Discrete approach in gradation surfaces computation

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Abstract: - Gradation surfaces as a new empirical color management system offer vast opportunities for practitioners in printing industry. This simple and obvious approach helps to improve color synthesis for a particular printing system. However, mathematical apparatus of differential geometry that is used in the approach may consider intricate. Moreover, experiments show difficulties in calculations of gradation surfaces. This require a simplification of the method. We offer a new discrete method of calculation for such an approach. The method show satisfactory accuracy and wide prospects of further applications. All geometric objects are considered in the CIE Lab space where the space's metric is determined by the magnitude of the color difference CIE dE

Keywords: - Color managmenet system, color prediction model, color managment, empirical surface color prediction.

1 Introduction

Color prediction models (CPMs) help in adjusting colors and improving color management in print. Today, there are many CPMs exist. The models predict the resulting color in print based on a set of inks as inputs of a particular reflectance or tristimulus equation. *Empirical* surface color prediction models take into account superposition of ink halftones and do not deal with the light propagation and fading within the print. The models demonstrate the relationship between reflected light and surface coverages by colorants. *Physically inspired* models engage a more detailed analysis of light-print interaction based on mathematical prediction of how light paths go within a halftone print and what resulting fade is. *Ink spreading* models characterize the effective surface of an ink dot after it has been printed at a given nominal surface coverage compared to the effective surface coverage that forms the physical dot gain. The models accounting for ink spreading in all ink superposition conditions rely on ink spreading curves mapping nominal surface coverages to effective surface coverages for the surface coverages of single ink halftones, as well as ones superposed with one and two solid inks. The further development of color prediction models deal with spread-based and light propagation and transportation probability. *Spectral reflection prediction* models study the impact of different factors influencing the range of printable colors (the inks, the substrate, the illumination conditions, and the halftones) and create printer characterization profiles for the purpose of color management [1]. These models together with ink-spreading models take into account physical dot gain and able to predict reflectance spectra as a function of ink surface coverage for 2–4 inks (binary and ternary color systems). The models uses multiple tone reproduction (ink spreading) curves (TRC) to characterize the physical dot gain of the ink halftones on the substrate and in all solid ink superposition conditions [2–4]. Different color prediction models have been successfully applied to color reproduction management in various contexts [5–9]. The major drawback of the mentioned models is the fact that all of them are computationally capacious, as n colorants require a solution of system of 2n equations; therefore, they cannot be implicated into real workflow.

Empirical approaches are more promising; however, they usually require a significant amount of print tests to do. Most practitioners prefer using relatively simple methods for setting up printing systems by analyzing gradation scales and applying gradation-based techniques that are known as an indispensable attribute of color printing systems settings [10, pp. 88–89]. At the same time, the authors expresses doubts about the rational use of these characteristics in digital printing technology. The main problem is the fact that using the gradation curves in conventional 2D embodiment significantly reduces the quantity and quality of information extracted from them. In the work [11], 3D gradation trajectories are introduced as a further development of the gradation curves approach. Implication the mathematical apparatus of differential geometry for gradation trajectories analysis in 3D CIE Lab space allows reveal their intrinsic features of curvature and torsion. These features are applied to define the ink limits in ink-jet printing systems and to create the empirical approach based on trajectories' curvature and torsion behavior analysis.

Gradation surfaces of two colors are introduced as a mater stretched on two adjacent gradation trajectories [12]. In the case of printing colorants (CMY), their paired double overlays (binaries) correspond to additive primary colors (RGB). Since RGB and CMYK spaces are both device-dependent models, there has been no simple or general conversion formula that converts between them, as concern grey balance at least. We might suggest the way to develop such a conversion based on the ideas of gradation trajectories and surfaces.

The gradation trajectory of double overlays is a surface constructed on the basis of gradation trajectories of two colorants. Take as an example the pair of *color1* (n) and *color2* (m). Each colorant is able to take a halftone value from 0 (pure substrate) to 1 (full dye). Thus,

$$\begin{gathered} n \in [0;1]\, for\ color1 \\ m \in [0;1]\, for\ color2 \end{gathered}$$

In the case of continuous halftones, we obtain a square region of allowable recipes on the plane (n, m). In real print, a frequent grid occurs instead of a solid square. Each reproducible tone has its own recipe (n, m) and a set of $L^*a^*b^*$-coordinates. A smooth change in tone causes a smooth change of $L^*a^*b^*$-coordinates. We can represent it in (2), which describe smooth surfaces in $L^*a^*b^*$-space.

$$a = a\,(n,m);\ b = b\,(n,m);\ L = L\,(n,m)$$

The equation (2) seems to be equal to equation of the surface in the Cartesian space [13]. Thus, a gradation surface is a locus of points in the $L^*a^*b^*$-space that satisfies the system of conditions (1) and (2). Weather the printing system is preliminary characterized, it might me assumed that gradation trajectories of a single color channel effectively described by polynomials of the degree not higher than third (3). For b and L coordinates, equations have the same type. Note a_0, b_0, L_0 are the coordinates of the substrate.

$$\begin{gathered} a_{color1} = a_0 + \sum_{i=1}^{3} a_{color1,i} \cdot n^i \\ a_{color2} = a_0 + \sum_{i=1}^{3} a_{color2,i} \cdot m^i \end{gathered}$$

We designate a binary gradation surface as "stretched" on the gradation trajectories of generatrix pairs of colorants so that it contain them inside. In the system (2) it will be reflected as special cases, as for instance, the gradation trajectory *color1* is contained in the gradation surface of *binar1* tones (4).

$$a = a\,(n,0);\ b = b\,(n,0);\ L = L\,(n,0)$$

Weather gradation surface is stretched on the gradation trajectories then, taking into account (3), we can assume the explicit form of (2) is (5). For b and L coordinates, equations have the same type.

$$a_{binar1} = a_0 + \sum_{i=1}^{3} \sum_{j=0}^{i} a_{i-j,j} \cdot n^{i-j} \cdot m^j$$

The required degree of the polynomial (3rd) was approved experimentally. The graph of the analytic surface (see Fig.1) is deviated from the experimental data by a distance smaller than the measurement error of the spectrophotometer.

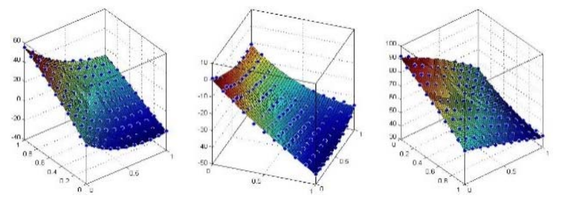

Fig. 1. Approximations of $L^*a^*b^*$-coordinates (2) by (5): left – a, central – b, right – L; dots are the measured values; colored surfaces are approximations

Further, we introduced a concept of geodesic lines. Characterization of a printing machine is made by a uniform distribution of points along the gradation curve. Therefore, it would be logical to assume that this principle should also be adhered to in the case of a double halftone surface. Since the figure of double superimposition in color space is not a curve, but a surface, the points should be located evenly within the surface – at an equal distance from each other, as if by a grid. It is precisely this goal that an element of differential geometry, such as geodesic lines, perfectly fit. A geodesic line is an analog of a straight line on a plane for a surface, i.e. straight line, which, by the shortest path, connects two points on the surface. The basic property of a geodesic line: on any sufficiently small piece of surface through two points the only one arc of the geodesic line can be drawn, just as on a plane through two points the only one straight line do.

The following is an algorithm for finding out a geodesic in general form. In our case, we consider a regular piece of the surface B (*binar1*) defined by vector equations (2). The first fundamental form of the surface B is the following (6):

$$\begin{gathered} dB^2 = E(n,m)dn^2 + 2F(n,m)dndm + G(n,m)dm^2 \\ E(n,m) = \left(\frac{da}{dn}\right)^2 + \left(\frac{db}{dn}\right)^2 + \left(\frac{dL}{dn}\right)^2 \\ F(n,m) = \frac{da}{dn}\frac{da}{dm} + \frac{db}{dn}\frac{db}{dm} + \frac{dL}{dn}\frac{dL}{dm} \\ G(n,m) = \left(\frac{da}{dm}\right)^2 + \left(\frac{db}{dm}\right)^2 + \left(\frac{dL}{dm}\right)^2 \end{gathered}$$

A regular piece of the surface B with the first fundamental form is a two-dimensional Riemannian space referred to the coordinates (n, m). If we consider a surface as a Riemannian space, then vectors, tensors, scalar products, and covariant differentiation can be defined on it [14]. The three-index Christoffel symbols have the following form for the surface B (7):

$$\Gamma_{11}^{1} = \left\{ {1 \atop 1\;\;1} \right\}_B, \quad \Gamma_{11}^{2} = \left\{ {2 \atop 1\;\;1} \right\}_B,$$
$$\Gamma_{12}^{1} = \left\{ {1 \atop 1\;\;2} \right\}_B = \left\{ {1 \atop 2\;\;1} \right\}_B, \quad \Gamma_{12}^{2} = \left\{ {2 \atop 1\;\;2} \right\}_B = \left\{ {2 \atop 2\;\;1} \right\}_B,$$
$$\Gamma_{22}^{1} = \left\{ {1 \atop 2\;\;2} \right\}_B, \quad \Gamma_{22}^{2} = \left\{ {2 \atop 2\;\;2} \right\}_B.$$

For any geodesic m = m (n), the corresponding function m(n) satisfies the differential equation (8).

$$\frac{d^2m}{dn^2} = \Gamma_{22B}^{1}\left(\frac{dm}{dn}\right)^3 + \left[2\Gamma_{12B}^{1} - \Gamma_{22B}^{2}\right]\left(\frac{dm}{dn}\right)^2 + \left[\Gamma_{11B}^{1} - 2\,\Gamma_{12B}^{2}\right]\frac{dm}{dn} - \Gamma_{11B}^{2}$$

The gradation trajectory of *binar1* is the curve in the *L*a*b**-space containing two points (n,m)=(0,0) as pure substrate and (n,m)=(1,1) as binary dye. Moreover, this curve must lie on the surface of *binar1* halftones B. Thus, the gradation trajectory of *binar1* is the geodesic satisfying equation (8) with boundary conditions m(0)=0, m(1)=1 (Fig.2). For instance, the gradation trajectory of Blue color obtained by this manner does not meet the traditional representation of equal recipes everywhere except full dye. Such an approach ensures the invariance of the hue in the entire range of gradations of Blue color.

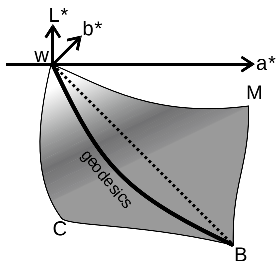

Fig. 2. The gradation surface of Blue (B) halftones stretched on gradation trajectories of cyan and magenta (CM); red curve is a geodesic; black line is a nominal line of Blue color by recipe C=M; W is a white point (substrate)

For convenience of the following exposition, we replace the notation. Instead of the variables m, n (1) we will use the corresponding letter to denote the relative proportion of the ink. Then, the obtained geodesic equations corresponding to the Red (R), Green (G) and Blue (B) colors, respectively, will take the form (9).

$$1. Y = \varphi_R(M).\ 2. Y = \varphi_G(C).\ 3. M = \varphi_B(C)$$

The following order of mutual dependence of variables is accepted: that variable which costs earlier in abbreviation CMY, will be considered independent. Therefore, the variable C is always independent, and Y is always dependent. M (magenta) is considered independent in the calculation of the Red geodesic, and acts as dependent when calculating the Blue one.

The proposed approach is elegant and easy-to-understand, however, it shows difficulties in practical implications. This work develops a discrete approach for gradation surfaces calculation as a simplification of the method. We offer an empirical method for such a calculation, as well as its validation.

2 Approach

The empirical approach we describe as follows. Preliminary linearization → Print specially developed test chart → Measure Lab coordinates of the chart with a spectrophotometer → Sorting data by color channels in order to extract R,G,B tone surfaces → Surfaces fitting by a polynomial of third degree (3)–(5) → Calculation of the geodesics from 0 to the full dye for R,G,B tones → Use geodesics obtained as arguments for (2) → Define equal lightness levels → Calculation of grey balance formulations by (10) → Print test chart to assess the result. All our models are built with help of MatLab package.

3 Experimental

For the experiment, we use the 4-color (CMYK) printer Konica-Minolta C6000L. Print mode: 600×600 dpi. Substrate: coated paper FancyEmboss 110 g/m². The measurement tools: spectrophotometer x-Rite iOne iSis + x-Rite ProfileMaker package.

A halftone scale contains 14800 fields. The scale represents the multidimensional grid of test values. The total number of patches is 11 (0, 0.1, 0.2…1) values per channel to the power of the number of colorants (four). They are synthesized using Argyll CMS and a TestChartGenerator in ProfileMaker for the automatic iOne iSis spectrofotometer. Such a frequent grid allow to distinguish all possible surfaces within device' color space. We utilized surfaces of binaries only, i.e. it formed 121 patch per each surface. Thus, 330 patches were actually used. Matrix variables, which contained n, m, L, a, b data of each color patch as columns, are imported in MatLab, where they are carried out in further mathematical processing.

The proposed method of gradation surfaces is based on the interpolation of experimental data by polynomials (5). The approximation of dependences (2) by polynomials (5) is implemented in MatLab package using the fit function. Final evaluation is done by preparation of a new arbitrary scale that is further printed out and measured (see Figure 3b).

4 Results and discussion

Results of geodesics calculations by (9) are shown in Table 1. Evidently, gradations of Red, Green, Blue tones do not match equal recipes of process inks. Figure 3a shows the dependencies of lightness (CIE L) on the part of tone for independent variables: Magenta for Red, Cyan for Green, Cyan for Blue. Trends in the Figure show the linear dependency that confirm the printing system linearity. Moreover, such linearity is convenient for definition of equal lightness levels.

The maximum possible value of the neutral color recipe (without the involvement of K-black) is determined by the component that has the highest brightness of the full tone (red). The errors in terms of deviations from the linear trends we associate with the rounding error, because it is impossible to put the formulation into the source code with a precision of better than a few percent. The developed and printed test chart for grey balance evaluation is shown in Figure 3b.

Figure 4 shows the lines of red, green and blue tones in the CIE Lab space and a line of synthesized neutral colors.

Analysis of the nature of the deviation of chromaticity from the neutral tone indicates the accumulation of a systematic error. This is probably due to inadequate accuracy of the condition of equal lightness when the tone of red, green and blue increases. Nevertheless, as can be seen from Figure 5, the line of neutral colors is close to the vertical.

Table 1. Calculated geodesics.

Red		Green		Blue	
Magenta	Yellow	Cyan	Yellow	Cyan	Magenta
0.00	0.00	0.00	0.00	0.00	0.00
0.05	0.06	0.05	0.04	0.05	0.05
0.10	0.11	0.10	0.07	0.10	0.11
0.15	0.17	0.15	0.11	0.15	0.16
0.20	0.22	0.20	0.15	0.20	0.22
0.25	0.28	0.25	0.19	0.25	0.28
0.30	0.33	0.30	0.23	0.30	0.34
0.35	0.38	0.35	0.27	0.35	0.40
0.40	0.43	0.40	0.31	0.40	0.46
0.45	0.48	0.45	0.36	0.45	0.53
0.50	0.53	0.50	0.41	0.50	0.59
0.55	0.58	0.55	0.46	0.55	0.65
0.60	0.63	0.60	0.51	0.60	0.71
0.65	0.68	0.65	0.57	0.65	0.77
0.70	0.72	0.70	0.62	0.70	0.82
0.75	0.77	0.75	0.68	0.75	0.87
0.80	0.82	0.80	0.75	0.80	0.91
0.85	0.86	0.85	0.81	0.85	0.94
0.90	0.91	0.90	0.87	0.90	0.96
0.95	0.95	0.95	0.93	0.95	0.99
1.00	1.00	1.00	1.00	1.00	1.00

For convenience, we show the projections of points of neutral tone on the chromaticity plane, which are grouped near zero, with the exception of the latter. A visual comparison of synthesized neutral fields with blacks matched in brightness (K) has shown an interesting phenomenon: it is impossible to determine which field is composite, and what is black, in spite of some difference in the shades.

5 Conclusion

A new empirical method for gradation surfaces computation in 3D CIE $L^*a^*b^*$-space has been proposed in this work. Gradation trajectories in terms of gradation surfaces, as well as the method of their analytical description, are described in discrete manner.

Gradation trajectories are the global features of a printing process that are depended on type of a substrate and properties of the ink only. They are not affected by rasterizing method, number of passes and measuring technique.

Gradation surfaces evaluation with the help of gradation trajectories of inks binaries (R, G, B) instead of inks themselves (C,M,Y) might be utilized as a powerful and fast-acting tool for ink-jet systems characterization.

The further development of the approach implies introduction of 3D gradation surfaces as a method describing interconnection between two or even three colorants, especially in the case of regular and light inks pairs and not only for ink-jet but also for all kinds of print.

References

1. Bala, R.: Device characterization. Digital Color Imaging Handbook. G. Sharma (ed.). CRC Press, Boca Raton, FL, 269–379 (2003)
2. Balasubramanian, R.: Optimization of the spectral Neugebauer model for printer characterization. J Elec.Imag. 8, 156–166 (1999)
3. Hersch, R.D., Crété, F.: Improving the Yule–Nielsen modified spectral Neugebauer model by dot surface coverages depending on the ink superposition conditions. Proc. SPIE 5667, 434–445 (2005)
4. Wyble, D.R., Berns, R.S.: A critical review of spectral models applied to binary color printing. Col.Res.&App. 25, 4–19 (2000)
5. Garg, N.P., Singla, A.K., Hersch R.D.: Calibrating the Yule–Nielsen Modified Spectral Neugebauer Model with Ink Spreading Curves Derived from Digitized RGB Calibration Patch Images. J.Imag.Sci.Tech. 52(4), 040908–040908-5 (2008)
6. Arney, J.S., Engeldrum, P.G., Zeng H.: An expanded Murray-Davis model of tone reproduction in halftone imaging. J.Imag.Sci.Tech. 39, 502–508 (1995)
7. Livens, S.: Optimisation of Printer Calibration in the Case of Multi Density Inks. Conference on Color in Graphics, Imaging, and Vision, CGIV 2002. Final Program and Proceedings, 633–638 (2002)
8. Chagas, L., Blayo, A., Giraud P.: Color Profile: methodology and influence on the performance of ink-jet color reproduction. IS&T's NIP20. 2004 International Conference on Digital Printing Technologies, 655–659 (2004)
9. Wu, Y-J.: Reducing Ink-jet Ink Consumption with RIP software for POP Display Media. Digital Fabrication and Digital Printing. NIP30 Technical Program and Proceedings, 108–111 (2014)
10. Kipphan, H.: Handbook of Print Media. Springer-Verlag Berlin-Heidelberg. 1207p. (2001)
11. Milder, O.B., Tarasov, D.A., Titova, M.Yu.: Inkjet Printers Linearization Using 3D Gradation Curves. CEUR Workshop Proceedings. 1814. Proceedings of the 1st International Workshop on Radio Electronics & Information Technologies (REIT 2017), 74–83 (2017)

12. Milder O., Tarasov D. Gradation Surfaces as a Method for Multi-color Ink-Jet Printers Color Specifications Management. In: Paul M., Hitoshi C., Huang Q. (eds) Image and Video Technology. PSIVT 2017. Lecture Notes in Computer Science, vol 10749 pp 53-61 (2018)
13. Pogorelov, A.V.: Differential geometry. Noordhoff, 171pp. (Transl. from Russian) (1959)
14. Korn, G.A., Korn, T.M.: Mathematical Handbook for Scientists and Engineers: Definitions, Theorems, and Formulas for Reference and Review. Courier Corporation, 1130p. (2000)