

## Mathematical models in neuroscience

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*Abstract:* In this paper, several mathematical models in neuroscience are provided. Firstly, the semantic system of the brain and the semantic logic, i.e., the logic that is specific to the human brain are discussed. Then, a mathematical-physical model to explain the mirror neurons paradigm is developed. Precisely, considering that any biological structure can be assimilated to a fractal (both structurally and functionally), a mathematical-physical model is proposed in order to explain the mirror neurons paradigm. Extending de Broglie's idea concerning the wave-corpuscle duality by means of the information (in its implicit and explicit) form, we are lead to assume the existence of a fractal medium, which can store and transmit information in the form of a natural field (called a fractal field). In consequence, the mirror neurons transmitting mechanism can be explained by this spontaneous symmetry breaking, in which the specific neuronal network and specific logics appear.

*Key-Words:* Neuroscience; semantic system; brain; neuronal network; spectral network; mirror neurons.

### 1 Introduction

The evolution of science in the second half of the 20<sup>th</sup> century led to the development of fractals and of fractal geometry, of topology and chaos theory, as well as of nonlinear dynamics, which started to better explain various phenomena from different fields, which had previously been described only by Newtonian dynamics.

All these theories were grouped into something that, in the last decades, came to be called the science of complexity or the complex systems theory whose principles can be applied through specific properties at any scale or reality level, from the string theory to the cosmologic models.

Many studies have appeared, trying to apply this theory to biological systems, human body and human mind. The development of research in studying the brain and its functioning has crossed a series of stages in the 20<sup>th</sup> century, from the era of the great anatomical discoveries to phrenology and

towards the behaviourist stage and then the cognitivist one, whereas in the last decades neurosciences attempted to include the phenomenology of psychological reality into an interdisciplinary approach.

In the last years, however, neurosciences have had to open up even more to interdisciplinarity in order to include Quantum Physics, Information Technology and even Cosmology scientists, as well as traditional specialists in Psychology, Neurology and Psychopathology. This need for a wide interdisciplinary comes from the necessity to apply the principles of complex systems theory to brain activity. In order to achieve that, it is necessary to overcome the concept according to which the psychological activity is only the product of neuron activity.

In the past decade, the human genome project has experienced such an approach, which proved successful. In 2013 it was opened a European

offensive on knowledge of the human brain, called the Human Brain Project and also a Human Brain Mapping Initiative in USA. In both projects there is a large concern of universities and private entities in a wide range of interdisciplinary research. The goal of the Human Brain Project is to pull together all our existing knowledge about the human brain and to reconstruct the brain, piece by piece, in supercomputer-based models and simulations. The models offer the prospect of a new understanding of the human brain and its diseases and of completely new computing and robotic technologies.

One of the major obstacles to understanding the human brain is the fragmentation of brain research and the data it produces. In neuroscience, the project will use neuro-informatics and brain simulation to collect and integrate experimental data, identifying and filling gaps in our knowledge and prioritising future experiments.

The principles of complex systems theory can be applied through specific properties at any scale or reality level, from the string theory to meteorology and to the cosmologic models. The spectral component associated and related to the material, corpuscular one (the neuronal and non-neuronal structures of the brain) must be at least as important as the corpuscular part, which is structured and was studied in the last hundred years. Another reality which coexists with us is the a-spatial and a-temporal reality, described by the wave formula, in which the undertaking of the spatial-temporal information is made by light through modulation of its frequency. The electromagnetic theories of consciousness propose that consciousness can be understood as an electromagnetic phenomenon.

The analysis of the Toda network [1,11] (with its fractal but also structural-functional specific), allows for modelling the neuronal network, under two components: the structural, corpuscular network and the functional, network.

## 2 The semantic system of the brain and the semantic-emergent logic

Research undertaken in the last century made it possible that semiology, as a philology field, which studied the significance of the natural language, be extrapolated through semiotics as a science of the sign significance, in very many fields of the social sciences and humanities, but also in the natural sciences and information technology. We are currently speaking about the semantic web, about semantic algorithms and semantic logic, with an

aim to building programs and systems which relate to artificial intelligence.

Meanwhile, the information semantics is a research field of the human cognition, starting from the reality that language is the expression of thought, and the study of language semantics represents an analysis of the specifically human processing. The syntax, the semantics, the pragmatics, along with the hermeneutical and holistic approach represent logical mechanisms which can explain the superior psychological processes, as well as abstractization, conceptualization, generalization, symbol representation and metaphor. All these have always been used in the narrative process, but they were not analyzed from a holistic perspective which belongs to the structuring of reality, both the physical and the psychological one.

The new approaches in the cognitive psychology and in neurosciences in general, have information in the foreground, but also the way in which, starting from data, information becomes an aggregate of knowledge which can describe reality.

Apart from the quantitative aspect which the information contains (that is present in Shannon and Weaver's information theory [26], Stonier [23] etc.), the qualitative component of the information is very important from a psychological viewpoint. This qualitative component is given by its significance. This is the reason why semiology and semiotics, meaning and semantics are all notions which are currently being revisited from a different perspective, not only the philological one. This led to the development of semiology, as an analyzing strategy for the text. To date, semantic information is a phrase which is being analyzed both by philosophers and information technology specialists (semantic information, semantic web, ontologies) (see also Barabási [8]).

The semantic aspects of information become increasingly important for psychologists, too, because the structure and the way in which the nervous system functions ask for this type of approach. The nervous structures, from the end ramifications to the cerebral cortex, contain superposed nervous nuclei, of an increasing complexity, from the spine to the bulb and the brain stem, then to the diencephalon and the sub-cortical centres. The information undergoes a completion phase, from the simple binary data, combined in frequencies and amplitudes, to increasingly complex informational structures which reach the level of the brain cortex in order to create images through mapping; these are then used for our representation of reality.

The meaning of language is represented in various regions of the cerebral cortex, better known

as the semantic system. Until now, a small part of the semantic system was mapped, so the semantic selectivity of most areas remained unknown.

Gallant *et al.* [17] systematically mapped the semantic selectivity in various regions of the cortex, using "voxel-wise" in their research through fMRI. The subjects were exposed to some narratives that they listened to for some hours, thus highlighting the organization of the semantic system in stable *patterns* from one person to the other. Then they used narrative generative models in order to create a detailed semantic atlas. The results suggest that many areas of the semantic system represent information which is connected to specific semantic domains or groups of concepts which are in relation and which are positioned in cerebral areas connected to the multiple significances which can be contained by notions and concepts.

The aim of [17] was to structure the way in which the brain represents significance ("the semantic content") of language. The majority of the previous studies regarding language and the brain relied on words and isolated sentences. Gallant *et al.* [17] used narrative scenario stimuli because the aim was the outline of the whole range of semantic concept within one single study. This made it possible to outline a semantic map for each individual, map which could show which areas in the brain react to words with similar significance or semantic content. Another aim of this study was the creation of a semantic atlas through combining data from multiple subjects, showing which parts of the brain represented similar information but in different contexts/different thematic areas.

The study [17] did not have the aim of testing only one hypothesis or addressing only one simple question. The wish was that of exhaustive making of the representation of significance or of the semantic information in narrative language, along the whole cerebral cortex. The resulting maps show that the semantic information is represented in complex patterns distributed along several large areas of the cortex. Moreover, each of these regions contains many distinct areas which are selective for special types of semantic information, for example the people, the numbers, the visual properties or the places. It was also discovered that these cortical maps are relatively similar with different people, up to the smallest details.

These semantic maps provide us for the first time with a detailed map of the way in which significance is represented on the whole surface of the human cortex. Instead of the limiting of the language to only a series of areas of the brain, one discovers that language activates relatively vast

areas of the brain. One also discovers that these representations [17] are bilateral: the reactions of the cerebral right hemisphere are approximately as great and as varied as the reactions from the left hemisphere.

The continuation of the research in this field, as well as our approach connected to the semantic logic [1,11] can lead to new theories connected to the functioning of the mental component of the psychological system.

We have currently two separate approaches, which are complementary: a semantic system of the brain, described through the modern techniques of the fMRI, statistical programs and information technology adaptations on the one hand, and on the other hand a study of the semantics of the language and a semiological study of signs in general, which were made during the last century. Considering that the language lies at the basis of superior psychological processes (the language logic expresses the logic of thought), it is important to develop a theory which should explain this link between thinking and language. If the new highlight [17] of the semantic system of the brain describes the location of polysemantic significance units of the words in the neuronal structures (semantic hardware), then it is necessary to analyze the programme which uses this neuronal structure (semantic software).

The authors' view is that this programme implies the description of a special logic, the semantic logic (see also Martin [18]). Out of coherence reasons, this logic is used both in the structuring of the brain as an emergent complex system and in structuring the semantic mind. This is the *semantic-emergent logic* [12].

There is thus a piece of information which we call semantic-emergent information. The logic based on which this information is structured will bear the name of *semantic-emergent logic*. It is different from the bivalent logic, but also from the fuzzy logic, because the values which can be true cannot be probabilistically assessed, they are conditioned by their semantic value.

Significance is achieved through reference of informational contents from less complex levels to more complex levels, achieving unity from an informational viewpoint between digital and analogous, between discreet and continuous, between fragmentary and holistic.

The sweep between increasingly complex structures is based on the correlation between the corpuscle and the corresponding wave. Starting from this point, this phenomenon which is present in the wave-corpuscle duality (Heisenberg [16]), a matrix system to describe the corpuscle was conceived

(which is equivalent to Schrödinger's wave equation) and which includes information which alternates between the spatially and temporally structured information in the corpuscle and the same information distributed in the "content" of the wave. Thus, the topological diversity of information between the two poles, the fragmentary, discreet, digital one on one hand and the holistic, continuum and analogical one in the wave on the other hand.

While the corpuscles keep individual properties which lead to explicit links (strong forces, weak forces, electromagnetic forces, quantum gravitation), the corresponding waves of these particles overlap through modulation into waves which contain the whole information, continually dispersed (see also Pribram [19]). This aspect which comes from the properties of the waves on the one hand and of the corpuscles on the other hand, as well as that of the wave-corpuscle duality, lead to the conclusion that all corpuscle structures, from the nuclear to the atomic, molecular and macromolecular ones, contain the information both in the content of the structure of these corpuscular formation and in the corresponding waves, where information is to be found in the modulation of waves along with forming the spatial-temporal structures.

To conclude, any corpuscular structure from the three-dimensional space-time presents an informational equivalent which is dispersed in the modulated waves, correlated with the corpuscular spatial-temporal structure. The correlation between wave and corpuscle is kept along with the forming of corpuscular structures and participates to their making. This is due especially to fragmentary, discreet, structural information, which is correlated to the continuous, holistic information and which is dispersed in the modulated wave which results from the integration of all the waves corresponding to the respective corpuscles.

### **3 A mathematical-physical model in neuroscience: The mirror neurons paradigm**

In 1980's and 1990's Rizzolatti *et al.* [20-22] placed electrodes in the ventral pre-motor cortex of the macaque monkeys to study neurons specialized for the control of hand and mouth actions (for instance, taking hold of an object and manipulating it).

Rizzolatti *et al.* [20-22] found that some neurons respond both when the monkey picked up the food or when the monkey saw a person pick up a piece of food. Later on, they discussed the role of the mirror-neuron system in action recognition,

proposing that the human Broca's region was the homologue region of the monkey ventral pre-motor cortex. All these studies pointed out the presence of mirror neurons responding to hand actions.

Ferrari *et al.* [13,14] described the presence of mirror neurons responding to mouth actions and facial gestures, which suggests that mirror neurons respond during both action and observation of action. A wide network of brain areas shows mirror properties in humans (e.g., the somato-sensory cortex). This fact allows the observer feel what it feels like to move in the observed way).

So, a mirror neuron is a neuron that fires both when an animal (or human being) acts and when the animal (the human being) observes the same action performed by another. In this way, the neuron "mirrors" the behavior of the other, as though the observer were itself acting.

As a consequence, mirror neurons may be important for understanding the actions and intentions of other people, for simulating the behavior of the others, for developing the emotional capacities (as empathy), for providing the physiological mechanisms to the perception/action coupling; for learning new skills by imitation, for a better understanding of certain cognitive disorders - autism etc.

In some experiments, mirror neurons activate before the monkey or the human being observes the action. In this way, mirror neurons could be the neural basis for predicting the subsequent actions of a person and one could explain in this way the action prediction/intuition.

In what follows, we shall explain our mathematical-physical models concerning the mirror neurons paradigm.

Let us admit that the human brain is a fractal in the sense of its neuronal network. As a physical object, a fractal (in the sense of Mandelbrot) has a smooth structure at arbitrary small scales. It is too irregular to be described in a traditional Euclidean geometrical way. It is self-similar (at least stochastic or approximate), i.e., its parts are (almost) identical with the whole.

According to de Broglie's idea [10] (the wave-corpuscle duality), any particle in motion generates a corresponding wave. As consequence, one can find in brain a structural (corpuscular) (neuronal) part and a spectral one. These two parts must be in fractal coherence (Agop *et al.* [1-5], Crumpei *et al.* [1,11], Toda [24,25]). While the structural (neuronal) (corpuscular) part is in correspondence with the determinism (causality), the spectral (undulating) one is in correspondence with the

potentiality (non-determinism) (Atmanspacher [6,7], Bohm [9]).

Determinism is causal and temporal, being explicated in the three-dimensional Euclidean space  $\mathbb{R}^3$ . Determinism follows the sense of evolution through the second law of Thermodynamics. On the contrary, potentiality is a-causal and a-temporal, being explicated in a multidimensional space (e.g., Riemann space, Hilbert space etc.).

There exists a permanent exchange of information (emergent information) between the corpuscular part and the spectral one, the goal being to maintain the structure and the coherence of the system. As consequence, the discrete, fragmentary, digital information found in particles is the same with the continuous, analogical, holistic information from the corresponding fractal field. Thus, information can be found fragmentarily, discrete in every particle, but also analogically, continuously, in the corresponding fractal field and this principle remains valid at all scales and levels of reality.

Information generates the fractal potential (Agop *et al.* [1-5]), which in turn generates the fractal fundamental field. This field (which possesses a fractal force) could mean (according to the scale resolution) a gluon field, a gravitational field (linked by mass to structurally), a meson field, a field of strong interactions, an electromagnetic field etc.

Although the two components (the structural and the spectral) seem to be disjoint, they can be correlated based on a fractal theory developed on multidimensional manifolds by means of an informational energy (Agop *et al.* [1-5] etc.).

Based on [15], one can easily understand the generation mechanism of the metric of the "fractal field". If we describe this field through the amplitude and phase of its harmonics, then these "physical objects" constitute "ensembles" that are transitivity manifolds of the Barbilian group. The invariant metric of this group generates the associated Lagrangean and this leads to the "fractal field equations". Practically speaking, we discuss here about holography in its most adequate sense.

In this way, the points from the n-dimensional manifold (multidimensional level) which are in different states and phases - the macaques' spectral network - lead to the Barbilian group formalism (as a group of synchronization of all physical objects having the same impulse - the macaques) which in turn leads to the invariant metric. Based on the least, one gets the Lagrangean and so finally the fractal field equations.

Through the Barbilian group (and thus through the fractal field), the macaques are elements of a same set, being synchronized (correlated) in any moment. Thus, the Barbilian group is the macaques' synchronization group (i.e., macaques' fractal coherence). Selecting the scale resolution (by means of the compactification of the dimensions and three-dimensional projection, which means in fact Reality), one selects and makes explicit the synchronization field that allows the interaction of the macaques.

The superposition of all zooms we make (assimilated to so called pre-fractals, that is, the phases preceding the structuring of the intention) finally imposes this goal, of structuring of the intention, which leads to the passage from non-determinism to determinism. The transmission of the intentions is made instantaneously, spectrally by means of the n-dimensional manifold built varying the initial conditions. As a consequence, all receptor objects (macaques, human beings etc.) respond accordingly, by activating their mirror neurons.

The following conclusions are thus obvious:

- i) The transmission of the intentions is made spectrally to all physical objects (in this case the macaques or the human beings) from the manifold;
- ii) Through the Barbilian group (and thus through the fractal field), the macaques are elements of a same set, being synchronized (correlated) in any moment. Thus, the Barbilian group is the macaques' synchronization group;
- iii) Synchronization means here fractal coherence;
- iv) The fractal field is given by the fractal potential given by information;
- v) One passes from non-determinism to determinism when holography appears. Thus, all the macaques respond (that is why their neuron(s) fire(s));
- vi) When the initial conditions (amplitude and phase) vary and when we construct the differential manifolds, we obtain holography, which is instantaneously transmitted to all macaques. Here, the hologram means the mental superposition of all the zooms we make (assimilated to the so called pre-fractals), that finally lead to structure the intention. The intention is then transmitted through holography and the receptor manifests accordingly;
- vii) The variation of the amplitude leads to the variations of the frequency and phase. From the structural (deterministic) point of view, the receptor could perceive this as a code (a language), which represents the variation in frequency (i.e., modulation), that is, the information transmission. In this way, the conceptualization of the action is made.

## 4 Conclusion

The meanings of words and language are represented in a semantic system distributed across much of the cerebral cortex. Each semantic concept is represented in multiple semantic areas and each semantic area represents multiple semantic concepts. The semantic system of the brain shows that words are grouped by meaning, thus revealing how complicated and widespread the word maps in our heads are. The semantic logic and the semantic system of the brain could generate new strategies in the cognitive-behavioral approach by using and reevaluating the therapeutic stories, metaphors, symbols and rituals in a creative way.

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