Optimization Problem of Local Constraints for Coherent Topology Behavior

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Abstract: - In the paper is presented some researches regarding the problem of local constraints, effective stiffness criterion and strength model, integrated relaxed stress constraint in the light of the newest approaches on handling stress constraints. By their very nature stress constraints are local constraints which result in large scale optimization problems that are often difficult to solve. As is known stress constraints induce the "singularity phenomenon" and low density regions sometimes remain highly strained. The limit of the stress state in the microstructure tends to a value, higher than the stress limit. The procedure cannot remove these regions, despite the fact that, removing totally the material, the stress constraint would not be active. The study analyzes this problem in order to obtain a coherent topology and convergent behavior of the numerical solver.

Key-Words: - handling stress, topology, optimization, constraint, stress.

1 Introduction

In structural design, the topology generation for the structure is traditionally associated with the conceptual phase of design [1]. The designer may consider many alternative topologies iteratively. Once an appropriate topology has been selected, only certain parameters such as cross-section of the elements of a truss or parameters describing the boundary of the geometry are varied This approach may yield suboptimal shapes due to the inability of the approach to modify the topology during the optimization. The truly optimal shape may require the creation of one or more new boundaries to change the topology of the component being designed.

A more effective approach is Bendsoe-s and Kikuchi's (1988) who developed Khon and Strang non-homogenous problem and assumed that the material is porous and solved the optimal distribution of porosity. A design domain is defined as the space within the structure has to fit. This domain is divided into a rectangular mesh. The loads to be carried by the structure and the support conditions are prescribed by the designer.

The optimization problem can be formulated in terms of compliance (the minimization of the strain energy), restricted by varies conditions: volume, mass or equivalent stress. The shape and topology synthesis problem involves solving for a domain that optimizes some structural property for given loads and boundary conditions. One of the most challenging problems is to represent the domain as a variable. This is done [1], [5] describing it within the feasible domain so that the design variable – the shape density function has a value greater than the threshold value. The contours of this function corresponding to a threshold value are defined as the boundaries of the shape so that the regions where the value of the function is below the threshold are not part of the geometry.

Topology optimization has been extended from compliance design to other criteria [2, 3, 6]. Recently Cheng and Guo [3] proposed a new strategy – the ε relaxation technique – in order to approach the singularity phenomenon. Different integrated stress criteria developed further are presented in order to reduce computing time.

2 Optimization problems

The most common structural design problem requires finding the optimal topology of a structure, such that it can support the applied loads using a minimum amount of material. For a linear elastic structure built of a material with ultimate strength σ_l, the material distribution problem can be written as [4]

$$\left.\begin{aligned} &\min_{0\le\rho(x)\le 1} \int_\Omega \rho(x)dx \\ &\text{subject to}: \sigma_{VM}(\rho(x),x) \le \sigma_l \quad \text{if} \quad \rho(x) > 0 \end{aligned}\right\} \tag{1}$$

where $\rho\ (x)$ is the material density at point x and σ_{VM} $(\rho\ (x),\ x)$ is the von Misses stress at point x. The design problem is usually solved by discrediting the design domain with a number N of finite elements, using the shape density function as design variable, as in compliance design based optimization problems. Due to the uneven contours and convergence problems induced by the classical 0/ 1 integer problem, authors used several methods to transform it in a continuous one Bendsoe and Kikuchi (homogenization method) [1], Allaire and Kohn (penalty exponential function). This allows the variable ρ to take intermediate values and use of sensivity analysis and numerical methods to solve the problem.

3 The numerical model

The modeling of material properties in elements with intermediate densities is based on the power-law approach (SIMP method), which introduce a penalty exponential factor ($\eta > 1$) for the basic physical and mechanical properties of the material quantified with the design variable ρ.

$$E^* = \rho^\eta E^0, \tag{2}$$

where "*" denotes an effective or overall value and "^{O}" indicates the value for solid material. The use of η allows obtaining more accurate solution in terms of void/ fully dense (solid) material. The most efficient value for η is 3 [1], [4].

For the stress, a model of the strength properties is given by a similar power law with the exponent η. This allows reformulating the failure criterion [4]

$$\frac{\sigma^*_{VM}}{\rho^\eta} = \sigma_l. \tag{3}$$

4 Stress criteria

Stress constraints induce the "singularity phenomenon" [3]. Low density regions sometimes remain highly strained. When the density decreases to zero in these regions, the limit of the stress state in the microstructure tends to a value, higher than the stress limit. The procedure cannot remove these regions, despite the fact that, removing totally the material, the stress constraint would not be active. To circumvent this situation, Cheng and Guo replaced the solution of the singular problem with a sequence of perturbed non-singular problems which can be solved with usual optimization algorithms: the ε relaxation method. This method, initially developed for truss optimization problems, does not cover the continuum type topology optimization problems. The reduction of the perturbation parameter ε leads to constraint violations and slows down the convergence of the procedure. Duysinix and Sigmund [4] proposed a set of perturbed constraints, similar to the original relaxation technique.

$$\left.\begin{aligned} &\rho\left(\frac{\sigma^*_{VM}}{\rho^\eta\sigma_l} - 1\right) \le \varepsilon - \varepsilon\rho \\ &1 \ge \rho \ge \varepsilon^2 \end{aligned}\right\}. \tag{4}$$

The perturbation vanishes for $\rho = 1$ so that the solution remains feasible when ε is reduced. The permissible stress is increased for low densities, as shown by the rewritten form of equation (4).

$$\frac{\sigma^*_{VM}}{\rho^\eta} \le \sigma_l\left(1 - \varepsilon + \frac{\varepsilon}{\rho}\right). \tag{5}$$

The permissible stress is increased for low a density which gives the possibility to create or remove holes without violating the stress constraint.

To control the local stress state, the stress is treated as a local constraint, for every finite element, as Duysinix and Bendsoe did . Due to the significant increase of the optimization problem induced by this approach, Duysinix and Sigmund proposed to include both the ε – relaxation technique that alleviates the singularity phenomena and the use of effective stress criterion into the global stress constraint. There are two global measures of the relaxed distributed stress criterion (5).

The first global measure is the “p-norm” of the relaxed stress criterion.

$$\left[\sum_{e=1}^{N}\left(\max\left\{0,\frac{\sigma^{*}_{VM,e}}{\rho_e^{\eta}\sigma_l}+\varepsilon-\frac{\varepsilon}{\rho_e}\right\}\right)^{p}\right]^{\frac{1}{p}}\leq 1 .(6)$$

The second global stress constraint is the “p- mean” of the relaxed stress criterion

$$\left[\frac{1}{N}\sum_{e=1}^{N}\left(\max\left\{0,\frac{\sigma^{*}_{VM,e}}{\rho_e^{\eta}\sigma_l}+\varepsilon-\frac{\varepsilon}{\rho_e}\right\}\right)^{p}\right]^{\frac{1}{p}}\leq 1 .(7)$$

For a given *p*, the maximum stress value is always bounded from above by the p-norm and from below by the p-mean. Negative values of the relaxed criterion only appear for low stressed elements which can be truncated without influencing the global constraint, which remains continuous up to the *p-1* derivative – smooth enough. From the numerical experiments of Duysinix and Sigmund, the choice of parameter *p* must result from a compromise of the high values for the control the maximum value of the stress criterion and the p-norm and p-mean ill-conditioning when *p* increases. Good results were obtained with $p = 4$ [4].

5 Conclusions

The use of integrated equivalent stress constraints proposed by Duysinix and Sgmund proves an alternative to the use of local stress constraints for continuum-type structures. A theoretical study of the two integrated constraints shows that they bound the maximum value of the criterion, which is the limit value of the two p-norms and p-means as p grows to infinity. The p-mean function converges by lower value towards the infinite mean while the p-norm provides upper bounds to the maximum stress criterion. There is no relation identified yet between the p-norms and p-means to the maximum value and the number of elements considered in the constraint. With only one integrated constraint function is difficult to control the large number of elements which are close to the admissible stress. The convergence process becomes oscillatory and the constraint violation increases. For practical applications the parameter *p* was taken to 4 in order to consider a large influence of all active local stress criteria in the global constraints.

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