Thermodynamic process of an ideal heat transfer process in a system with a solar collector as a higher temperature tank

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Abstract: - This paper presents a mathematical calculation for the temperature of a heat tank in a system with a solar collector and this is done by a first order differential equations. The obtained formula describes the temperature in the time domain and allows us to determine in advance the final temperature of the process. This is very important in a case when there is heating of the tank not only by solar energy, but also by means of some other energy source, for example, some type of fuel. Then we can plan the energy consumption and make savings, because in such a way we are able to predetermine the temperature at the end of the process. In this paper we make a mathematical derivation that is model and example that enables development of other formulas for another different processes.

Key-Words: - heat-transfer process; solar collector; energy source; calculation; computer optimisation

1 Introduction

The work was motivated by the need to design a heating system where we can easily determine how efficient is a solar thermal system and what are its characteristics i.e. how quick it can heat a certain heat tank and to which temperature. So far, no known formula could calculate this. Knowing the characteristics of the system in the time domain allows us to select the power of the thermal solar collector as well as the mass of a heat tank so that it meets the desired requirements.

Using first-order differential equations, we will derive a general formula for the ideal process of heat transfer from a warmer tank of mass m_2, which is a solar collector for heat energy of power P, to a colder heat tank of mass m_1. Our intention is to derive a formula that can later be brought closer to the real process with additional coefficients. Based upon a such a mathematical model, formulas for much faster processes can be derived and allow computers to predict process flow in a single iteration in microseconds without the need to perform long numerical simulations involving millions of iterations that are time consuming.

This model allows us to predict the ongoing process with a simulator derived by formula in a time domain that describes the process. With this we are opening the possibility of iterations in the program to make parameter changes and to find the best that will meet our final goal. In this way we can make energy savings in process where the computer controls the parameters along the optimal curve and knows exactly how the process curve ends, with just one iteration.

In this paper we make a mathematical derivation which is model and example that enables development of other formulas for other different processes.

2 Derivation of a formula describing the temperature of a heat tank in an ideal process

We will calculate the formula for the temperature of the heat tank shown in Fig.1.

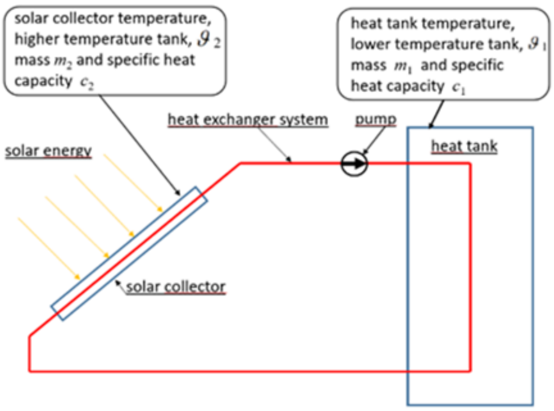

Fig. 1. Scheme of thermodynamic process of heat transfer from the collector to the heat tank

The system consists of a warmer tank which is a thermal solar collector of mass m_2 with a substance of specific heat capacity c_2 and a colder tank which is a heat tank of mass m_1 of a substance of specific heat capacity c_1 and a system of exchangers passing through them within which the liquid flow q_m of specific heat capacity c_3.

2.1. Defining variables in a process model:

Index 1 - lower temperature,

index 2 - higher temperature,

ϑ_{1p} - initial temperature of colder tank, [K]

ϑ_{2p} - initial temperature of warmer tank, [K]

ϑ_1 - temperature of colder tank variable in time, [K]

ϑ_2 - temp. of warmer tank variable in time, [K]

c - specific heat capacity of the substance in the system when the same substance is used in all parts of the system, [J/kgK]

c_1 - specific heat capacity of the substance in the colder tank, [J/kgK]

c_2 - specific heat capacity of the substance in the warmer tank, [J/kgK]

c_3 - specific heat capacity of the substance inside the tube and heat exchanger, [J/kgK]

q_m - fluid flow rate in pipes per second, [kg/s]

m_1 - mass of colder tank medium, [kg]

m_2 - mass of warmer tank medium, [kg]

P – power of solar collector, [W]

2.2. Substitution of differentials

Differential substitution is derived from the differential equation of the system [1][2][3].

$$q_m \cdot c_3 \cdot \left(\vartheta_2 - \vartheta_1\right) = m_1 \cdot c_1 \cdot \frac{d\vartheta_1}{dt} = -m_2 \cdot c_2 \cdot \frac{d\vartheta_2}{dt} \tag{1}$$

$$q_m \cdot c_3 \cdot \left(\vartheta_2 - \vartheta_1\right) dt = m_1 \cdot c_1 \cdot d\vartheta_1 = -m_2 \cdot c_2 \cdot d\vartheta_2 \tag{2}$$

From the equality (3)

$$m_1 \cdot c_1 \cdot d\vartheta_1 = -m_2 \cdot c_2 \cdot d\vartheta_2 \tag{3}$$

we define temperature change differentials in tanks determined by their masses,

and the differential for temperature change in a colder tank is equal to:

$$d\vartheta_1 = -\frac{m_2 \cdot c_2}{m_1 \cdot c_1} d\vartheta_2 \tag{4}$$

while the temperature change differential in a warmer tank is also defined by the power of the solar collector.

$$d\vartheta_2 = -\frac{m_1 \cdot c_1}{m_2 \cdot c_2} d\vartheta_1 + \frac{Pdt}{m_2 \cdot c_2} \tag{5}$$

From the differential we get a mathematical evidence of the temperature of the colder tank:

$$\vartheta_1 = \vartheta_{1p} + \int_0^t d\vartheta_1 = \vartheta_{1p} - \frac{m_2 \cdot c_2}{m_1 \cdot c_1} \int_0^t d\vartheta_2 \tag{6}$$

The temperature of the warmer tank is:

$$\vartheta_2 = \vartheta_{2p} + \int_0^t d\vartheta_2 =$$

$$= \vartheta_{2p} + \frac{P}{m_2 \cdot c_2} \cdot \int_0^t dt - \frac{m_1 \cdot c_1}{m_2 \cdot c_2} \int_0^t d\vartheta_1 =$$

$$= \vartheta_{2p} + \frac{P}{m_2 \cdot c_2} \cdot t - \frac{m_1 \cdot c_1}{m_2 \cdot c_2} \int_0^t d\vartheta_1 \tag{7}$$

2.3. Determination of the general formula for the temperature of a colder tank

Here we determine the general formula for the temperature of a colder tank of an ideal heat transfer process in a system with a collector that is a tank of a higher temperature.

Differential equation for cooler tank temperatures is:

$$\frac{d\vartheta_1}{dt} = \frac{q_m \cdot c_3}{m_1 \cdot c_1} \left(\vartheta_2 - \vartheta_1\right) \tag{8}$$

By substituting the temperatures ϑ_1 and ϑ_2 defined in (5) and (6), we obtain the differential equation for the lower temperature expressed only by the lower temperature variable:

$$\frac{d\vartheta_1}{dt}=\frac{q_m\cdot c_3}{m_1\cdot c_1}\left[\vartheta_{2p}+\frac{P}{m_2\cdot c_2}\cdot t-\frac{m_1\cdot c_1}{m_2\cdot c_2}\int_0^t d\vartheta_1-\left(\vartheta_{1p}+\int d\vartheta_1\right)\right]$$

$$\frac{d\vartheta_1}{dt}=\frac{q_m\cdot c_3}{m_1\cdot c_1}\left[\vartheta_{2p}+\frac{P}{m_2\cdot c_2}\cdot t-\vartheta_{1p}-\left(\frac{m_1\cdot c_1}{m_2\cdot c_2}+1\right)\int d\vartheta_1\right]$$

$$\frac{d\vartheta_1}{dt}=\frac{q_m\cdot c_3}{m_1\cdot c_1}\left[\vartheta_{2p}-\vartheta_{1p}+\frac{P}{m_2\cdot c_2}\cdot t-\left(\frac{m_1\cdot c_1+m_2\cdot c_2}{m_2\cdot c_2}\right)\int d\vartheta_1\right] \quad (9)$$

When we solve the integral at the end we get:

$$\int d\vartheta_1=\vartheta_1+k_1 \quad (10)$$

Let's include it in the equation and get:

$$\frac{d\vartheta_1}{dt}=\frac{q_m\cdot c_3}{m_1\cdot c_1}\left[\vartheta_{2p}-\vartheta_{1p}+\frac{P}{m_2\cdot c_2}\cdot t-\left(\frac{m_1\cdot c_1+m_2\cdot c_2}{m_2\cdot c_2}\right)\cdot\left(\vartheta_1+k_1\right)\right]$$

$$\frac{d\vartheta_1}{dt}=-\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot\left(\vartheta_1+k_1\right)+\frac{q_m\cdot c_3}{m_1\cdot c_1}\cdot\left(\vartheta_{2p}-\vartheta_{1p}+\frac{P}{m_2\cdot c_2}\cdot t\right)$$

$$\frac{d\vartheta_1}{dt}+\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot\left(\vartheta_1+k_1\right)=\frac{q_m\cdot c_3}{m_1\cdot c_1}\cdot\left(\vartheta_{2p}-\vartheta_{1p}+\frac{P}{m_2\cdot c_2}\cdot t\right)$$

$$\frac{d\vartheta_1}{dt}+\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot\vartheta_1=\frac{q_m\cdot c_3}{m_1\cdot c_1}\cdot\frac{P}{m_2\cdot c_2}\cdot t+\frac{q_m\cdot c_3}{m_1\cdot c_1}\cdot\left(\vartheta_{2p}-\vartheta_{1p}\right)-\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot k_1$$

$$\frac{d\vartheta_1}{dt}+\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot\vartheta_1=\frac{q_m\cdot c_3}{m_1\cdot c_1}\cdot\frac{P}{m_2\cdot c_2}\cdot t+\frac{q_m\cdot c_3}{m_1\cdot c_1}\cdot\left(\vartheta_{2p}-\vartheta_{1p}-\frac{\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_2\cdot c_2}\cdot k_1\right)$$

$$\frac{d\vartheta_1}{dt}+\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot\vartheta_1=\frac{q_m\cdot c_3}{m_1\cdot c_1}\cdot\left(\frac{P}{m_2\cdot c_2}\cdot t+\vartheta_{2p}-\vartheta_{1p}-\frac{\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_2\cdot c_2}\cdot k_1\right) \quad (11)$$

Using equation (12) to solve the 1st order differential equation

$$\frac{dy}{dx}+p(x)\cdot y=q(x) \quad (12)$$

From (11) we define polynomials *p(t)* (13), *q(t)* (14) and a polynomial $\mu(t)$ (15):

$$p(t)=\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2} \quad (13)$$

$$q(t)=\frac{q_m\cdot c_3}{m_1\cdot c_1}\cdot\left(\frac{P}{m_2\cdot c_2}\cdot t+\vartheta_{2p}-\vartheta_{1p}-\frac{\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_2\cdot c_2}\cdot k_1\right) \quad (14)$$

$$\mu(t)=e^{\int p(t)dt}=e^{\int\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}dt}==e^{\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t} \quad (15)$$

we get the equation for the temperatures of the colder tank written in the form:

$$\vartheta_1=\frac{1}{\mu(t)}\cdot\left[\int q(t)\cdot\mu(t)dt+konst\right] \quad (16)$$

After including all polynomials, the first-order differential equation to be solved takes the form:

$$\vartheta_1=\frac{1}{e^{\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}}\cdot\left[\int\frac{q_m\cdot c_3}{m_1\cdot c_1}\cdot\left(\frac{P}{m_2\cdot c_2}\cdot t+\vartheta_{2p}-\vartheta_{1p}-\frac{\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_2\cdot c_2}\cdot k_1\right)e^{\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}dt+k_2\right]$$

$$\vartheta_1=\frac{1}{e^{\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}}\cdot\left[\int\frac{q_m\cdot c_3}{m_1\cdot c_1}\cdot\frac{P}{m_2\cdot c_2}\cdot t\cdot e^{\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}dt+\int\frac{q_m\cdot c_3}{m_1\cdot c_1}\cdot\left(\vartheta_{2p}-\vartheta_{1p}-\frac{\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_2\cdot c_2}\cdot k_1\right)e^{\frac{q_m\cdot c_3\cdot\left(m_1\cdot c_1+m_2\cdot c_2\right)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}dt+k_2\right] \quad (17)$$

The part within the first-order differential equation needs to be solved by partial integration

$$\int \frac{q_m \cdot c_3}{m_1 \cdot c_1} \cdot \frac{P}{m_2 \cdot c_2} \cdot t \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} dt = \\ = \frac{q_m \cdot c_3}{m_1 \cdot c_1} \cdot \frac{P}{m_2 \cdot c_2} \cdot \int t \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} dt \tag{18}$$

Partial integration:

$$\int u(x) \cdot v'(x) dx = u(x) \cdot v(x) - \int v(x) \cdot u'(x) dx \tag{19}$$

We define *u(t) i v(t)*:

$$u = t \quad , \quad dv = e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} dt$$

$$du = dt \ , \quad v = \frac{m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t}$$

Then we solve the partial integral:

$$\frac{q_m \cdot c_3}{m_1 \cdot c_1} \cdot \frac{P}{m_2 \cdot c_2} \cdot \int t \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} dt =$$

$$= \frac{q_m \cdot c_3}{m_1 \cdot c_1} \cdot \frac{P}{m_2 \cdot c_2} \cdot \left[\begin{array}{l} t \cdot \frac{m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} \\ - \int \frac{m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} dt \end{array} \right] =$$

$$= \frac{q_m \cdot c_3}{m_1 \cdot c_1} \cdot \frac{P}{m_2 \cdot c_2} \cdot \left[\begin{array}{l} t \cdot \frac{m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} \\ - \frac{m_1^2 \cdot c_1^2 \cdot m_2^2 \cdot c_2^2}{q_m^2 \cdot c_3^2 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} \end{array} \right] =$$

$$= \frac{P \cdot t}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} \\ - \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} \tag{20}$$

and we include the solution of partial integration in the equation and get:

$$\vartheta_1 = \frac{1}{e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t}} \cdot \left[\begin{array}{l} \frac{P \cdot t}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} - \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} + \\ + \frac{q_m \cdot c_3}{m_1 \cdot c_1} \cdot \left(\vartheta_{2p} - \vartheta_{1p} - \frac{(m_1 + m_2)}{m_2} \cdot k_1 \right) \cdot \frac{m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} \cdot e^{\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} + k_2 \end{array} \right]$$

$$\vartheta_1 = \frac{P}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot t - \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} + \\ + \frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p} - \frac{m_1 \cdot c_1 + m_2 \cdot c_2}{m_2 \cdot c_2} \cdot k_1 \right) + k_2 \cdot e^{-\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} \tag{21}$$

The solution of the differential equation for a colder tank finally has the form:

$$\vartheta_1 = \frac{P}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot t - \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)^2} + \frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right) - k_1 + k_2 \cdot e^{-\frac{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t} \quad (22)$$

3 Determining coefficients k_1 and k_2

It is necessary now to determine the coefficients k_1 and k_2, and they should be determined within the boundary conditions.

The first boundary condition is defined at the initial moment:

$$\text{for } t = 0 \Rightarrow \vartheta_1(0) = \vartheta_{1p}$$

This means that the parts containing t are equal to zero, the temperature has an initial value of ϑ_{1p}, and the exponential part containing t is equal to 1, so after inclusion *t=0* in equation (22) we get the equation of the first boundary condition:

$$\vartheta_{1p} = -\frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)^2} + \frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right) - k_1 + k_2 \quad (23)$$

If we single out the coefficients on the left side, then the equation has the form:

$$-k_1 + k_2 = \vartheta_{1p} - \frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right) + \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)^2} \quad (24)$$

If we take the final boundary condition for $t = \infty$, we will define it so that we know the solution (and not the infinite temperature). We will do this by introducing a special case when the power of the solar collector $P = 0$, so we will know the final boundary condition, that will not be infinite temperature but the known temperature of the mixture, the temperatures in the warmer and colder tanks will be the same and will be:

$$\vartheta_1(\infty) = \frac{\vartheta_{1p} \cdot m_1 \cdot c_1 + \vartheta_{2p} \cdot m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \quad (25)$$

In the solution of the differential equation (22) when we include that $t = \infty$, the exponential part disappears, is equal to zero, and with it the coefficient $k_2 = 0$, also the term containing *P=0* is equal to 0:

$$\vartheta_1(\infty) = \frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right) - k_1 \quad (26)$$

We include (25) on the left side of the equation, so the equation of the second boundary condition takes the form:

$$\frac{\vartheta_{1p} \cdot m_1 \cdot c_1 + \vartheta_{2p} \cdot m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} = \frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right) - k_1 \quad (27)$$

Also, when the coefficient is separated to the left side we get:

$$k_1 = \frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right) - \frac{\vartheta_{1p} \cdot m_1 \cdot c_1 + \vartheta_{2p} \cdot m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \quad (28)$$

From the two boundary conditions we have a system of two equations with two unknown variables:

$$-k_1 + k_2 = \vartheta_{1p} - \frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right) + \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)^2}$$

$$k_1 = \frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right) - \frac{\vartheta_{1p} \cdot m_1 \cdot c_1 + \vartheta_{2p} \cdot m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2}$$

When we solve the equation system we get the coefficients:

$$k_1 = -\vartheta_{1p} \quad (29)$$

$$k_2 = -\frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right) + \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)^2} \quad (30)$$

By including the coefficients and after sorting, we obtain the final form of the formula for the temperature of a warmer tank in the ideal process of heat transfer from a collector of mass m_2 with a substance of specific heat c_2 to a heat tank of mass m_1 with a substance of specific heat capacity c_1, via a heat exchanger with a specific heat capacity c_3 and a flow rate q_m:

$$\begin{aligned}\vartheta_1 = {} & \frac{P}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot t - \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)^2} + \frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right) + \\ & + \vartheta_{1p} + \left(- \frac{m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right) + \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)^2} \right) \cdot e^{-\frac{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t}\end{aligned} \tag{31}$$

$$\begin{aligned}\vartheta_1 = {} & \frac{m_1 \cdot c_1 \cdot \vartheta_{1p} + m_2 \cdot c_2 \cdot \vartheta_{2p}}{m_1 \cdot c_1 + m_2 \cdot c_2} + \frac{P \cdot t}{m_1 \cdot c_1 + m_2 \cdot c_2} - \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)^2} + \\ & + \left(\frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)^2} - \frac{m_2 \cdot c_2 \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right)}{m_1 \cdot c_1 + m_2 \cdot c_2} \right) \cdot e^{-\frac{q_m \cdot c_3 \cdot \left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t}\end{aligned} \tag{32}$$

We can write the same formula in different ways, for example:

$$\begin{aligned}\vartheta_1 = {} & \vartheta_{1p} + \frac{P \cdot t + c_2 \cdot m_2 \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right)}{c_1 \cdot m_1 + c_2 \cdot m_2} - \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(c_1 \cdot m_1 + c_2 \cdot m_2\right)^2} + \\ & + \left(\frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(c_1 \cdot m_1 + c_2 \cdot m_2\right)^2} - \frac{c_2 \cdot m_2 \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right)}{c_1 \cdot m_1 + c_2 \cdot m_2} \right) \cdot e^{-\frac{q_m \cdot c_3 \cdot \left(c_1 \cdot m_1 + c_2 \cdot m_2\right)}{c_1 \cdot m_1 \cdot c_2 \cdot m_2} \cdot t}\end{aligned} \tag{33}$$

$$\begin{aligned}\vartheta_1 = {} & \frac{P \cdot t + c_1 \cdot m_1 \cdot \vartheta_{1p} + c_2 \cdot m_2 \cdot \vartheta_{2p}}{c_1 \cdot m_1 + c_2 \cdot m_2} - \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(c_1 \cdot m_1 + c_2 \cdot m_2\right)^2} + \\ & + \left(\frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(c_1 \cdot m_1 + c_2 \cdot m_2\right)^2} - \frac{c_2 \cdot m_2 \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right)}{c_1 \cdot m_1 + c_2 \cdot m_2} \right) \cdot e^{-\frac{q_m \cdot c_3 \cdot \left(c_1 \cdot m_1 + c_2 \cdot m_2\right)}{c_1 \cdot m_1 \cdot c_2 \cdot m_2} \cdot t}\end{aligned} \tag{34}$$

$$\begin{aligned}\vartheta_1 = {} & \vartheta_{1p} + \frac{P \cdot t}{c_1 \cdot m_1 + c_2 \cdot m_2} + \\ & + \left(\frac{c_2 \cdot m_2 \cdot \left(\vartheta_{2p} - \vartheta_{1p}\right)}{c_1 \cdot m_1 + c_2 \cdot m_2} - \frac{P \cdot m_1 \cdot c_1 \cdot m_2 \cdot c_2}{q_m \cdot c_3 \cdot \left(c_1 \cdot m_1 + c_2 \cdot m_2\right)^2} \right) \cdot \left(1 - e^{-\frac{q_m \cdot c_3 \cdot \left(c_1 \cdot m_1 + c_2 \cdot m_2\right)}{c_1 \cdot m_1 \cdot c_2 \cdot m_2} \cdot t} \right)\end{aligned} \tag{35}$$

If we assume that each part of the system, a warmer tank, a colder tank and a system of exchangers have the same type of fluid, and thus the same specific heat capacity, then we get

$$\vartheta_1 = \frac{m_1 \vartheta_{1p} + m_2 \vartheta_{2p}}{m_1 + m_2} + \frac{P \cdot t}{c \cdot \left(m_1 + m_2\right)} - \frac{P \cdot m_1 \cdot m_2}{q_m \cdot c \cdot \left(m_1 + m_2\right)^2} + p + \left(\frac{P \cdot m_1 \cdot m_2}{q_m \cdot c \cdot \left(m_1 + m_2\right)^2} - \frac{m_2 \left(\vartheta_{2p} - \vartheta_{1p}\right)}{m_1 + m_2} \right) \cdot e^{-\frac{q_m \left(m_1 + m_2\right)}{m_1 \cdot m_2} t} \tag{36}$$

The time constant of the system in which each part of the system may be of another substance is:

$$\tau_0 = \frac{c_1 m_1 c_2 m_2}{q_m c_3 \left(c_1 m_1 + c_2 m_2 \right)} \quad (37)$$

And the time constant of a system made of the same heat medium in each part of the system, ie. equal specific heat capacities $c = c_1 = c_2 = c_3$ is equal to:

$$\tau_0 = \frac{m_1 m_2}{q_m \left(m_1 + m_2 \right)} \quad (38)$$

4 Conclusion

Based on the derived formula in the time domain for the temperature of the heat tank in the ideal process, with realistic coefficients by which we can describe the losses, the formula can also describe the real process. Our derived formula makes it possible to predict tank temperatures at a certain interval, and if we are talking about a heat source that uses some other form of energy, the formula can provide savings and optimize fuel consumption. Based upon a such a mathematical model, formulas for much faster processes can be derived and allow computers to predict process flow in a single iteration.

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