Stabilized Column Generation for the Crew Pairing Problem with Time Windows

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Abstract:- Column generation has proven to be efficient in solving the linear programming relaxation of large-scale instances of the crew pairing problem with time windows. However, difficulties arise when the instances are highly degenerate. Recent research has been devoted to accelerate column generation while remaining within the linear programming framework. This paper presents an efficient approach to solve the linear relaxation of the crew pairing problem with time windows. It combines column generation, preprocessing variable fixing, and stabilization. The outcome shows the great potential of such an approach for degenerate instances.

Keywords:- column generation, crew pairing problem, linear programming.

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1. Introduction

In the airline industry, optimization and automation of the crew pairing is a major financial and organizational challenge. The problem is to cover cost of all flights of the company, programmed over a given time, with crews trained staff cockpit (pilot, co-pilots) and flight attendants (stewardesses, stewards) . At intervals of several days (of the order of the week), each crew from the base to which he is assigned, connects a number of flights and returned to base. This sequence of flights back to the base is called rotation. The crew pairing of an airline is extremely restricted by international regulations, national and domestic labor, and the limited availability of resources.

These constraints make the problem particularly difficult to solve. The use of models and optimization software for this problem enables large companies to make substantial financial gains. It is not uncommon that a reduction of one percent on the total cost of rotations translates into tens of millions of dollars of savings for large companies [1], where research, basic and applied abundant on the subject. The crew pairing problem with time windows $(CPPTW)$ can be formulated as a feasible flow problem minimum cost in a multiple network, with additional variables and time windows.

Finally, note that resource constraints make the problem $(CPPTW)$ *NP-hard*. This places them beyond the resolution capabilities of even the most specialized software available today. To be able to treat them, methods of decomposition of the space of solutions are used. Decomposition often causes problems with an inordinate number of variables, hence the growing interest in the so-called column generation method. Like iterative methods, column generation can suffer from a convergence problem. Several methods to improve the convergence of column generation have been proposed in the literature, the best known and most used are the stabilization methods [14, 15] which operate on the overall process with the objective of reducing the number of iterations by reducing the oscillations of the values of the dual variables.

2. Presentation of the Problem

The $(CPPTW)$ is an important optimization problem that is part of the airline crew scheduling procedure and can be modeled, if the cost function is linear, the Linear Programming in mixed variables. We have a feasible flow problem minimum cost on all subnets with varying binary variables and continuous flow of resources:

$$\min \sum_{r \in R} c_r \; x_r \tag{1}$$

$$\text{st}: \sum_{r \in R} x_r = 1 \quad \text{for } i \in N = \{1, \ldots.., n\} \tag{2}$$

$$x_r \in \{0,1\} \quad \text{for} \quad r \in R \tag{3}$$

Or R designate all eligible rotations satisfying the resource constraints and sequence between flights, c_r represents the *cost* of the rotation $r \in R$, and the binary variable x_r indicates binary choice whether or not the pairing r in the solution.

2.1 Algorithm

This iterative process of solving the *master problem* and the *sub-problem* is stopped when all tours are positive reduced cost in solving the problem by a sign that the continuous optimum is reached.

A variant of this method to accelerate the process in practice, is to add at each iteration a subset of complementary routes of negative reduced *cost* instead of the single best route of the *sub-problem* . The desired maximum size of this subset of columns may be set to inbound to evolve during the algorithm. The overall complexity of the method is highly dependent on the complexity of the *sub-problem* that resource constraints make it NP-*hard*. However, it is often possible to solve in a reasonable time by an implicit enumeration of R by exploiting the graph structure of the *sub-problem* and using variants of shortest path algorithms.

The main steps of our approach are summarized below:

$Master\ problem\ Sub\ problem$ - Projection $vertex$.
- DPA-L.
- DPA-LND.

Generated the solutions.

3. Numerical Results

We implemented our algorithm using Java programming language. For the simulation, we used a CPU Intel Core i9-9900KF (8 cores), 3.60GHz, RAM 32 GB, running under Windows 10 (64 bit). Linear programs for restricted master problems are solved with ILOG CPLEX 20.1. The results for the instances Solomon's with reduced time windows are shown in $Table1$, we reported the iteration number (N_i), the lower bound (L_i^b) and the upper bound (U_i) of the objective, the computational time in seconds (T_i), the number of generated columns (C_i), where $i = 1$ for classic method and $i = 2$ for our method. To obtain the upper bound, we used the branch-and-bound method. Nevertheless, comparison of the two algorithms is achieved using their computed lower bounds.

The comparison between the different methods and our approach has revealed that it has provided good results.

4. Conclusion

In this section devoted to solving the crew pairing problem with time windows, we have mainly developed approaches to column generation and decomposition master problem and sub-problem. We separated the Crew Pairing Problem in two phases. The difficulty of solving sub-problem is directly related to the number of resources, we particularly studied the techniques of reduction of space resources, and the concept of reduction is a key element of the effectiveness of the overall resolution of issue. Indeed, if in a strategic planning perspective the computation time may be less critical than the overall cost of rotations, however in an operational setting the gain on the time resolution of sub-problem becomes a major issue. The prospects of research on this problem are numerous. These re-optimization problem of growing interest among engineers in charge of planning in the large transport companies and open up avenues of research particularly interesting and promising.

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Table 1: Comparison of two approaches for solving the CPPTW for Solomon's instances with 100 customers.

	classic method				our method				Comparison	
Instance	L_1^b	T_1	N_1	C_1	L_2^b	T_2	N_2	C_2	Gap	T_1/T_2
c201	589,1	44,9491	418	28845	589,1	11,6671	528	6526	0,00	3,85
c202	589,1	179,6573	552	44694	589,1	61,4668	722	12701	0,00	2,92
c203	585,767	1223,024	995	75865	585,767	446,2332	1488	32852	0,00	2,74
c204	582,383	4893,6566	1630	136556	582,226	990,1879	2056	58070	0,03	4,94
c205	582,369	151,9166	472	42390	582,363	44,6547	617	15218	0,00	3,40
c206	575,993	346,5503	614	56564	575,845	61,8254	652	16738	0,03	5,61
c207	570,524	367,8769	544	52839	570,52	74,188	691	18018	0,00	4,96
c208	570,255	423,4152	635	59277	570,278	65,6802	614	17867	0,00	6,45
r201	1080,749	9,8496	119	9102	1080,771	6,4248	231	3947	0,00	1,53
r202	933,446	93,7052	241	15273	933,458	23,7589	353	6624	0,00	3,94
r203	756,739	451,4435	402	28667	756,731	87,1933	565	11958	0,00	5,18
r204	640,238	7638,2746	638	56850	640,272	559,2558	863	22660	-0,01	13,66
r205	838,773	76,568	242	19494	838,772	29,6386	372	8314	0,00	2,58
r206	749,068	474,0996	370	28002	749,066	84,8317	538	12161	0,00	5,59
r207	668,711	3609,1966	568	50978	668,711	281,5144	703	19200	0,00	12,82
r208					610,278	1197,6882	1001	31544		0,00
r209	750,455	445,0778	292	28162	750,454	82,0649	484	11179	0,00	5,42
r210	753,985	205,3728	309	23599	753,993	63,3126	494	11025	0,00	3,24
r211	650,834	1793,735	573	47048	650,845	167,9773	741	17532	0,00	10,68
rc201	1107,012	13,2285	149	9682	1107,011	6,554	227	3656	0,00	2,02
rc202	880,343	127,7105	265	17952	880,329	29,5938	339	6574	0,00	4,32
rc203	693,53	902,8656	475	35439	693,1	146,6249	591	13829	0,06	6,16
rc204	607,663	14066,2658	815	78145	606,758	587,2997	854	25540	0,15	23,95
rc205	967,105	59,4491	230	15595	967,097	19,7616	304	5936	0,00	3,01
rc206	852,167	130,5895	305	24025	852,178	30,1325	348	7607	0,00	4,33
rc207	767,951	567,5535	417	29098	767,982	80,8772	456	10815	0,00	7,02
rc208	627,276	3223,8542	638	44904	627,155	223,5683	726	17074	0,02	14,42