Reliability prediction of tensile strength of a glass fiber reinforced polyamide using response surface method

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Abstract: - The present paper consists firstly in developing mathematical models to predict tensile strength of a glass fiber reinforced polyamide. Based on response surface method (RSM) a mathematical model has been determined, in which three factors with three levels are implemented. Glass fiber content, temperature and strain rate are chosen as the main input parameters in this study. The tensile strength is considered as output response which is evaluated through experimental tests. Secondly the reliability of tensile strength is proposed based on the developed mathematical models, where the dispersions of: (i) the Glass fiber content and (ii) the temperature are taken into account using the Strength Load method with the Monte Carlo simulation. The proposed approach can be used as a powerful and an interesting method for engineering design, to determine with more secure the tensile strength behavior of a glass fiber reinforced polyamide.

Key-Words: - Reliability approach, Response surface methodology, Thermoplastic composites, Monte-Carlo simulation.

1 Introduction

Glass fibre reinforced polyamides are widely used in many industrial applications, such as: stressed functional automotive parts (fuel injection rails, steering column switches) and safety parts (sports and leisure). It received increasing attention in these applications due to their stiffness, toughness, light weight and their resistance to tensile loading. Injection moulding process is considered as the most conventional methods used for processing fibre reinforced thermoplastics compounds. It improves the mechanical properties over the unreinforced ones [1]. However, voids are always present in these injected materials in which cracks can be initiated and propagated in one of three regions: the matrix, the fibre or the fibre/matrix interface [2]. In this context, several works have dealt with the mechanical properties of thermoplastic composites containing short fibres. These properties result from a combination of the fibre, the matrix properties and the ability to transfer stresses across the fibre/matrix interface, but it also depends on many variables such as fibre ratio, diameter, length, orientation and the strain rate which are of prime importance to the final properties of the thermoplastic composites [3,4]. Behaviour evolution of composite materials is phenomena affected by high uncertainties where the deterministic approach fail to estimate exactly the damage fracture. In this area, reliability approaches become more and more considered as an engineering design in industrial application [5, 6]. However, few studies have dealt with the case of reliability approach through composite materials [7, 8]. The objectives of the present paper consist in:

(i) Developing an analytical model for correlating the interactive and higher-order influence of various parameters such as the glass fibre content, temperature and strain rate on tensile strength of thermoplastic composites using response surface methodology (RSM).

(ii) The RSM technique coupled with experimental tests is used to investigate the effect between the different factors on the tensile strength of a glass fibre reinforced polyamide.

(iii) Developing a probabilistic approach for evaluating the tensile strength reliability of a glass fibre reinforced polyamide by taking into account the dispersions of : (*) the Glass fibre content and (**) the temperature. The 'Strength-Load' method coupled with the Monte Carlo simulation is implemented for computing the tensile strength reliability.

2 Reliability Assessment Method:

The failure probability and the reliability are used to quantify risks that the behaviours of the fracture exceed a given criterion. The Strength-Load approach is considered to be one of the most reliably computational methods to evaluate the risk [9, 10].

To compute the reliability, one considers a vector of random variables {X} representing uncertain structural quantities. Let xi be an element of the random vector {X}, with a probability density function $f_{Xi}(xi)$. A performance function G({X}), separating the security and the failure fields is written as follows:

$$G(\{X\}) = S(\{X\}) - L(\{X\}) \quad (1)$$

Where G({X}) = 0 is the limit state function, S({X}) is the strength function and L({X}) is the load function [9]. In that case, if the inequality G({X}) > 0 is satisfied, this indicates a structural safety condition. In the opposite case, if G({X}) < 0, this means a failure of such a structure **(**Figure.1).

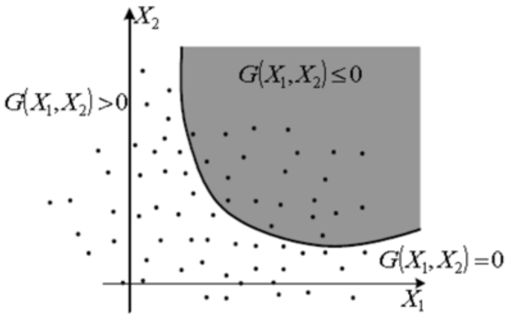

Figure 1: Probability density function of G({X}) [10].

The failure probability P_f (G({x})<0) is given by:

$$P_f=\int_{G\le 0} fx(x)dx \quad (2)$$

With $f_{Xi}(xi)$ is the joint density function (JDF) of G({x}).

To compute the failure probability Pf is very difficult with analytical method this is due to the difficulty to know $f_{Xi}(xi)$, and then we can use the Monte Carlo Simulation: which is widely used in the case of high number of random variables and also when $\boldsymbol{f_{Xi}}(\mathbf{xi})$ is practically difficult to find.

Using several random sampling, the Monte Carlo method aims to simulate a high number of load and strength values according to their JDF. For a total number of simulation N, all events are represented by the computed values of G({x}). It is well established that the failure event frequency, defined by G({x}) < 0, extends towards the failure probability Pf when N →+∞ [11]. Generally N is taken equal to 10^4, it is an acceptable computational cost especially in the case of explicit function [11].

The failure probability Pf is then expressed as follows:

$$Pf = \lim_{N\to+\infty} \frac{\text{Number of failure events } (G(\{x\})< 0)}{N} \quad (3)$$

Finally, the reliability R is given by the following relationship:

$$R = 1 - Pf \quad (4)$$

In this paper, we used the Monte Carlo Simulation to compute the reliability but an explicit relationship of the limit state function G({x}) is needed, for this reason a response surface methodology is performed to determinate the limit state function G({x}).

3 Response surface methodology

The main idea of the surface response method is to construct a polynomial approximation of the limit state function based on the results obtained by the design of experiments (DoE) method where a full factorial design plan is used [12]. It is a sequential experimentation strategy for building and optimizing the empirical model. Consequently, RSM is a collection of statistical and mathematical procedures that are useful for the analysis of problems, in which the objective is to optimize this response [13, 14]. In the RSM, the quantitative form of relationship between desired response and independent input variables could be written as:

$$Y = f(X1, X2, X3 \ldots\ldots\ldots Xn) \pm \varepsilon \quad (5)$$

X1, X2 ...Xn are the independent input variables where the response Y depends on them and the experimental error term, denoted as ε. The appearance of the response function is a surface as plotting the expected response of G({x}). This approximation of G({x}) will be proposed using the second order polynomial regression model, which called the quadratic model.

In the case of the quadratic response surface with three random variables, the response is represented as follows:

$$G(\{x\}) = a_0 + \sum_{i=1}^{n} a_i X_i + \sum_{i=1}^{n} a_{ii} X_i^2 + \sum_{i<1}^{n} a_{ij} X_i X_j \qquad (6)$$

Where a_i represents the linear effect of X_i, a_{ii} represents the quadratic effect of X_i^2 and aij reveals the linear-by-linear interaction between X_i and X_j. These coefficients of the mathematical model are determined by the regression tool. Then response surface G({x}) contains the cross product terms, linear terms and squared terms. Using this quadratic mathematical model of G({x}) in this study is not only to locate regions of the desired target, but also to investigate over the entire factor space. The necessary data for building the response models are collected by the design of experiments.

4 Experimental work

4.1 Design

In this study, experiments were designed based on the experimental work given by Mouhmid et al. [15, 16]. The main objective of the factorial experiments consists in studying the relationship between the response as a dependent variable and the different parameter levels. This approach helps to understand better how the change in the levels of each parameter can affect the response. It has been reported [16] that the factorial experiments provide an opportunity to study not only the individual effects of each factor but also their interactions. The design required 27 experiments with 3 levels. The design was generated and analyzed using MINITAB 16.0 statistical package. Glass fibre content, temperature and strain rate are chosen as the main input parameters in this study.

4.2 Experimental procedure

The material used in the present study was short glass fibre reinforced polyamide P66. The geometry of the test specimen is shown in Figure 2, in which tensile tests were carried out with an Instron machine (10 kN) equipped with a temperature controlled chamber.

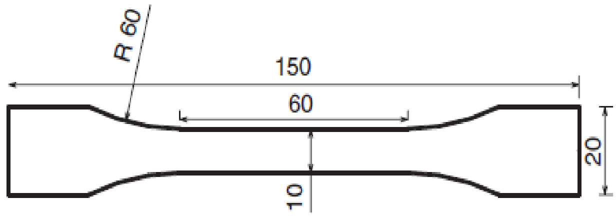

Figure 2: Tensile specimen dimensions (mm).

Tests were carried out at different parameters such as: glass fibre content, temperature and strain rate. The glass fibre content is selected to be between 0 and 30 %, strain rate ranges from 1 to 49 mm/min and temperature is considered to be between 20 and 80° C. The experiments have been carried out according to the designed experimentation as illustrated in Table 1.

N° essai	$\dot{\varepsilon}$	T	Vf	Tensile strength
1	1	20	0	65
2	1	20	15	78
3	1	20	30	85
4	1	50	0	57
5	1	50	15	75
6	1	50	30	82
7	1	80	0	48
8	1	80	15	65
9	1	80	30	74
10	25	20	0	70
11	25	20	15	80
12	25	20	30	90
13	25	50	0	59
14	25	50	15	78
15	25	50	30	88
16	25	80	0	49
17	25	80	15	68
18	25	80	30	78
19	55	20	0	73
20	50	20	15	83
21	50	20	30	95

22	50	50	0	60
23	50	50	15	79
24	50	50	30	90
25	50	80	0	49
26	50	80	15	69
27	50	80	30	80

Table 1: Experimental results.

3 Results and discussion

3.1 Development a mathematic model

In this section, the influence of both intrinsic parameters (glass fibre content) and extrinsic parameters (strain rate and temperature) on the tensile strength has been investigated.

The response surface method coupled with the experimental results is implemented for predicting the mathematical relationships that link tensile strength as a function of the glass fibre content, temperature and strain rate.

The analytical expression, obtained from analyzing the influences of the various dominant parameters on the tensile strength is given by:

Tensile strength $= 66.82 + 0.2155\dot{\varepsilon} - 0.0523\,T + 0.965 v_f - 0.0009\,\dot{\varepsilon}*\dot{\varepsilon} - 0.0024\,T*T - 0.0133\, v_f*v_f - 0.0015\,\dot{\varepsilon}*T + 0.0018\,\dot{\varepsilon}*v_f + 0.0048\,T*v_f$. (7)

The analysis of variance (ANOVA) has been performed to justify the goodness of fit of the mathematical models. The confidence level of 99.8% for tensile strength is in reasonable agreement with the adjusted measures to noise ratio. In order to validate and check the accuracy of the constructed RS model, 27 verification points are generated. Figure 3 shows the verification point's results obtained using Experiment and RS models.

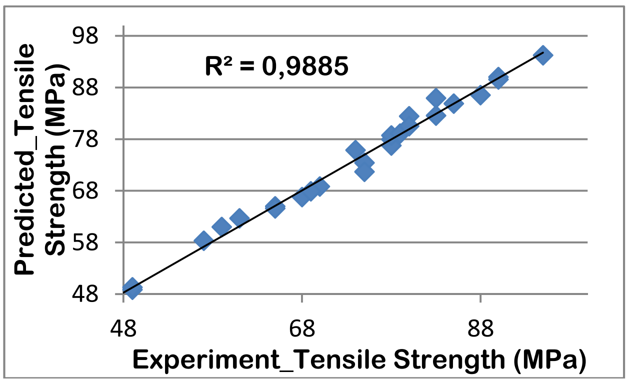

Figure 3: Goodness of fit of RS model.

The results of the RS models are in a good correlation with experimental results and provide accurate and satisfactory results. All the above consideration indicates an excellent adequacy of the developed empirical relationships. Additional, the Pareto chart indicates main and interaction effects considered statistically significant by ANOVA (Fig. 4).

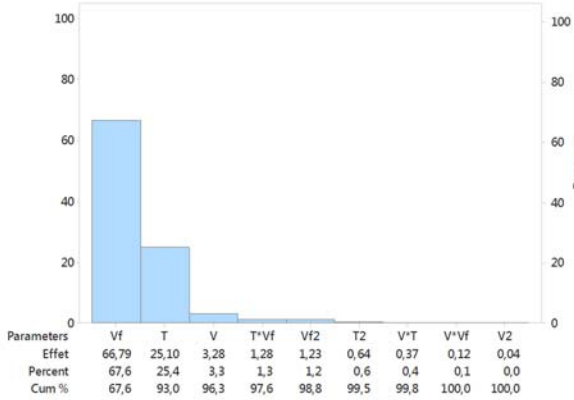

Figure 4: Pareto chart of standardized effects on the tensile strength.

As apparent, the glass fibre content has the strongest effect of 66% on the tensile strength. The temperature contributes 25% on the tensile strength. From this analysis, it is confirmed that the effect of the strain rate was trivial, with a percentage contribution equals to 3.5% on the tensile strength. This observation is in good agreement with the previous results [17], in which they have found that the effect of strain on the strength tensile was negligible on the glass fibre reinforced polyamide composites.

3.2 Effect and sensitivity analysis of input factors

The effect of strain rate, temperature and glass fibre content, on the tensile strength is illustrated in Figure 5.

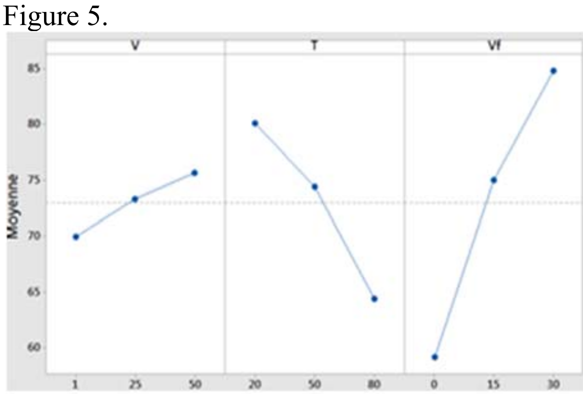

Figure 5: Main effect plot of V($\dot{\varepsilon}$), T and v_f on the value of the tensile strength.

It is observed that the glass fibre content has the highest significant effect on both the tensile strength. Moreover, the temperature was found as a significant parameter in which an increase in the temperature value leading to decrease the tensile strength. However, an increasing of the strain rate value leading to increase in the tensile strength. Comparing with temperature and glass fibre content, the effect of the strain ratio is trivial. In conclusion, the optimal combination of different parameters to achieve high tensile strength is A3B1C3.

3.2 Reliability application

Fibre content and temperature are the main parameters that should be in better control of the production process in order to reduce the failure probability for tensile strength.
The experimental results are coupled with the reliability-approach using the Response Surface Method. The basic idea of this method is to approximate the system response by an explicit function and to determine their distribution as a function of the required variables. The dispersion parameter is characterized by their coefficient of variation which is calculated as the ratio between the mean values and the standard deviation. MATLAB software is used to determine the distribution of tensile strength as a function of both temperature and fibre content. Then, the Monte Carlo simulation is implemented for evaluating the failure probability using Equations (1), (2), (3) and (4).
Figures 6-7 illustrate the loading dispersion zones: case 1: tensile strength as a function of fibre content in which temperature is assumed to be normally distributed (Cov=5%), case 2: tensile strength as a function of temperature in which fibre content is assumed to be normally distributed (Cov=5%).

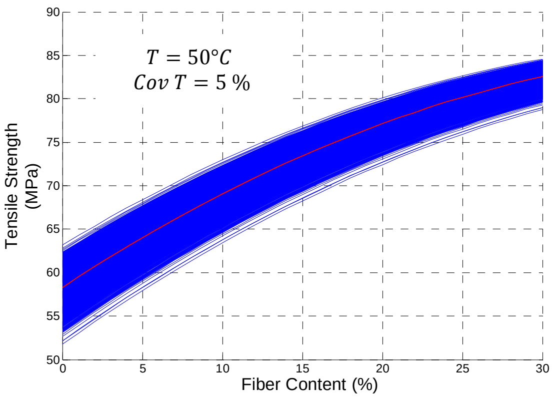

Figure 6: Evolution of σr versus fiber content v_f (Random case) with Cov=5%.

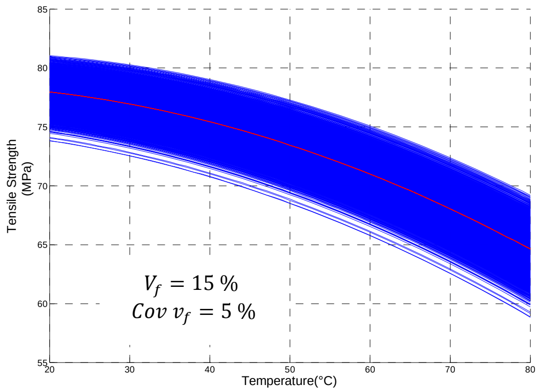

Figure 7: Evolution of σr versus temperature (Random case) with Cov=5%.

3.3 reliability results

(i) From Figures 8-9, we can deduce the significant effect of the two factors T and v_f on the reliability index which is calculated using the Monte Carlo Simulation.

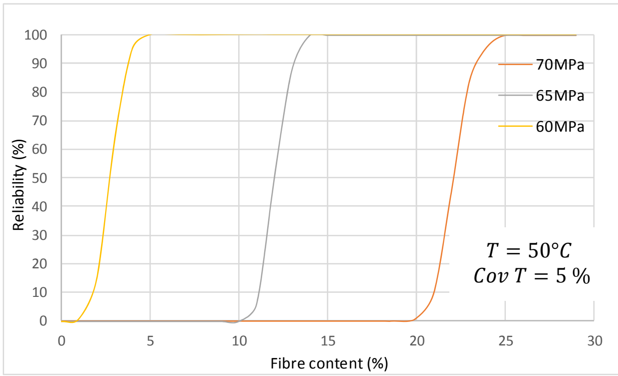

Figure 8: Reliability versus Glass fibre content with different critical tensile strength.

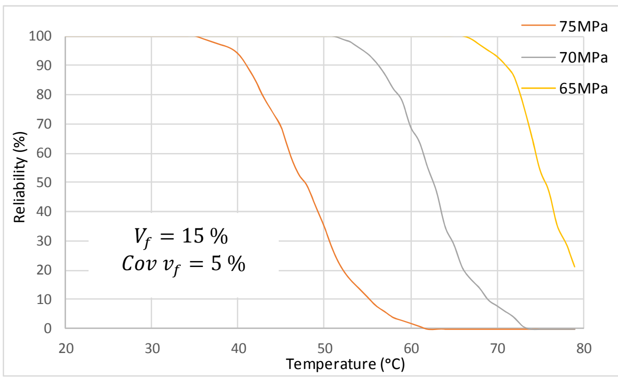

Figure 9: Reliability versus Temperature with different critical tensile strength.

It is interesting to note that the reliability decreases when the critical tensile strength increases. Then, for a low glass fiber content value, it is noted that the reliability is equal to zero. As against ,when this value is increased, the reliability tends to 1.As a consequence, we can conclude that the ratio of fiber has a very important effect on tensile strength such that if we add the fiber the reliability increases and is always in the field of safety. For example, if you need a tensile strength superior of 80 MPa, you must have a 30 % minimum of glass fiber content for keeping in safe zone. For a low value of temperature, it is noted that the reliability is equal to 1 against if the temperature exceeds 50 °C (the glass transition temperature Tg of the polyamide 66 practically equal to 50 °C) the reliability decreases and tends to 0. Therefore, it can be concluded that temperature of loading for a glass fiber reinforced polyamide has a very significant effect on the tensile strength.

(ii) The developed probabilistic approach takes into account the dispersion of (i) the fiber content and the temperature in which the tensile strength is considered as output parameters. In order to study the effect of different input parameters on the tensile strength of glass reinforced polyamide, the distributions of the tensile strength are evaluated for different Cov values (Figure 10).

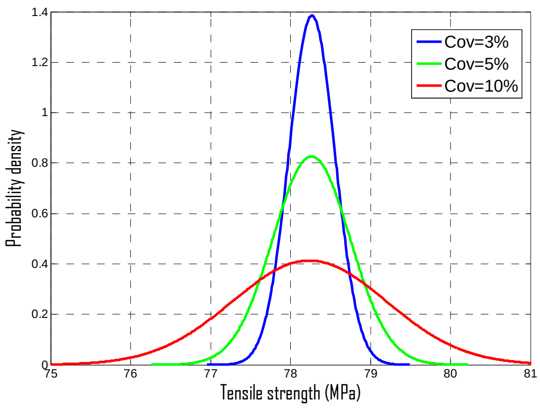

Figure 10: Tensile strength distribution.

It is observed that the probability density values rises by the decrease of the input parameter distribution. As a result, we found a dispersion value of tensile strength equal to 0.3%, 0.6% and 1.3% respectively in 3%, 5% and 10% dispersion of input parameters.

(iii) We assume that all the probabilistic variables follow a normal distribution characterized by means and Coefficient of Variation (CoV) values. Two parameters are considered as probabilistic design parameters with different (CoV) values:

Figures 11-12 presents the computed reliability curves plotted for the case of a CoV of 3%, the case of a CoV of 5% and the case of 10% of fiber content and temperature. It shows the dispersion of the 10^4 design points for the three cases of study.

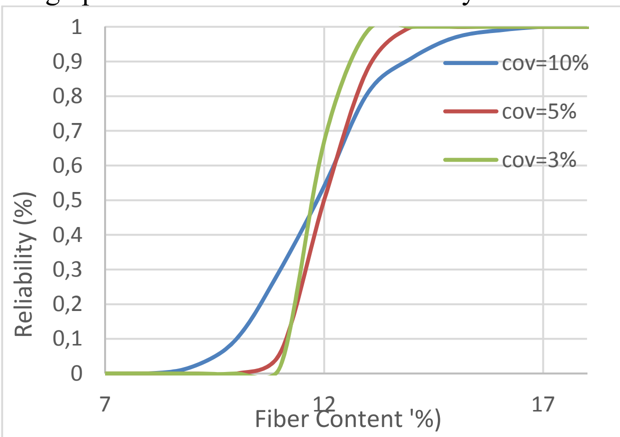

Figure 11: Reliability versus fiber content with different Cov.

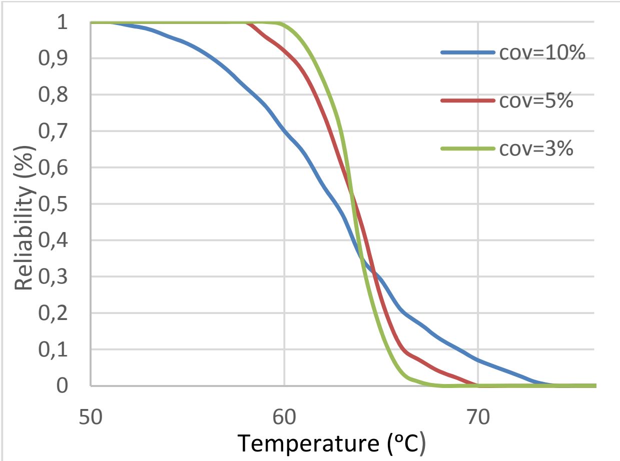

Figure 12: Reliability versus Temperature with different Cov.

For v_f=13%, the reliability R is approximated equal to 100% in the first case of study. For the second and third cases, the reliability decreases respectively to 92.3% and 70.7 %. Then for T=60°C, a reliability of 99.8% is estimates for the first case of study. For the second and third cases, the reliability decreases respectively to 97.1% and 86.7 %. We observe a change of the range between the extreme values of the glass fiber content, corresponding to 0% and 100% reliabilities. It is observed that the scatter of the glass fiber content increases when the Cov value of the input temperature increases. This result is consistent with the physical observations.

4 Conclusion

In this paper, the Response Surface Method (RSM) is used to calculate the performance function

G({X}) and their corresponding design points based on experimental results. The basic idea of this method is to approximate the system response by an explicit function of random variables. An empirical relationship was developed based on the RSM approach for correlating the tensile strength with predominant process parameters. A probabilistic approach based on the developed mathematical models and Monte Carlo simulation for predicting the tensile strength reliability of a glass fibre reinforced polyamide is proposed.

The following conclusions can be drawn:

Based on the experimental observations and response surface methodology, we can investigate the tensile strength of our material. This result can be used also, to characterize qualitatively the effect of the different temperature and fiber content modifications.

A maximum tensile strength of 95 MPa was obtained under the condition of 20̊ C temperature, 49 mm/min strain rate and 30% glass fiber content reinforced PA66 and that confirmed by RSM method. Based on the reliability prediction of tensile strength, the glass fiber content must be superior of 15% and the temperature must be inferior of 50̊ C for keeping in safety condition.

A probabilistic approach has been developed to evaluate the tensile strength reliability as a function of the temperature and fiber content. This approach leads to improve the deterministic models by taking into account the various dispersions of: (i) temperature and (ii) fiber content which are very significant and rarely considered. This observation clearly shows the capability of the probabilistic model to correctly take into account the statistical distribution of the input parameters in composite materials.

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