Study regarding the Accuracy of Measurement by Factors as Repeatability and Reproducibility

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Abstract: Each piece of measuring system is subject to a certain amount of variation. The following paper can be used as a guidelines in determining the precision, major problems, amount of variation of all gauging systems used throughout the manufacturing process. It will also provide a controlled structure for qualification and re-qualification of any such measuring systems used to perform measurement of product and/or process parameters.

Key-Words: accuracy, stability, system error, acceptability.

1 Introduction
Manufacturers may believe that they have an applicable and functional gauging system based on the perceived accuracy or design, but often the accuracy of the gauge is wrong. The most common situation is that a major portion of the specified tolerance is lost to measurement errors, incorrect usage, or equipment variation.
To avoid this situation is in our best interest to conduct a study of the measuring system to determine if it is both “correct” and “repeatable”.

1.1 Gauge system error
Four are the variation factors that are characterizing variable gauges no matter what the gauges are:
♦ Accuracy
♦ Stability
♦ Repeatability
♦ Reproducibility
Accuracy: is defined as the difference between the observed average of measurements and “the true average” of the same measurements. To get “the true average”, you need to use the highest precision measuring device available.

"The true average"
Accuracy
Measurement distribution
Observed average

Fig 1 Accuracy
Stability: is defined as the periodic variation that occurs due to environmental changes, power fluctuations, wear, or deterioration of gauge. It is measured as the difference between two observed averages of measurements taken in the same conditions in two different periods of time.

Repeatability (Equipment Variation): is defined as the amount of variation in the gauge when the same parts and part characteristics are measured several times by the same person.

Reproducibility (Operator Variation): is defined as the amount of variation in the measurements when different persons use the same gauge on the same parts and part characteristics.

2 Percent repeatability and reproducibility

All the variation factors were presented in the previous paragraph. The first two factors (accuracy and stability) are usually quite small, so we really don’t need to concern ourselves with them. The opposite is true for the last two factors (repeatability and reproducibility): they are generally large and will cause problems with product’s acceptance or rejection. That is why further studies will take care only about the last two factors mentioned. The percent Repeatability and Reproducibility (% R&R) is the percentage of the specified tolerance that is lost to gauging error.

2.1 Determination of gauge acceptability
Team planning and preparation is of absolute importance for the success of the gauge study. The characteristics of the gauge, number of inspectors, the significance of the characteristic being checked and the number of parts to measure are all items that need to be considered before the final plan is implemented.

Following are recommended practices for the study:

♦ Number of inspectors/testers: three (3) or two (2) should be used for part measurement
♦ Number of trials: the test is designed for three (3) or two (2) trials. Three is recommended when prior gauge capability is unknown. Two can be used when prior gauge capability studies have shown an R&R of less than 20%.
♦ Number of parts: usually ten (10), but five (5) if previous gauge capability studies have shown an R&R of less than 20%.
♦ Measuring ability: the gauge should have graduations that allow at least one-tenth (1/10) of the tolerance of the characteristic to be read directly.

After the plan is completely defined, the sample units must be measured. Data is computed using the following formulas:

♦ Each part is measured three or two times.

Measurements give a certain amount of variation characterized by the average R. Averages R_i are calculated also for each inspectors/testers. Average R is the average of the inspectors /testers averages.

\[ R = \frac{(R_1 + R_2 + R_3)}{3} \]  
for three inspectors/testers  
(1)

\[ R = \frac{(R_1 + R_2)}{2} \]  
for two inspectors/testers  
(2)

♦ Repeatability (Equipment Variation: EV) is given by the following formula:

\[ EV = K_1 \times R \]  
(3)

where \( K_1 \) is 4.56 for 2 trials or 3.05 for 3 trials

♦ Reproducibility (Operator Variation: OV) is given by the following formula:

\[ OV = K_2 \times R \]  
(4)

where \( K_2 \) is 3.65 for 2 operators/testers or 2.70 for 3 operators/testers

♦ Total Repeatability and Reproducibility (R&R) is given by the following formula:

\[ R \& R = \sqrt{(R \text{, repeatability})^2 + (R \text{, reproducibility})^2} \]  
(5)

Gauge acceptability determination:

\[ \% R \& R = \frac{R \& R}{\text{Specified Tolerance}} \]  
(6)

The criteria for determining the acceptability of a gauging system is dependent on the percentage of the specification tolerance that is used by the system error. Currently, the standards used for gauge acceptability is as follows:

♦ \( \% R \& R \leq 10\% \) Acceptable  
(7)

♦ \( 10\% \leq \% R \& R \leq 30\% \) Conditionally acceptable dependent on the importance of application, gauge cost, engineering approval, etc.  
(8)

♦ \( \% R \& R > 30\% \) Unacceptable. Repair or use other measurements means  
(9)

The study can provide also the team members with significant information about the causes of gauge errors. For example if the lack of reproducibility is large when compare to repeatability, some possible causes are:

♦ User training in method of using and reading the gauge is needed

Calibrations of the gauge dial could be more clearly defined.

If the lack of repeatability is large when compare to reproducibility, the reasons may be:

♦ The gauge could be redesigned for easy of use

Gauge maintenance might be required

3 Case studies

The present case is referring to an electronic product. The flag was raise when discrepancies were notice between the acceptance/rejection rate for different test equipment. The test parameter under discussion is a product critical parameter. A complete test equipment evaluation was requested by Customer. All data was computed using a special soft.

Because no other R&R studies were performed, initially we chose the following R&R strategy:

♦ Number of testers: 2
♦ Number of trials: 3
♦ Number of parts: 10
♦ Parameter specification limits:
  ♦ upper specification limit: 25
  ♦ lower specification limit: 17.5
  ♦ specified tolerance: 7.5
♦ Measuring ability: \( >1/10 \) of the tolerance

The measurements are presented in Table 1.
Table 1 Tester measuring values.

<table>
<thead>
<tr>
<th>Data</th>
<th>Tester No: 1</th>
<th>Tester No: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trial</td>
<td>Range</td>
</tr>
<tr>
<td>Sample #</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>24.01</td>
<td>24.00</td>
</tr>
<tr>
<td>2</td>
<td>23.98</td>
<td>23.98</td>
</tr>
<tr>
<td>3</td>
<td>23.98</td>
<td>23.98</td>
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<tr>
<td>4</td>
<td>23.98</td>
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</tr>
<tr>
<td>5</td>
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<td>6</td>
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<tr>
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<td>23.96</td>
<td>23.95</td>
</tr>
<tr>
<td>10</td>
<td>23.96</td>
<td>23.96</td>
</tr>
</tbody>
</table>

| Range average $R_1$ | 0.006 | Range average $R_2$ | 0.027 |
| Sample average $X_1$ | 23.97 | Sample average $X_2$ | 22.56 |

Average range $R$ 0.016
X-bar range $X$ 1.414

Data were computed and the test evaluation by R& R point of view is given in Table 2 (initial status).

Studying the data presented in table 2 we can draw some conclusions:

♦ Test equipment repeatability and reproducibility (R&R) is unacceptable. More than 68% of the tolerance is lost due to the gauge R&R.
♦ As the table 2 states, test repeatability is acceptable. About 0.67% of the tolerance is lost due to the equipment variation.
♦ The major problem is the reproducibility. Reproducibility is about 68%.

Table 2 Tolerance analysis

<table>
<thead>
<tr>
<th>Tolerance analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repeatability (EV)</td>
</tr>
<tr>
<td>Reproducibility (AV)</td>
</tr>
<tr>
<td>Repeatability and Reproducibility (R&amp;R)</td>
</tr>
<tr>
<td>Control limit for individual ranges</td>
</tr>
</tbody>
</table>

Note: any ranges beyond this limit may be the result of assignable causes. Identify and correct. Discard values and recomputed statistics.

The explanations are inside the following chart (the XBar-R chart for measurements):

♦ There are two different areas in XBar chart:
♦ for tester number 1, with an average of the observed measurements of 23.96;
♦ for tester number 2, with an average of observed measurements of 22.55;
♦ this drift between testers is the cause for unacceptable percent R&R;
♦ both tester is working with acceptable variances (there are no major variances inside these two areas).
All data shows very clear that testers need to be recalibrated. After this activity was completed another R&R study was performed to verify the new tester status.

The R&R strategy which was followed is:

- Number of testers: 2
- Number of trials: 2
- Number of parts: 5
- Parameter specification limits:
  - upper specification limit: 25
  - lower specification limit: 17.5
  - specified tolerance: 7.5
  - Measuring ability: >1/10 of the tolerance

The measurements are presented in Table 3.

Data were computed and the test evaluation by R&R point of view is given in Table 4 (final status).

### Table 3 Measurement with new tester status.

<table>
<thead>
<tr>
<th>Data</th>
<th>Tester 1</th>
<th></th>
<th>Tester 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trial</td>
<td></td>
<td>Trial</td>
<td></td>
</tr>
<tr>
<td>Sample #</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>Range</td>
</tr>
<tr>
<td>1</td>
<td>24.01</td>
<td>24.00</td>
<td>0.01</td>
<td>23.87</td>
</tr>
<tr>
<td>2</td>
<td>23.93</td>
<td>23.99</td>
<td>0.06</td>
<td>23.88</td>
</tr>
<tr>
<td>3</td>
<td>24.00</td>
<td>24.00</td>
<td>0.00</td>
<td>23.87</td>
</tr>
<tr>
<td>4</td>
<td>24.00</td>
<td>23.99</td>
<td>0.01</td>
<td>23.88</td>
</tr>
<tr>
<td>5</td>
<td>24.02</td>
<td>24.00</td>
<td>0.02</td>
<td>23.88</td>
</tr>
<tr>
<td>6</td>
<td>Range average R&lt;sub&gt;1&lt;/sub&gt;</td>
<td>0.020</td>
<td>Range average R&lt;sub&gt;2&lt;/sub&gt;</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td>Sample average X&lt;sub&gt;1&lt;/sub&gt;</td>
<td>23.994</td>
<td>Sample average X&lt;sub&gt;2&lt;/sub&gt;</td>
<td>23.905</td>
</tr>
</tbody>
</table>

**Average range**

0.039

**X-bar range**

0.089

### Table 4 Evaluation by R&R point of view.

<table>
<thead>
<tr>
<th>Tolerance analysis</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Repeatability (EV)</td>
<td>0.178</td>
</tr>
<tr>
<td>Reproducibility (AV)</td>
<td>0.32</td>
</tr>
<tr>
<td>Repeatability and Reproducibility (R&amp;R)</td>
<td>0.366</td>
</tr>
<tr>
<td>Control limit for individual ranges</td>
<td>0.127</td>
</tr>
</tbody>
</table>
Studying the data presented in able 4 we can draw some conclusions:

♦ Test equipment repeatability and reproducibility (R&R) is acceptable. Less than 5% of the tolerance is lost due to the gauge R&R. As Fig 6 shows, both testers are setup at the same value 23.9. Final test status is characterized by acceptable variances in and within testers. No drift is found in the final measurements.

We can compare the measurements given by both testers (see the Xbar-MR chart presented below):

![Individuals (X) Chart](chart.png)

**Fig 6. Final results.**

### 4 Conclusions

In order for equipment to be qualified, any system variation must be within an acceptable tolerance, based on the specification of the process being measured. The R&R study presented in this paper is a method for easy equipment evaluation. This technique, combined with other statistical methods, can suggest if any assignable causes are present and can completely evaluate the measurement equipment.

References:


