$$\left.\begin{aligned} &\rho\left(\frac{\sigma^*_{VM}}{\rho^\eta \sigma_1}-1\right) \le \varepsilon-\varepsilon\rho \\ &1 \ge \rho \ge \varepsilon^2 \end{aligned}\right\}. \quad (4)$$

The perturbation vanishes for $\rho = 1$ so that the solution remains feasible when ε is reduced. The permissible stress is increased for low densities, as shown by the rewritten form of equation (4)

$$\frac{\sigma^*_{VM}}{\rho^\eta} \le \sigma_1(1-\varepsilon+\frac{\varepsilon}{\rho}). \quad (5)$$

The permissible stress is increased for low densities which give the possibility to create or remove holes without violating the stress constraint.

To control the local stress state, the stress is treated as a local constraint, for every finite element, as Duysinix and Bendsoe did. Due to the significant increase of the optimization problem induced by this approach, Duysinix and Sigmund proposed to include both the ε – relaxation technique that alleviates the singularity phenomena and the use of effective stress criterion into the global stress constraint. There are two global measures of the relaxed distributed stress criterion (5).

The first global measure is the "p-norm" of the relaxed stress criterion

$$\left[\sum_{e=1}^{N}\left(\max\left\{0,\frac{\sigma^*_{VM,e}}{\rho_e^\eta \sigma_1}+\varepsilon-\frac{\varepsilon}{\rho_e}\right\}\right)^p\right]^{\frac{1}{p}} \le 1. \quad (6)$$

The second global stress constraint is the "p- mean" of the relaxed stress criterion

$$\left[\frac{1}{N}\sum_{e=1}^{N}\left(\max\left\{0,\frac{\sigma^*_{VM,e}}{\rho_e^\eta \sigma_1}+\varepsilon-\frac{\varepsilon}{\rho_e}\right\}\right)^p\right]^{\frac{1}{p}} \le 1. \quad (7)$$

For a given p, the maximum stress value is always bounded from above by the p-norm and from below by the p-mean. Negative values of the relaxed criterion only appear for low stressed elements which can be truncated without influencing the global constraint, which remains continuous up to the $p-1$ derivative – smooth enough. From the numerical experiments of Duysinix and Sigmund, the choice of parameter p must result from a compromise of the high values for the control the maximum value of the stress criterion and the p-norm and p-mean ill-conditioning when p increases. Good results were obtained with $p = 4$ [4].

5 Conclusions

The use of integrated equivalent stress constraints proposed by Duysinix and Sigmund proves an alternative to the use of local stress constraints for continuum-type structures. A theoretical study of the two integrated constraints shows that they bound the maximum value of the criterion, which is the limit value of the two p-norms and p-means as p grows to infinity. The p-mean function converges by lower value towards the infinite mean while the p-norm provides upper bounds to the maximum stress criterion. There is no relation identified yet between the p-norms and p-means to the maximum value and the number of elements considered in the constraint. With only one integrated constraint function is difficult to control the large number of elements which are close to the admissible stress. The convergence process becomes oscillatory and the constraint violation increases. For practical applications the parameter p was taken to 4 in order to consider a large influence of all active local stress criteria in the global constraints.

The producer guaranty about the reliability indicators of the cutting tools raises the customers' reliance and offers an advantage against the concurrence. Therefore, the setting of a unitary method for reliability indicators estimation is a very actual and timely attempt.

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