kW$$

The difference between P_m and P_g is the losses on the stator and rotor windings, that is,

$|P_{loss}|$= $|P_{cu,r}|$ + $|P_{cu,s}|$ or $|P_{loss}|$ = $|P_m|$ - $|P_g|$

$|P_{loss}|$=$P_{cu,r}$ + $P_{cu,s}$=4.63+23.1=71.73kW, alternatively

$|P_{loss}|$=$|P_m|$ - $|P_g|$=1619.7-1547.96 = 71.0 kW

The losses in the windings are around 72kW. The efficiency of the DFIG is then,

η = Pg/Pm=1547.96/1619.7=95.57%

Following the same procedure, the calculated rotor voltage, rotor current, the equivalent impedance of the RSC and power conversions in the super synchronous (1950 and 1700 rpm) modes, sub synchronous mode (1250 and 1050 rpm) are done and the results are shown in Figure 7. For the convenience of comparison, the calculated results for the DFIG operating in the synchronous (1500 rpm) are also used. From the manufacturer of wind turbine generator, speed versus generated power is obtained & combined summary is given in the paper here with by.

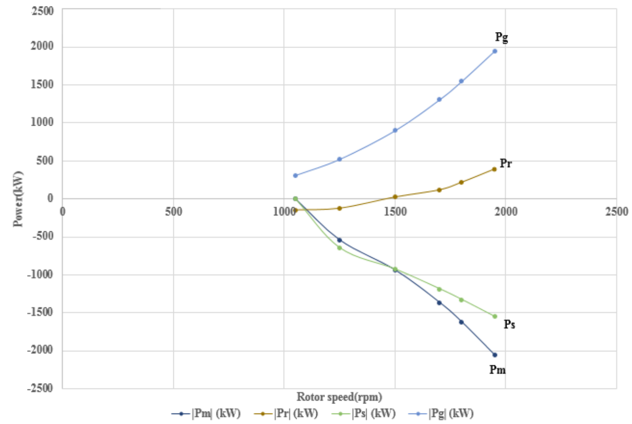

Figure 7 Mechanical, rotor, stator & grid power of DFIG operating at super & sub synchronous speeds

3.3.1. Summary

- As it is described above and seen in Figure 8, Req and Xeq are very nearly the same & follow the same pattern. Therefore, the equivalent impedance of the RSC of DFIG in WECS can be approximately equals √2*R_{eq} or √2*X_{eq} at steady state operating conditions except at synchronous speed. X_{eq} is zero at synchronous speed.
- At synchronous speed, the rotor voltage phase angle & rotor current phase angles are the same(Figure 9), indicating the rotor power factor(pf) angle of zero. The rotor current phase angle is increasing when the generator is running from sub synchronous speed to super synchronous speed, while the rotor voltage phase angle varies slowly. In addition, the rotor pf angle is drastically decreasing positively during sub synchronous & nearly remaining constant during super synchronous mode of operation due to the slight phase change between rotor voltage & current.
- For a given rotor speed, the rotor copper winding loss is higher than the stator copper losses due to higher rotor currents. In connection, the total loss of windings increases as a DFIG runs from sub synchronous to super synchronous speed.
- At synchronous speed, P_s is equal to the mechanical power and the rotor power is at minimum value.
- At rated speed, P_g from the analysis is 1547.9 kW where as from the factory test result that conducted at 1802 rpm speed @ 703V is 1540 kW [8].

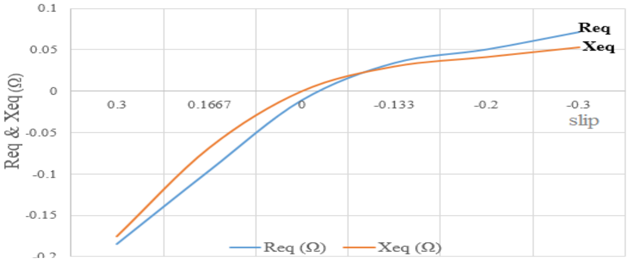

Figure 8. Rotor Equivalent resistance reactance

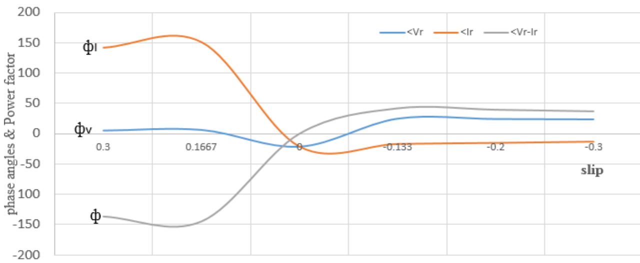

Figure 9. RSC phase angles

In large megawatt wind generators, R_s is normally very small. To simplify the analysis, it can be neglected [11]. The simplified method thus can be used in cases where it becomes difficult and unable to find the parameters of DFIG in accurate method.

When the generator operates with unity stator power factor, its air-gap power can be calculated by

$$P_{ag} \approx 3(V_S - I_S R_S) I_S \approx 3 V_S I_S \qquad (40)$$

From Equations (19) and (40), the stator current is:

$$I_S = \frac{T_m \omega_S / p}{3 V_S} \qquad (41)$$

With I_s known, the equivalent impedance of the RSC can be calculated, and the steady-state performance of the DFIG be analyzed. Compared with I_s given in Equation (22), the calculation of I_s by equation (41) is simpler. More importantly, this method can facilitate the analysis of the DFIG wind energy systems with non-unity stator pf, which is discussed hereby.

3.4. Analysis of DFIG under Leading & Lagging Power Factor Operation

When the generator operates with a leading or lagging power factor, I_s can be calculated by

$$I_S = \frac{T_m \omega_S / p}{3 V_S \cos\varphi_S} \qquad (42)$$

With I_s calculated, the Z_{eq} of RSC can be obtained following the same procedures as of before.

3.4.1. Leading PF Operation ($\cos\varphi_s$= 0.95)

3.4.1.1. Super Synchronous Mode (1950 rpm)

When the generator operates at 1950 rpm, I_s is:

$$I_S = \frac{T_m \omega_S / P}{3 V_S \cos\varphi_S} = \frac{-10084 \ * 50\pi}{3 * 398.4 * 0.95} = -1395.1 \text{A}$$

Here, Tm=-$(1950/1800)^2$*8.5922 = -10.04 kNm

$$\overline{V}m = \overline{V}s + \bar{I}s(R_S + j\omega_S L_{lS}) = 416.3 < 12.1^0 \text{ V}$$

$$\bar{I}m = \frac{\overline{V_m}}{j\omega_S L_m} = 69.87 - j325.91 \text{ A}$$

$I_r = I_s - I_m$ = -1464.97 + j325.91A=1500.<-12.54^{0}A

$$\overline{V_r} = s\overline{V_m} - \overline{I_r}(R_r + js\omega_S L_{lr}) = 126.43 < 26.95^0 \text{ V}$$

$$\overline{Z}eq = \frac{\overline{V}r}{\bar{I}r} = 0.0650 + j0.0536\Omega$$

Req=0.0650 Ω Xeq=0.0536 Ω

$$P_m = 3 I_r^2 (R_r + R_{eq}) \frac{1-s}{s} = -2227.54 \text{ kW}$$

Alternatively, $P_m = T_m . \omega_m$ = -2059.2 kW

When power factor reduces from unity to 0.95, I_s & I_r increases. However, the torque unaltered as it doesn't depend on the power factor. This resulted in larger variation of P_m as calculated in two ways. P_r then is,

$P_r = 3I_r^2 R_{eq}$ =439.22 kW

$P_{cu,r} = 3I_r^2 R_r$ =74.83 kW

$P_{cu,s} = 3I_s^2 R_s$ =36.5 kW

$$P_S = 3 V_S I_S \cos\varphi_S = -1583.94 \text{ kW}$$

$$|P_g| = |P_S| + |P_r| = 1583.94 + 439.22 = 2023.16 \text{ kW}$$

$$|P_{loss}| = |P_m| - |P_g| = 2059.2 - 2023.16 = 36.04 \text{ kW}$$

Efficiency= 98.25%

Following the same procedure, the calculation was done on other speeds.

4. Dynamic Modeling of DFIG

4.1. αβ Model

In developing the dynamic αβ model of the DFIG, space vector theory is applied to the basic electric equations of the machine. Figure10 shows the three different rotating reference frames typically utilized to develop space vector-based models of the DFIG. The stator reference frame (αβ) is a stationary, the rotor reference frame (DQ) rotates at ω_m and the synchronous reference frame (dq) rotates at ω_s. Subscripts "s", "r" and "a" are used to denote that one space vector is reference to the stator, rotor and synchronous reference frames, respectively. By using clark, parke and inverse transformations, a space vector can be represented in any of these frames [10].

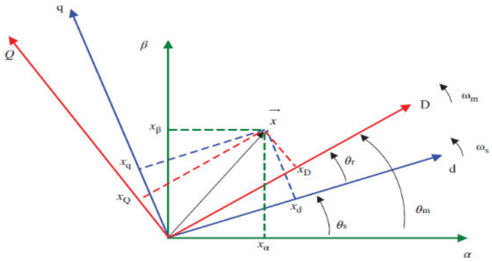

Figure 10. Reference frames

The general voltage equation is:

$$V = I.R + \frac{d|\psi|}{dt} \qquad \vec{V}_s{}^s = \vec{I}_s{}^s R_s + \frac{d\vec{\psi}_s{}^s}{dt}$$

$$\vec{V}_r{}^r = \vec{I}_r{}^r R_r + \frac{d\vec{\psi}_r{}^r}{dt} \tag{43}$$

In stationary reference frame:

$$\vec{V}_s{}^s = [\vec{I}_s{}^s][R_s] + \frac{d|\vec{\psi}_s{}^s|}{dt} \Rightarrow \begin{cases} u_{\alpha s} = r_s i_{\alpha s} + \dfrac{d\psi_{\alpha s}}{dt} \\ u_{\beta s} = r_s i_{\beta s} + \dfrac{d\psi_{\beta s}}{dt} \end{cases} \tag{44}$$

$$\vec{V}_r{}^r = [\vec{I}_r{}^r][R_r] + \frac{d|\vec{\psi}_r{}^r|}{dt} \tag{45}$$

Writing the rotor voltage in stator reference frame

$\vec{V}_r{}^r = T\left[\vec{V}_r{}^s\right]$ Where, 'T' is transformation factor,

$$T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{46}$$

Then, $\vec{V}_r{}^r = T.[\vec{V}_r{}^s] = T.[\vec{I}_r{}^s][R_r] + \dfrac{d|T.\vec{\psi}_r{}^s|}{dt}$

Multiplying both sides by T^{-1} and taking $T.\,T^{-1}=1$

$$[T^{-1}]T.[\vec{V}_r{}^s] = [T^{-1}]T.[\vec{I}_r{}^s][R_r] + [T^{-1}]\frac{d|T.\vec{\psi}_r{}^s|}{dt}$$

$$\omega_r = s\omega_s \to f_r = sf_s \tag{47}$$

$$[T^{-1}.\frac{dT}{dt}] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\omega_m \qquad \text{Thus,}$$

$$\vec{v}_r{}^s = [\vec{i}_r{}^s][R_r] + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\omega_m.\vec{\psi}_r{}^s + \frac{d\vec{\psi}_r{}^s}{dt}$$

$$\vec{v}_r{}^s = \begin{cases} v_{\alpha r} = R_r i_{\alpha r} + \dfrac{d\psi_{\alpha r}}{dt} + \omega_m \psi_{\beta r} \\ v_{\beta r} = R_r i_{\beta r} + \dfrac{d\psi_{\beta r}}{dt} - \omega_m \psi_{\alpha r} \end{cases} \tag{48}$$

In the same way, the stator & rotor flux expressions in space vector form in a stationary reference frame:

$$\begin{bmatrix} \psi_s{}^s \\ \psi_r{}^s \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_r \end{bmatrix} . \begin{bmatrix} i_s{}^s \\ i_r{}^s \end{bmatrix} \to \begin{cases} \psi_{\alpha s} = L_s i_{\alpha s} + L_m i_{\alpha r} \\ \psi_{\beta s} = L_s i_{\beta s} + L_m i_{\beta r} \\ \psi_{\alpha r} = L_m i_{\alpha s} + L_r i_{\alpha r} \\ \psi_{\beta r} = L_m i_{\beta s} + L_r i_{\beta r} \end{cases} \tag{49}$$

$$\begin{bmatrix} i_s{}^s \\ i_r{}^s \end{bmatrix} = \frac{1}{L_m{}^2 - L_s.L_r} \begin{bmatrix} -L_r & L_m \\ L_m & -L_s \end{bmatrix} \begin{bmatrix} \psi_s{}^s \\ \psi_r{}^s \end{bmatrix} \tag{50}$$

Where $L_s = L_{ls} + L_m$ and $L_r = L_{lr} + L_m$

$v_{\alpha s}$; $v_{\beta s}$; $v_{\alpha r}$; $v_{\beta r}$; $i_{\alpha s}$, $i_{\beta s}$, $i_{\alpha r}$, $i_{\beta r}$ and $\psi_{\alpha s}$, $\psi_{\beta s}$, $\psi_{\alpha r}$, $\psi_{\beta r}$ are voltages [V], currents [A] and flux linkages [Wb] of the stator and rotor in α and β-axis, R_s and R_r are the resistances of the stator & rotor windings [Ω], L_s, L_r, L_m are the stator, rotor and mutual inductances [H]. $L_l s$, $L_l r$ are the stator and rotor leakage inductances [H], ω is the speed of the reference frame [rad/s], ω_m is the mechanical angular velocity of the generator rotor [rad/s]. On the other hand:

$$P_s = \frac{3}{2}(v_{\alpha s} i_{\alpha s} + v_{\beta s} i_{\beta s}) \tag{51}$$

$$Q_s = \frac{3}{2}(v_{\beta s} i_{\alpha s} - v_{\alpha s} i_{\beta s}) \tag{52}$$

While the electromagnetic torque, created by the DFIG, can be calculated by:

$$T_{em} = \frac{3}{2} p I_m \{\vec{\psi}_r \vec{i}_r\} = \frac{3}{2} p(\psi_{\beta r} i_{\alpha r} - \psi_{\alpha r} i_{\beta r}) \tag{53}$$

By adding the mechanical motion equation that describes the rotor speed behavior:

$$T_{em} - T_m = J\frac{d\Omega_m}{dt} \tag{54}$$

With J, the inertia of the rotor and T_m, the load torque applied to the shaft. The models of the DFIG above can be used for computer-based simulations. Lastly, it has to be noticed that the model parameters of the machine, R_s, R_r, L_{ls}, L_{lr}, and L_m are the same for both steady state and dynamic models.

4.2.dq model

The space vector model of the DFIG can be also represented in a synchronously rotating frame by multiplying the voltage expressions by $e^{-j\theta s}$ & $e^{-j\theta r}$, respectively, the dq voltage equations can be: [10]

$$\vec{V}_s^{\,a} = \vec{I}_s^{\,a} R_s + \frac{d\vec{\psi}_s^{\,a}}{dt} + j\vec{\omega}_s^{\,a} \rightarrow \begin{cases} v_{ds} = R_s i_{ds} + \frac{d\psi_{ds}}{dt} - \omega_s \psi_{qs} \\ v_{qs} = R_s i_{qs} + \frac{d\psi_{qs}}{dt} + \omega_s \psi_{ds} \end{cases} \quad (55)$$

$$\vec{V}_r^{\,a} = \vec{I}_r^{\,a} R_r + \frac{d\vec{\psi}_r^{\,a}}{dt} + j\vec{\omega}_r^{\,a} \rightarrow \begin{cases} v_{dr} = R_r i_{dr} + \frac{d\psi_{dr}}{dt} - \omega_r \psi_{qr} \\ v_{qr} = R_r i_{qr} + \frac{d\psi_{qr}}{dt} + \omega_r \psi_{dr} \end{cases} \quad (56)$$

Where ω_r, the induced rotor voltages have frequency of $\omega_m = \omega_s - \omega_r$ or $\omega_r = s\omega_s \rightarrow f_r = sf_s$

Similarly, the fluxes yield:

$$\begin{bmatrix} \vec{\psi}_s^{\,a} \\ \vec{\psi}_r^{\,a} \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_r \end{bmatrix} \cdot \begin{bmatrix} \vec{i}_s^{\,a} \\ \vec{i}_r^{\,a} \end{bmatrix} \rightarrow \begin{cases} \psi_{ds} = L_s i_{ds} + L_m i_{dr} \\ \psi_{qs} = L_s i_{qs} + L_m i_{qr} \\ \psi_{dr} = L_m i_{ds} + L_r i_{dr} \\ \psi_{qr} = L_m i_{qs} + L_r i_{qr} \end{cases} \quad (57)$$

For a sinusoidal supply of voltages, at steady state, the dq components of the voltages, currents, and fluxes will be constant values, in contrast to the αβ components that are sinusoidal magnitudes.

The instantaneous active and reactive power of DFIG can be converted into a synchronously rotating d-q reference frame by [12]

$$P_s = \frac{3}{2}(v_{ds} i_{ds} + v_{qs} i_{qs}) \quad (58)$$

$$Q_s = \frac{3}{2}(v_{qs} i_{ds} - v_{ds} i_{qs}) \quad (59)$$

5. Vector Control of DFIG

In DFIG wind energy systems, the stator voltage oriented control is achieved by aligning the d-axis of the synchronous reference frame with V_s.

The resultant d and q-axis stator voltages are: [2].

$V_{qs} = \psi_{ds} = 0$ and $V_{ds} =$ (60)

$\omega_s = 2\pi fs$ (61)

The stator voltage vector angle Φ_s, is referenced to the stator frame, which varies from zero to 2π when V_s rotates one revolution in space. The rotor rotates at speed ω_r. The rotor position angle Φ_r is also referenced to the stator frame. The angle between the stator voltage vector and the rotor is the slip angle, defined by $\Phi_{sl} = \Phi_s - \Phi_r$ (62)

Since the DFIG operates with unity power factor, the stator current vector I_s is aligned with V_s but with opposite direction (DFIG in generating mode). The dq-axis components can be controlled independently by the rotor converters. The P_s & Q_s of the stator are controlled by the RSC by considering the delay time. Therefore, it is worthwhile to investigate the controllability of P_s and Q_s by the rotor voltage and current.

Where ψ_{ds} and ψ_{qs} are the dq-axis stator flux linkages,

$$\psi_{ds} = L_s i_{ds} + L_m i_{dr}; \psi_{qs} = L_s i_{qs} + L_m i_{qr} \quad (63)$$

From which the dq-axis stator currents are calculated,

$$i_{ds} = \frac{\psi_{ds} - L_m i_{dr}}{L_s}; i_{qs} = \frac{\psi_{qs} - L_m i_{qr}}{L_s} \quad (64)$$

In DFIG wind energy system, I_r is controlled by the RSC. Referring to the IG model, the stator voltage vector for the steady-state operation of the generator:

$$V_s = i_{ds} R_s + j\omega_s \psi_s \quad (65)$$

The representation in dq-axis is

$$(V_{ds} + jV_{qs}) = (i_{ds} + ji_{qs})R_s + j\omega_s(\psi_{ds} + j\psi_{qs}) \quad (66)$$

From which the dq-axis stator flux linkages are

$$\psi_{ds} = \frac{V_{qs} - R_s i_{qs}}{\omega_s}; \psi_{qs} = -\frac{V_{ds} - R_s i_{ds}}{\omega_s} \quad (67)$$

Using the stator voltage oriented control (vqs = 0),

$$P_s = \frac{3}{2}(V_{ds} i_{ds}); Q_s = -\frac{3}{2} V_{ds} i_{qs} \quad (68)$$

$$P_s = \frac{3}{2} V_{ds} (\frac{\psi_{ds} - L_m i_{dr}}{L_s}); Q_s = -\frac{3}{2} V_{ds} (\frac{\psi_{qs} - L_m i_{qr}}{L_s} \quad (69)$$

$$\begin{cases} i_{dr} = -\frac{2L_s}{3V_{ds} L_m} P_s + \frac{1}{L_m} \psi_{ds} \\ i_{qr} = \frac{2L_s}{3V_{ds} L_m} Q_s + \frac{1}{L_m} \psi_{qs} \end{cases} \quad (70)$$

$$i_{dr} = -\frac{2L_s}{3V_{ds} L_m} P_s - \frac{R_s}{\omega_s L_m} i_{qs}$$

$$i_{qr} = \frac{2L_s}{3V_{ds} L_m} Q_s + \frac{R_s}{\omega_s L_m} i_{ds} - \frac{V_{ds}}{\omega_s L_m} \quad (71)$$

For Vqs=0 & Neglecting the R_s, we have

$$i_{dr} = \frac{2L_s}{3V_{ds} L_m} P_s \rightarrow a; \quad i_{qr} = \frac{2L_s}{3V_{ds} L_m} Q_s - \frac{V_{ds}}{\omega_s L_m} \rightarrow b \quad (72)$$

As of the above equations, for a given stator voltage, P_s & Q_s can be controlled by the dq-axis rotor currents independently. Figure 11 shows the DFIG modeling in Matlab-Simulink and Figure 12 shows simulated results by controlling P_s at 1MW & Qs at 0.

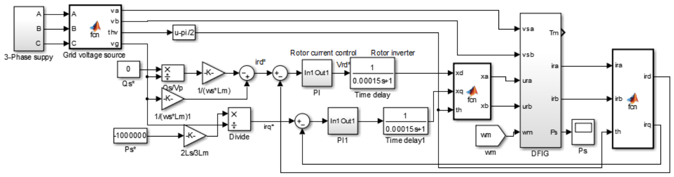

Figure11. DFIG Modeling for Ps & Qs control

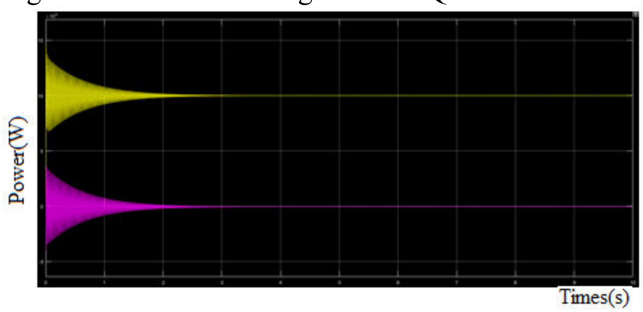

Figure12. Ps and Qs simulation results

6. CONCLUSION

The mechanical power, P_m of the wind turbine must be larger than power sent to grid in magnitude to supply losses. The rated wind turbine's power should be greater than $|P_g|$ of rated speed of generator (1800 rpm) as the DFIG acheives ± 30% slip. In this case, rated power of the gear box is 1.66MW [7]. This can be accepted as the calculated value is 1547.96 kW. The calculated value of the DFIG stator winding is 1326.5kW which is less than the rated power (1.56MW). This is also acceptable. As the rotor circuit of DFIG receives power from the grid through the converters during the sub synchronous mode of operation, the total power delivered to the grid is less as compared with super synchronous mode of operation. Therefore, the power delivered to the grid is $|P_g|=|P_s|-|P_r|$. $|P_s|$ is approximately equal to $|P_m| + |P_r|$ in the super synchronous operating mode, whereas in the sub synchronous operating mode $|P_s| \sim |P_m| - |Pr|$. The stator active power control and reactive power control with modelling has been done. When the pf of the DFIG reduces from unity to 0.95 in either lagging or leading, P_g is decreasing in sub synchronous mode and increasing in super synchronous mode. Rotor side control should be proceed as a further work.

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