Using Covariance Matrix and Fast Fourier Transform for Estimating Spectrum Hole Location in Cognitive Radio Network

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Abstract: This paper worked on Using Covariance Matrix and Fast Fourier Transform for Estimating Spectrum Hole Location in Cognitive Radio Network, Fast Fourier Transform (FFT) was used to split the Spectrum into sub-band channels. The paper adopted the use of Covariance Matrix in determining the actual range for the bounds. From the result it can be deduce that the areas where there are no signals at all are point where noise is prevalent, while the point between 0 and 20 on the principal component axis are the point where you have high possibility of spectrum hole availability, the point between 20 and 30 on the principal axis is the point with high possibility of error false alarm. Fast Fourier Transform was used to decompose the wideband spectrum to 64 Sub-band channels. the result of this shows that there exists a relationship between the covariance matrix and the sub-band energy. Two important properties of any square matrix are its trace, and its determinant. Analysis of Results obtained from Simulations shows that at signal-to-noise ratio (SNR) value of -10dB, spectrum holes were identified in 2 sub-band channels 13 and 53.

Key-Words: -Fast Fourier Transform, Covariance Matrix, Cognitive Radio, Spectrum, Ultra-Wide Band

1 Introduction
The recent increase in the demand for the already scarce wireless radio bandwidth has added weight to the increase in scarcity of radio spectrum. A recent study by Cisco [1] showed that global mobile data traffic grew by 71 percent in 2017 alone, reaching 11.5 exabytes per month of that year. The same study showed that nearly 650 million devices and connections were added in 2017. Another contributing factor to the problem of spectrum scarcity is the nature of spectrum allocation implemented by regulatory agencies, which causes inefficient utilization of radio spectrum [2]. Given a wideband spectrum range, as shown in Figure 1 [3], a scan reveals some bands are vacant, some bands are sparingly utilized, and others are entirely used.[[15][16]

Cognitive Radio is an emerging technique to increase the effective usage of the spectrum. Due to the rapid development in number of wireless devices and gadgets, there is a scarcity in radio frequency band. This lead to the spectrum sensing concept of identifying the unused spectrum holes. Spectrum sensing can be defined as the ability to detect the presence or absence of a licensed user in the channel, while the most challenging task is to sense in very low SNR regime [2][13]

The ultrawide band signal shown in Figure 1 is the concatenation of different signals with varying frequencies.[4][12][13]

The signal was generated using the relationship in equation 1 and fig 1.

\[ x(t) = x_1(t) + x_2(t) + x_3(t) + x_4(t) + x_5(t) + x_6(t) + x_7(t) + x_8(t) + x_9(t) + x_{10}(t) + x_{11}(t) \]  

(1)

between time and frequency in equation 1 and fig 1.
The signal in fig 1 and 2 are wideband signal embed with noise, signal which the frequency increases linearly over time [3][14]

![Figure 2 Input signal under observation](image)

Figure 2 Input signal under observation

This also shown in figure 5 and 6 which was computed using an exponential function with varying frequencies computed using an exponential function with varying frequencies.

![Figure 5 FFT Splitting of Wideband Signal](image)

Figure 5 FFT Splitting of Wideband Signal

2. Filter Bank using Fast Fourier Transform

The diagram in Fig 4 is the decomposed signal using an FFT (Fast Fourier transform) filter bank is shown in Fig 5 where the signals in each channel of the filter bank are.[3][15]

![Figure 4 The signal decomposition using an FFT (fast Fourier transform)](image)

Figure 4 The signal decomposition using an FFT (fast Fourier transform)

In this work considered splitting channels from channel 1 to Channel k shown in equation 2 and equation 3, the FFT filter bank considered has 64 sub-band channels. Thus, the first sub-band channel for the FFT filter bank in Fig 4 has an index of 1, while the last sub-band channel has an index of 64. With this arrangement, the decomposition of the input signal shown by the filter bank shown in Figure 4 will yield an array of coefficients spread across 64 sub-band channels as shown in Figure 7. The coefficients in each sub-band channel are computed using the relationships defined in equations.[4][15][6]
\[ channel_0 = b_0(n) = \sum_{m=n-(N-1)}^{n} a(m) e^{-jw_m} \]
\[ channel_1 = b_1(n) = \sum_{m=n-(N-1)}^{n} a(m) e^{-jw_m} \]
\[ \vdots \]
\[ channel_k = b_k(n) = \sum_{m=n-(N-1)}^{n} a(m) e^{-jw_m} \]

\[ x_1(t) = A_1 \sin(2\pi f_1 t) \]
\[ x_2(t) = A_2 \sin(2\pi f_2 t) \]
\[ x_3(t) = A_3 \sin\left[\phi_3 + 2\pi \left(f_3 t + \frac{\phi_3}{2}\right)\right] \]
\[ x_4(t) = A_4 \sin\left[\phi_4 + 2\pi \left(f_4 t + \frac{\phi_4}{2}\right)\right] \]
\[ x_5(t) = A_5 \sin\left[\phi_5 + 2\pi \left(f_5 t + \frac{\phi_5}{2}\right)\right] \]

\[ S(f, \alpha) = \sum_{r=-\infty}^{\infty} R_\alpha^y(r)e^{-j2\pi n r} \quad \text{where} \]
\[ R_\alpha^y(r) = E\left[y(n + r)y^*(n - \tau)e^{j2\pi n \alpha} \right] \]

The equation is the cyclic autocorrelation function, and the cyclic frequency is \( \alpha \). The cyclic spectral density function gives a maximum output at points where the fundamental frequencies of the received signal are equal to the cyclic frequency. [18][20]

**4 Cyclic Spectrum Estimation**

Communication systems are increasingly developed in a discrete domain employing digital conversion. In practice, cyclic spectrum estimation can be done using two algorithms, the FFT accumulation method (FAM) and Strip Spectral Correlation Analyzer (SSCA). In [12], authors discuss in detail different approaches of the two estimation methods: in the time domain and frequency domain. These two methods are based on changes in the smoothed cyclic cross periodogram which is defined as

\[ \int_{xyT}^\alpha (n, f) = \lim_{2n+1 \to \infty} \sum_{n=-N}^{N} \frac{1}{2n+1} X_T(n, f + \frac{\alpha}{2}) Y_T^*(n, f - \frac{\alpha}{2}) \]..........................(5)

where

\[ f_{xyT}^\alpha (n, f) = \lim_{2n+1 \to \infty} \sum_{n=-N}^{N} \frac{1}{2n+1} X_T(n, f + \frac{\alpha}{2}) Y_T^*(n, f - \frac{\alpha}{2}) \]

and are the complex envelopes of the signal \( x(n) \) and \( y(n) \) respectively and which are calculated as follows,

\[ R_{xx}^y = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} R_{xx}(t, \tau)e^{-j2\pi n \alpha} dt \]..........................(6)
Below is a 3D representation of a two sided display of the spectrogram as a waterfall plot.[8][9] The information here shows a breakdown in 2 domains both frequency and time. The figure 5 gives a clear visualisation of how the Signal is visualisation of how the graph is represented[6]

Considering the FFT in equation 1 for each of the 64 sub-band channels, a covariance matrix is generated from the reduced principal components using the relationship in (2)[7]:

5 Covariance Matrix

Covariance Matrix is used in specifically detecting hole location in the spectrum, this is after all the convergence has been realised. Employing a Power spectral Density map (PSD), the general representation of a covariance is given in equation 7 below

\[
\text{Covariance}(x, y) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \ldots \ldots 7
\]

- Covariance \((x, x)\) of spectrum data = \(\text{var}(x)\)
- Covariance \((x, y)\) = Covariance \((y, x)\)
- Let’s assume a spectral Data plot in figure 10 note that y axis represent the Energy level while the x axis represent the frequency.[10]
- The plot in figure 11 show a scattered graph of spectrum data, this platform will make it possible for covariance method to locate a point with high possibility of a spectrum hole.[11][21]

Covariance method is shown in equation 8

\[
\text{Covariance}(x, y) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \ldots \ldots 8
\]

- Diagonization will help further reduce the data using covariance matrix, there is high possibility of hole detection in the data within the diagonal of the matrix

\[
\text{Covariance}(\Sigma) = \begin{bmatrix}
\text{cov}(x_1, x_1) & \text{cov}(x_1, x_2) & \ldots & \text{cov}(x_1, x_m) \\
\text{cov}(x_2, x_1) & \text{cov}(x_2, x_2) & \ldots & \text{cov}(x_2, x_m) \\
\vdots & \vdots & \ddots & \vdots \\
\text{cov}(x_m, x_1) & \text{cov}(x_m, x_2) & \ldots & \text{cov}(x_m, x_m)
\end{bmatrix}
\]

\[
\text{Covariance}(\Sigma) = \frac{1}{n} (X - \bar{X})(X - \bar{X})^T; \text{where } X = \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_m
\end{bmatrix}
\]

\[
c = \text{randn}(5,5)
\]

\[
c = \begin{bmatrix}
0.5377 & -1.3077 & -1.3499 & -0.2050 & 0.6715 \\
1.8339 & -0.4336 & 3.0349 & -0.1241 & -1.2075 \\
-2.2588 & 0.3426 & 0.7254 & 1.4897 & 0.7172 \\
0.8622 & 3.5784 & -0.0631 & 1.4090 & 1.6302 \\
0.3188 & 2.7694 & 0.7147 & 1.4172 & 0.4889
\end{bmatrix}
\]

\[
\text{>> a_cov = a * a'}
\]

Undefined function or variable ‘a’.

\[
\text{>> cov = a * a'}
\]

Undefined function or variable ‘a’.

\[
\text{>> c_cov = c * c'}
\]

\[
c_{cov} = \begin{bmatrix}
-3.3292 & 14.2353 & -3.1405 & -2.3052 & 0.7867 \\
-2.4655 & -3.1405 & 8.4796 & 2.5011 & 3.2092 \\
-3.3249 & -2.3052 & 2.5011 & 18.1953 & 12.9338 \\
-4.3772 & 0.7867 & 3.2092 & 18.1953 & 10.5297
\end{bmatrix}
\]

\[
\text{>> a_cov = a * a'}
\]

Undefined function or variable ‘a’.

\[
\text{>> cov = a * a'}
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-4.3772 & 0.7867 & 3.2092 & 18.1953 & 10.5297
\end{bmatrix}
\]

Figure 10. PCA Graph of Spectrum Data with reduced noise
The covariance matrix in equation (9) is related to the elements $a_1, \ldots, a_n$. Two important properties of any square matrix are its trace, and its determinant.

The trace of the covariance matrix in (11) was computed using the relationship:[11]

$$\text{Trace}(R_z) = \sum_{i=1}^{n} z_{ii}$$

(11)

A comparison between the results obtained reveals that the energy in a sub-band channel and the trace of the covariance matrix generated from its principal components are related by the following relationship:[12]

$$E(a_i) \approx \alpha \left[ \text{Trace}(R_z) \right]$$

(12)

$$\text{ratio} = \frac{1}{\text{length}[\text{Trace}(R_z)]} \cdot \frac{\sum_{i=1}^{N} \text{Trace}(R_z)}{\text{length}[E(a_i)]}$$

(13)

where $\alpha > 1$ is a scaling factor. A simulation based on the relationships in (11) and (12) yields the results shown in Figure 13.
The result show value of -10dB, spectrum holes were identified in 2 sub-band channels 13 and 53 from figure 14.

To establish the correlation between the profile of the sum of trace elements and the energy in the sub-band channels, envelope detection is performed on the profiles in Figure 14. An observation of the two envelopes reveals that one envelope is exactly like the other, with the difference being only in magnitude.

An extraction of the envelopes as shown in Figures 15 reveals the differences in magnitude between the envelope of the sum of the trace elements in the covariance matrix in the slice of the tensor and the envelope of the energy in the sub-band channels.

The relationship in (6) can be used for computing the mean of ratio of differences in magnitude or the two envelopes.

![Figure 14: Envelope detection for sum of trace elements and sub-channel energies](image)

Figure 14: Envelope detection for sum of trace elements and sub-channel energies

An extraction of the envelopes as shown in Figures 15 reveals the differences in magnitude between the envelope of the sum of the trace elements in the covariance matrix in the slice of the tensor and the envelope of the energy in the sub-band channels.

Figure 15: Extracted envelopes

From the relationship in (6), the value of the ratio is computed to be approximately 25 i.e. \( \text{ratio} \approx 25 \).

From the foregoing therefore, in the quest to perform spectrum hole identification, one can use the energy in the sub-band channels before tensorization or the sum of the trace elements in each covariance matrix stored as a slice of the tensor.

### 6 Threshold Derivation for Spectrum Hole Identification

According to Qin et al, in a time-frequency decomposition of a signal or data set such as the case of wavelet packet decomposition, the adaptive threshold for each sub-band channel based on the energy of the sub-band channel is:

\[
\lambda_{i+1,2j} = \frac{\lambda_{i,j} + \alpha E_{i+1,2j}}{2} 
\]

\[
\lambda_{i+1,2j-1} = \frac{\lambda_{i,j} + \alpha E_{i+1,2j-1}}{2} 
\]

where \( \lambda_{i,j} \) is the threshold associated with the parent node, while \( \lambda_{i+1,2j} \) and \( \lambda_{i+1,2j-1} \) are the thresholds associated with the two sub-band channels from the parent node at the \((i+1)th\) level of DWPT decomposition. \( E_{i+1,2j} \) and \( E_{i+1,2j-1} \) are the energies associated with the two sub-band channels at the \((i+1)th\) level of DWPT decomposition. The factor \( \alpha \) is a tuning factor in the determination of the adaptive threshold, and as it tends to 1, the
energy of the received signal has a profound effect on the adaptive threshold; if it tends to zero, the threshold at \((i+1)\)th level becomes half the threshold at \(i\)th level.

This work considered an FFT filterbank. As such, the parameter \(\lambda_{i,j}\) in (13) and (14) will be a universal parameter since all the sub-band channels in an FFT filter bank originate from the parent node as shown in Figure 4. To derive \(\lambda_{i,j}\) we propose the following problem formulation in the next section.

7 Problem Formulation using Convex Optimization

Let \(x_1 \neq x_2\) be two points on \(\mathbb{R}^n\) which can be expressed as:

\[
y = \theta x_1 + (1 - \theta) x_2
\]

where \(\theta \in [0, 1]\) creates a line passing through \(x_1\) and \(x_2\). When \(\theta = 0\), \(y = x_2\); when \(\theta = 1\), \(y = x_1\). This implies that the value of \(\theta\) between 0 and 1 corresponds to the closed line segment between \(x_1\) and \(x_2\).

Definition 1
A set \(S\) is convex if \(\forall \theta \in [0,1]\) and \(\forall u, v \in S:\n
\[
\theta u + (1 - \theta) v \in S
\]

If \(U, V \in S\) and \(\theta \in [0, 1]\), then \(\theta u + (1 - \theta) v\) is the convex combination of \(U\) and \(V\).

Definition 2
Let \(E \subset \mathbb{R}^n\) be a set defined as:

\[
E = \{e_1, e_2, ..., e_n\}
\]

where \(e_1, e_2, ..., e_n\) are the respective sum of traces in each sub-band channel of the FFT decomposed signal. The set of all convex combinations of \(U\) and \(V\) is the set of points such that:

\[
\{w \in v: w = \theta + (1 - \theta)v, 0 \leq \theta \leq 1\}
\]

Definition 3
Let \(e\) be a vector of the sum of the traces in the covariance matrices \(\Theta\). Thus, we define \(e\) as follows:

\[
e = e_1, e_2, ..., e_{64}
\]

We define the maximum value of \(e\) in (17) as:

\[
E_{\text{max}} - \text{max}(e) = \text{max}\{e_1, e_2, ..., e_{64}\}
\]

Similarly, we define the minimum value of \(e\) in (12) as:

\[
E_{\text{min}} = \text{min}(e) = \text{min}\{e_1, e_2, ..., e_{64}\}
\]

Using the extrema values \(E_{\text{max}}\) and \(E_{\text{min}}\), we can define an interval such that:

\[
\left[e_{\text{min}}, e_{\text{max}}\right] \subseteq e
\]

If we have any two points \(c, d \in \left[e_{\text{min}}, e_{\text{max}}\right]\) in \(e\) such that \(c < d\), then for any \(\theta \in [0,1]\):

\[
e_{\text{min}} \leq c = (1 - \theta)c + \theta d < (1 - \theta)c + \theta d < (1 - \theta)d + \theta d \leq e_{\text{max}}
\]

Also, from [21], the threshold value using the Neyman-Pearson relationship for any spectrum sensing scheme based on energy detection is given as:

\[
P_f = Q\left(\frac{\Gamma}{\sigma_w^2} - 1\right)\sqrt{N}
\]

where \(N\) is the number of samples as in the DWPT sub-band channels, \(\sigma_w^2\) is the noise variance with zero mean, and \(\Gamma\) is the threshold.

From which we obtain:

\[
\left(\sigma_w^2\right)\left(\frac{Q^{-1}(P_f)}{\sqrt{N}}\right) + 1 = \Gamma
\]
\[ \lambda_{i+1,j-1} = \frac{\Gamma + \alpha E_{i+1,j-1}}{2} \tag{24} \]

8 Performance Evaluation using Receiver Operating Characteristics (ROC)

From the performance obtained as SNR is varied from -10dB to 10dB, the Receiver Operating Characteristics ROC curve for the variation of probability of detection (P_d) with respect to probability of false alarm (P_{fa}) is shown in Figure 17. It can be seen that as SNR is varied from -10dB to 10dB, the curve becomes sharper. This confirms the increase in sensitivity to spectrum hole identification with increase in SNR. Also, for different values of P_{fa}, the variation of P_d with respect to increase in SNR values is analyzed in the ROC curves shown in Figure 17. It can be seen at lower P_{fa}, better linearity was shown by P_d. This is what is expected of any good detection scheme.

![Figure 17 ROC curves for variation of P_d with respect to P_{fa}](image)

9 Discussion of the Result

The simulation in figure 18 shows the result of the estimated spectrum hole location is shown. From the graph it can be deduce that the areas where there are no signals at all are point where noise is prevalent, while that point between 0 and 20 on the principal component axis are the point where you have high possibility of spectrum hole availability, the point between 20 and 30 on the principal axis is the point

![Figure 18: Principal components computed from input signal at -10dB](image)

9.1 Measuring Performance using MSE

The results in table 1 of the performance measuring techniques is presented. The metrics of evaluating the performance of the techniques is based on the Mean Square Error (MSE) and Mean Square Error (MSE). This is given as

\[ MSE = \frac{1}{n} \sum_{i=1}^{n} (Y_i - Y_p)^2 \quad \ldots \ldots 24 \]

MSE= 0.71621875

The results of the evaluation with respect to the measurement of the errors and closeness of measurement in terms of the MSE, RMSE Hence, an improved performance is shown in the MSE where estimated accuracy is at 84.24% for MSE and 60.31% for RMSE at SNR= 10dB,
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10 Conclusion

The paper worked on Using Covariance Matrix and Fast Fourier Transform for Estimating Spectrum Hole Location in Cognitive Radio Network. It developed an enhanced technique which first used Fast Fourier Series Transform to split the Spectrum into sub-band channels. Covariance Matrix was used in determining the actual range for the bounds. 64 sub-band decomposition was achieved also using Fast Fourier Transform (FFT). It showed that there exists a relationship between the covariance matrix and the sub-band energy, the performance obtained as SNR is varied from -10dB to 10dB, the Receiver Operating Characteristics ROC curve for the variation of probability of detection (Pd) with respect to probability of false alarm (Pfa) is discovered. It can be seen that as SNR is varied from -10dB to 10dB, the curve becomes sharper. The confirms the increase in sensitivity to spectrum hole identification with increase in SNR.

References


