2.2 Moments of Output Signal to Interference Ratio

Moment of n-th order of ratio of two products of square rooted Gamma random variable and Nakagami-m random variable is:

$$m_n = \overline{w^n} = \int_0^\infty dw\, w^n p_w(w) =$$

$$= \frac{2}{\Gamma(c_1)\beta_1^{c_1}} \cdot \frac{2}{\Gamma(c_2)\beta_2^{c_2}} \cdot \frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} \frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} \cdot$$

$$\cdot \frac{1}{8}\Gamma(c_1+m_2)\beta_1^{c_1+m_2}\Omega_2^{c_1+m_2} \cdot m_2^{c_2-m_2}\,\Gamma(c_1+c_2)\frac{\beta_2^{m_1+c_2}}{m_2^{m_1+c_2}} \cdot$$

$$\cdot \frac{\Gamma(m_1+c_2)\Gamma(m_1+m_2)}{\Gamma(m_1+m_2+c_1+c_2)} \cdot \int_0^\infty dw\, w^{n+2m_1-1}$$

$$\cdot {}_2F_1\left(m_1+c_2, m_1+m_2, m_1+m_2+c_1+c_2, 1-\frac{m_1}{\Omega_1}\frac{\beta_2}{m_2}w^2\right). \quad (18)$$

By using replacement:

$$\frac{m_1}{\Omega_1}\frac{\beta_2}{m_2}w^2 = s, \quad (19)$$

it follows:

$$w^2 = \frac{\Omega_1 m_2}{m_1\beta_2}s, \quad wdw = \frac{1}{2}\frac{\Omega_1 m_2}{m_1\beta_2}ds. \quad (20)$$

After substituting, the expression for m_n becomes:

$$m_n = \frac{2}{\Gamma(c_1)\beta_1^{c_1}} \cdot \frac{2}{\Gamma(c_2)\beta_2^{c_2}} \cdot$$

$$\cdot \frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} \frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} \cdot \frac{1}{8}\Gamma(c_1+m_2)\beta_1^{c_1+m_2}\Omega_2^{c_1+m_2} \cdot$$

$$\cdot m_2^{c_2-m_2}\,\Gamma(c_1+c_2)\frac{\beta_2^{m_1+c_2}}{m_2^{m_1+c_2}} \cdot$$

$$\cdot \frac{\Gamma(m_1+c_2)\Gamma(m_1+m_2)}{\Gamma(m_1+m_2+c_1+c_2)} \cdot \int_0^\infty ds\, s^{\frac{n}{2}+m_1-1}$$

$$\cdot {}_2F_1(m_1+c_2, m_1+m_2, m_1+m_2+c_1+c_2, 1-s) \quad (21)$$

Now, by using the formula:

$${}_2F_1(a,b,c,-z)z^{-s-1} = \frac{\Gamma(a+s)\Gamma(b+s)\Gamma(c)\Gamma(-s)}{\Gamma(a)\Gamma(b)\Gamma(c+s)},$$

previous expression will be:

$$m_n = \frac{2}{\Gamma(c_1)\beta_1^{c_1}} \cdot \frac{2}{\Gamma(c_2)\beta_2^{c_2}} \cdot$$

$$\cdot \frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} \frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} \cdot \frac{1}{8}\Gamma(c_1+m_2)\beta_1^{c_1+m_2}\Omega_2^{c_1+m_2} \cdot$$

$$\cdot m_2^{c_2-m_2}\,\Gamma(c_1+c_2)\frac{\beta_2^{m_1+c_2}}{m_2^{m_1+c_2}} \cdot$$

$$\cdot \frac{\Gamma(m_1+c_2)\Gamma(m_1+m_2)}{\Gamma(m_1+m_2+c_1+c_2)} \frac{1}{2}\left(\frac{\Omega_1 m_2}{m_1\beta_2}\right)^{\frac{n}{2}+m_1}$$

$$\cdot \frac{\Gamma\left(m_1+c_2-\frac{n}{2}-m_1\right)\Gamma\left(m_1+m_2-\frac{n}{2}-m_1\right)\Gamma(m_1+m_2+c_1+c_2)\Gamma\left(\frac{n}{2}+m_1\right)}{\Gamma(m_1+c_2)\Gamma(m_1+m_2)\Gamma\left(m_1+m_2+c_1+c_2-\frac{n}{2}-m_1\right)} \quad (22)$$

2.3 Moment generating function of Output Signal to Interference Ratio

Moment generating function (MGF) of output SIR is:

$$M_w(s) = \overline{e^{sw}} = \int_0^\infty dw\, e^{sw} p_w(w) =$$

$$= \frac{2}{\Gamma(c_1)\beta_1^{c_1}} \cdot \frac{2}{\Gamma(c_2)\beta_2^{c_2}} \cdot \frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} \frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} \cdot$$

$$\cdot \frac{1}{8}\Gamma(c_1+m_2)\beta_1^{c_1+m_2}\Omega_2^{c_1+m_2} \cdot m_2^{c_2-m_2}\,\Gamma(c_1+c_2)\frac{\beta_2^{m_1+c_2}}{m_2^{m_1+c_2}} \cdot$$

$$\cdot \frac{\Gamma(m_1+c_2)\Gamma(m_1+m_2)}{\Gamma(m_1+m_2+c_1+c_2)} \cdot \int_0^\infty dw\, w^{2m_1-1}e^{sw}$$

$$\cdot {}_2F_1\left(m_1+c_2, m_1+m_2, m_1+m_2+c_1+c_2, -\frac{m_1}{\Omega_1}\frac{\beta_2}{m_2}w^2\right) =$$

$$= \frac{2}{\Gamma(c_1)\beta_1^{c_1}} \cdot \frac{2}{\Gamma(c_2)\beta_2^{c_2}} \cdot \frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} \frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} \cdot$$

$$\cdot \frac{1}{2}\Gamma(c_1+m_2)\beta_1^{c_1+m_2}\Omega_2^{c_1+m_2} \cdot m_2^{c_2-m_2}\,\Gamma(c_1+c_2)\frac{\beta_2^{m_1+c_2}}{m_2^{m_1+c_2}} \cdot$$

$$\cdot \frac{\Gamma(m_1+c_2)\Gamma(m_1+m_2)}{\Gamma(m_1+m_2+c_1+c_2)} \cdot \sum_{k_1=0}^\infty \frac{s^{k_1}}{k_1!}\int_0^\infty dw\, w^{2m_1-1+k_1} \cdot$$

$$\cdot {}_2F_1\left(m_1+c_2, m_1+m_2, m_1+m_2+c_1+c_2, 1-\frac{m_1}{\Omega_1}\frac{\beta_2}{m_2}w^2\right) \quad (23)$$

Now, let us introduce the integral J_3:

$$J_3 = \int_0^\infty dw\, w^{2m_1-1+k_1} \cdot$$

$$\cdot {}_2F_1\left(m_1+c_2, m_1+m_2, m_1+m_2+c_1+c_2, -\frac{m_1}{\Omega_1}\frac{\beta_2}{m_2}w^2\right) \quad (24)$$

By dint of exchange introduced by formulas (19) and (20):

$$\frac{m_1}{\Omega_1}\frac{\beta_2}{m_2}w^2 = s \ , \ w^2 = \frac{\Omega_1 m_2}{m_1\beta_2}s \ , \ wdw = \frac{1}{2}\frac{\Omega_1 m_2}{m_1\beta_2}ds \ ,$$

and after replacing the integral J_3 will be:

$$J_3 = \frac{1}{2}\left(\frac{\Omega_1 m_2}{m_1\beta_2}\right)^{m_1+\frac{k_1}{2}} \cdot \int_0^\infty ds\, s^{m_1+\frac{k_1}{2}-1} \cdot$$

$$\cdot\, {}_2F_1\left(m_1+c_2, m_1+m_2, m_1+m_2+c_1+c_2, -s\right) =$$

$$= \frac{1}{2}\left(\frac{\Omega_1 m_2}{m_1\beta_2}\right)^{m_1+\frac{k_1}{2}} \cdot$$

$$\cdot \frac{\Gamma\left(c_2-\frac{k_1}{2}\right)\Gamma\left(m_2-\frac{k_1}{2}\right)\Gamma\left(m_1+m_2+c_1+c_2\right)\Gamma\left(-m_1-\frac{k_1}{2}\right)}{\Gamma\left(m_1+c_2\right)\Gamma\left(m_1+m_2\right)\Gamma\left(m_1+m_2+c_1+c_2-m_1-\frac{k_1}{2}\right)} \qquad (25)$$

3 Numerical Results

The first moment of w (mean value $\overline{w}$) is given in the first four figures, and the second moment of w (squared average value $\overline{w^2}$) in the second four figures.

In Fig. 1, the main value $\overline{w}$ versus average signal power Ω_1 for Gamma long scale parameters $\beta_1=\beta_2=1$, Nakagami-m fading severity parameters $m_1=m_2=2$, Gamma fading severity parameters $c_1=c_2=2$ and variable interference signal power Ω_2 is presented. The first moment of w versus interference signal power Ω_2 for $\beta_1=\beta_2=1$, $m_1=m_2=2$, $c_1=c_2=2$ and changeable average desired signal power Ω_1 is shown in Fig. 2.

It is obvious from Fig. 1 that mean value $\overline{w}$ increases with increasing of average signal power Ω_1. The mean value is bigger for smaller values of average interference signal power Ω_2 for the same other parameters. One can see from Fig. 2 that the first moment of w decreases with enlarging of average interference power. This decline is more pronounced for small values of Ω_2.

In Fig. 3 and 4, the mean value $\overline{w}$ versus Gamma long term scale parameters β_1 and β_2, respectively, is plotted. In Fig. 3, the graph is drawn for $m_1=m_2=2$, $c_1=c_2=2$ and variable average signal power Ω_1, average interference power Ω_2 and scale parameter β_2. It is possible to see from this figure that the first moment of w rises with enlargement of Gamma long term fading parameter β_1. This is more expressed for bigger values of Gamma large scale fading parameter β_2.

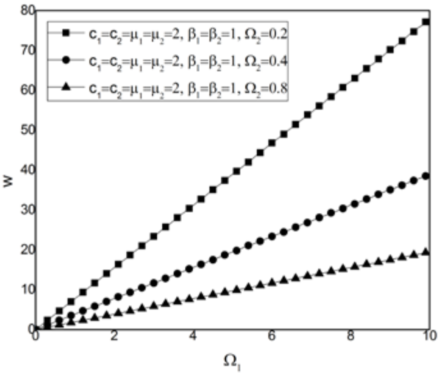
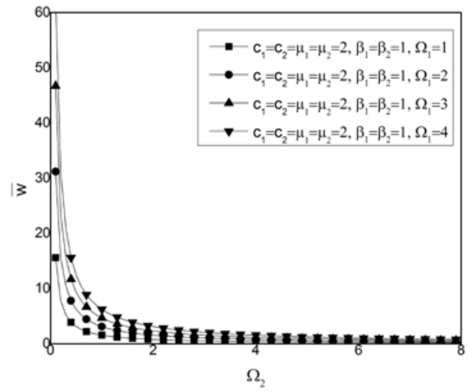


Fig. 1. Mean value $\overline{w}$ versus average desired signal power Ω_1 for Gamma long scale parameters $\beta_1=\beta_2=1$, Nakagami-m fading severity parameters $m_1=m_2=2$, Gamma fading severity parameters $c_1=c_2=2$ and variable average interference signal power Ω_2.



F

Fig.2. The first moment of w versus average interference power Ω_2 for $\beta_1=\beta_2=1$, $m_1=m_2=2$, $c_1=c_2=2$ and changeable average desired signal power Ω_1.

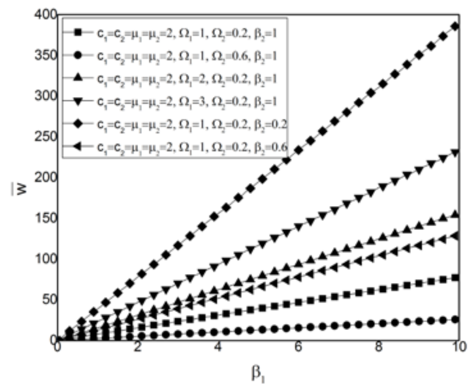


Fig.3. Mean value $\overline{w}$ versus Gamma long term scale parameter β_1 for $m_1=m_2=2$, $c_1=c_2=2$ and variable average signal power Ω_1, average interference power Ω_2 and scale parameter β_2.

From Fig. 4, it is visible that the first moment of w abates with an increase of Gamma long term parameter β_2. This is more expressed for smaller values of Gamma large scale fading parameter β_2. The mean value is bigger for bigger values of Ω_1 and smaller for larger value of Ω_2.

The second moment of w is introduced in Fig. 5 versus average signal power Ω_1 for $m_1=m_2=2$, $c_1=c_2=2$ and variable average interference power Ω_2 and scale parameters $\beta_1=\beta_2=1$. The squared average value $\overline{w^2}$ is presented in Fig. 6 versus interference power Ω_2 for $m_1=m_2=2$, $c_1=c_2=2$ and variable average interference signal power Ω_2 and scale parameters $\beta_1=\beta_2=1$.

It can be seen from these two figures that squared average value grows with rise of signal power and reduction of interference power. The least values are for bigger interference power Ω_2 and small signal power Ω_1.

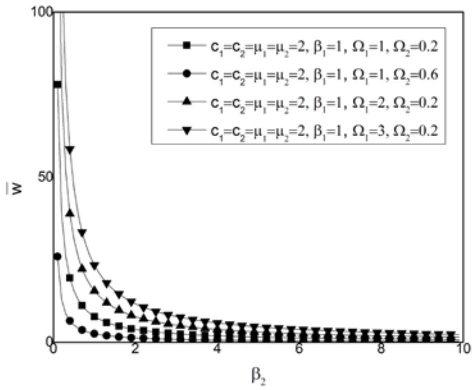


Fig. 4. The first moment of w versus scale parameter β_2 for $m_1=m_2=2$, $c_1= c_2=2$ and scale parameter $\beta_1=1$ and variable average signal power Ω_1 and average interference power Ω_2.

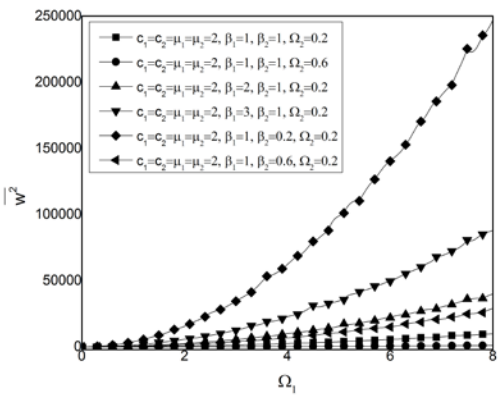


Fig.5. The squared average value $\overline{w^2}$ versus average signal power Ω_1 for $m_1=m_2=2$, $c_1=c_2=2$ and variable average interference power Ω_2 and scale parameters $\beta_1=\beta_2=1$.

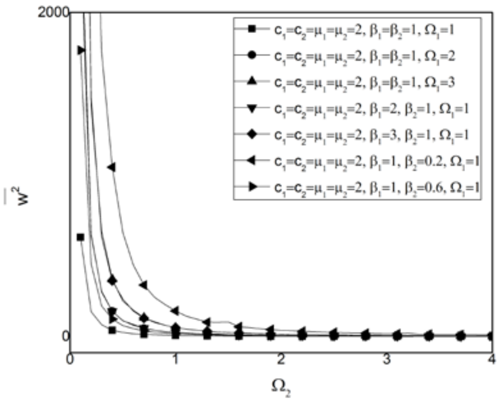


Fig.6. The second moment of w versus average interference power Ω_2 with $m_1= m_2=2$, $c_1= c_2=2$ and variable scale parameters β_1 and β_2 as well as average signal power Ω_1.

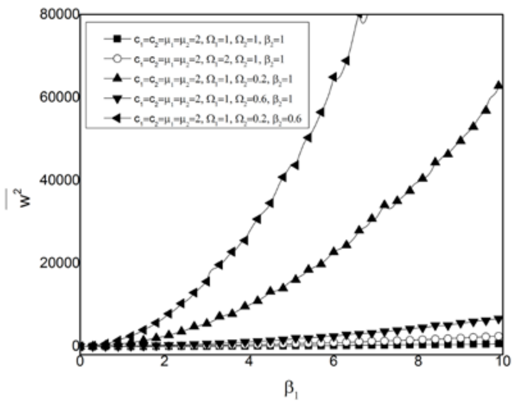


Fig. 7. The second moment of w versus scale parameter β_1 for changeable average signal power Ω_1, interference signal power Ω_2, and scale parameter β_2, with $m_1= m_2=2$, $c_1= c_2=2$.

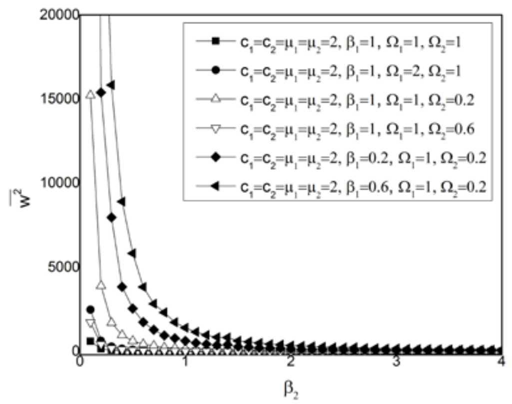


Fig.8. The second moment of w versus scale parameter β_2 for some values of average signal power Ω_1 and average interference signal power Ω_2, with $m_1= m_2=2$, $c_1= c_2=2$ and variable scale parameter β_1.

The second moment of w versus scale parameter β_1 for changeable average signal power Ω_1, interference power Ω_2, and scale parameter β_2, with $m_1= m_2=2$, $c_1= c_2=2$, is shown in Fig. 7. The squared average value $\overline{w^2}$ is plotted in Fig. 8 versus scale parameter β_2 for different values of average signal power Ω_1 and average interference power Ω_2, with $m_1=m_2=2$, $c_1=c_2=2$ and variable scale parameter β_1.

If scale parameter β_1 aggrandizes, the second moment of w also increases. The second moment declines with increment of scale parameter β_2. One can see from Figs. 7 and 8 that the squared average value has growth especially for big scale parameters β_1 for smaller β_2. The changes are more prominent for low values of β_2.

4 Conclusion

Wireless communication system operating over shadowed short term fading channel in the presence of cochannel interference affected to shadowed multipath fading is considered. Desired signal experiences Gamma long term fading and Nakagami-*m* short term fading and cochannel interference subjected to Gamma shadowed Nakagami-*m* multipath fading. In proposed model, desired signal envelope can be represented as product of square root of Gamma random variable and Nakagami-*m* random variable. Also, interference envelope can be represented as product of square root of Gamma random variable and Nakagami-*m* random variable. Signal to interference ratio can be calculated as the ratio of two products of square rooted Gamma random variable and Nakagami-*m* random variable.

For parameter m=1, Gamma shadowed Nakagami-*m* multipath fading channel becomes Gamma shadowed Rayleigh multipath fading channel. When parameter m goes to infinity, Gamma shadowed Nakagami-*m* multipath fading channel ensues pure Gamma long term fading channel. When Gamma shadowing severity parameter tends to be infinite, Gamma shadowed Nakagami-*m* multipath fading channel turns into pure Nakagami-*m* multipath fading channel. In a situation where both, Nakagami-*m* parameter m and Gamma shadowing severity parameter tend to infinity, Gamma shadowed Nakagami-*m* multipath fading channel is no fading channel.

In this paper, probability density function, moments and moment generating function of signal to interference ratio at the output of considered wireless communication system are calculated. By using these formulas, the outage probability and bit error probability can be calculated.

The first moment and the second moment increase as Nakagami-m parameter and Gamma long term fading parameter increase. The system performance are better for higher values of moments.

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