An Algorithm for Cost-Minimizing in Transportation via Road Networks Problem

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Abstract - This paper presents an algorithm for solving transportation via road network problem to distribute goods from a number of sources to a number of destinations. The objective is to minimize transportation cost. Since the trucks travel over the roads that are suitable for the cargo weight, then the cargo weight and the transportation distance become both independent variables in the standalone transportation and network problems. Therefore, the transportation cost and the network distance matrices will not be available for a prior, and the solution requires a merging of the transportation and network models in one framework. Herein, the solutions of the linear programming transportation model and the road network problem are simultaneously attained using the proposed iterative algorithm. Meanwhile, the computation of the transportation cost is obtained in the same framework. Based on the transportation prices of domestic agencies, a simulation of a numerical example is used to explain the proposed algorithm.

Keywords-: ***Transportation problem, Network problems, Linear programming, Floyd's algorithm, Cost Minimization***

I. INTRODUCTION

The transportation problem concept was originally formulated by Hitchcock, F. (1941), and since that time, several solution methods are developed. For instance, Sigrun Dewess (2014) presents a pivot generation approach that provides faster solution. Fast solution for a single-stage fixed charge transportation problem that is based on a heuristic algorithm is claimed by K. Antony Arokia, and Chandrasekharan Rajendran (2009). A genetic algorithm for a generalised transportation problem is introduced by W. Ho, P. Ji (2005) which is an extension of Hitchcock formulation. Zhi-Feng Hao, Han Huang, Xiao-Wei Yang (2006) presents a new particle swarm optimization of special structure and operators that are different than the classical particle swarm optimization. Deep analysis, including the sensitivity and the mixed constrained for transportation problems are introduced by Doustdagholi S. (2009), Badra M. N. (2007) and Adlakha V. (2006) respectively.

The researchers go further in studying the transportation problem in different applications, Tanveer Hussain (2010), Nana B Kudjo (2014), and Alfred Asase (2011). They are all assuming the availability of the initial cost matrix of their problems. In this paper, this assumption is relaxed.

Network optimization, as known today, has its origins in the 1940. The network model has been used in a different application such as highways, electric power systems, water delivery systems, and rail lines. In each of these problem settings, the purpose often is to send some goods (vehicles, electricity, material, or water) from one point to another as efficiently as possible; that is, along a shortest path, Alexander Schrijver (2010). There are several methods to find the shortest path; The Floyd-Warshall algorithm, developed in 1962, is preferred and widely used because of it is fastest among all, Weisstein, Eric (2009). Reem Z. Al-Douri (2014) has shown the powerful of using Floyd's algorithm for solving complicated networks in construction management. Since Floyd's algorithm provides in one run, all shortest distances and their paths between any two nodes, it is very suitable for solving the road network problem. Moreover, the algorithm keeps the distance calculations in an integer format.

The transportation cost is a complicated issue due to many reasons. Even in the same country and the same type of transporting means, the prices of transporting goods (materials, equipment) are often subject to change. For transportation by trucks, the changes of the international fuel and oil prices are probably the most important reason, Mark Berwick, Mohammad Farooq (2003). The second important reason is the type of the goods and their time-handling during charging, discharging and traveling as well. The transportation agencies calculate the cost according to different factors such as the weight, size or number of pieces; eventually, the number of trucks carrying the goods. Furthermore, the competition between the transportation agencies comprises an additional factor in calculating the cost. Sven B. Erlander (2010) has discussed in his book the issue of transportation planning in a theoretical framework such to obtain a cost minimizing.

II. BASIC USED MATHEMATICAL MODELS

In this section, the used mathematical models will be briefly introduced.

1. Transportation Model: Suppose that a system consists of m sources supplying n destinations. A flow on each arc incurs a unit variable cost q_{ij} (in unit of price). Furthermore, it is assumed that the variables x_{ij} be the amount of transported material from the source, i to the destination j. The available capacity at the ith source is, a_i, and the amount required at the jth destination is, bj. Mathematically, the transportation problem can be formulated as a linear programming model as, Hamdy A. Taha (2010):

$$min\,\{ z = \sum_{i=1}^{n}\sum_{j=1}^{m} q_{ij}x_{ij} \}$$

$Subjected\ to$:

$$\sum_{i=1}^{n} x_{ij} \le a_i, \qquad j = 1,2 \ldots m$$

$$\sum_{j=1}^{m} x_{ij} \ge b_i, \qquad i = 1,2 \ldots n$$

$$x_{ij} \ge 0 \tag{1}$$

where, z is the total cost that must be minimized and the coefficients q_{ij} are presented by the matrix Q,

$$Q = \begin{bmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \ddots & \vdots \\ q_{m1} & \cdots & q_{mn} \end{bmatrix} \tag{2}$$

The coefficients q_{ij} of the objective functions could represent units of delivery or traveling time (in hours), distance, transportation cost (in unit of price), number of vehicles, etc. In this work, the cost coefficients are not known in advance, nor an initial estimate is available.

2. Road Network Model: A road network consists of a set of n traffic intersections (cities, villages, etc.) and a set of ways; eventually, highways or subways connecting certain pairs of nodes. In network terminology, the traffic points are called nodes where each node represents a specific location. The ways are called arcs (or links), and each arc represents the connection between two different locations. Arcs are labeled by naming the nodes at either end. Many available methods are used to solve network problems. Floyd's algorithm is the most general one and can be programmed easily to handle large networks, Dykstra, D.P. (1984). Floyd's algorithm makes use of two matrices, the distance matrix D and the precedence matrix P, to solve all pair's shortest path problems. For n-nodes network, Floyd's algorithm is a closed-form algorithm that completes the solution in n iterations to give the shortest distance as well as the path. The input of the algorithm is the initial distance matrix of the network, D^0 and initial precedence matrix P^0. The solution requires the computation of the matrices D^1, D^2…. D^n and P^1, P^2… P^n using the following recurrent formulas, R. Panneerselvam (2003):

$$D_{ij}^{k} = min\{D_{ij}^{k-1}, (D_{ik}^{k-1} + D_{kj}^{k-1})\} \quad \forall\ i \ne j;$$

$$i, j, k = 1,2 \ldots n \tag{3}$$

$$P_{ij}^{k} = \begin{cases} P_{kj}^{k-1}, & D_{ij}^{k} \ne D_{ij}^{k-1} \\ P_{ij}^{k-1}, & otherwise \end{cases} \quad \forall\ i \ne j;$$

$$i, j, k = 1,2 \ldots n \tag{4}$$

It is given that,

$$D^0 = [d_{ij}^0],\ i, j = 1,2 \ldots n \tag{5}$$

$$P^0 = \begin{bmatrix} 0 & O_{1\times n-1} & & \\ 2 & 0 & 2O_{1\times n-2} & \\ 3 & 3 & 0 & 3O_{1\times n-3} \\ \vdots & \vdots & & \vdots \\ nO_{1\times n-1} & & & 0 \end{bmatrix} \tag{6}$$

where d_{ij}^0 is the distance between node i and node j, and O is a row vector whose entries are all ones.

The shortest distance between node i and node j can be found directly. In the distance matrix D^n, it is the distance d_{ij}, and the shortest path can be obtained from the precedence matrix P^n using a simple iterative known procedure.

In this work, initially, the D^0 matrix is not known because the cargo distribution from the sources to the destinations and hence the suitable roads are not specified yet. Therefore, since initially, both Q and D^0

matrices are not known, the problem of transportation via a road network becomes a non-convex problem.

3. Transportation Cost Model: Bear in mind the mentioned notes; a transportation cost model can be best developed according to agency invoices. The model will assume the following points:

a. The transportation means are trucks of different capacities.
b. The transportation cost is a function of the cargo weight and traveling distance. In addition, in each trip, the trucks carry a single type of goods.
c. The transportation cost is calculated according to the class of goods.
d. The transportation cost is calculated per unit weights; other units are converted to the unit of weight.
e. For different goods, the prices as function of cargo and distance are available from different transportation agencies.
f. The prices increase linearly outside the tables of prices. The prices are obtained from several transportation agencies.

The transportation costs of a certain class of goods, which is obtained from several agencies, are averaged and tabulated; usually, the prices are given in equal steps of distance d (in Kilometers) and cargo weights l (in tons). The least-square approximation technique is used to fit the transportation prices data into two-dimension polynomial, i.e.

$$C = f(d, l) = \sum_{i=0}^{p} \sum_{j=0}^{s} c_{ij} d^i l^j \, , c_{ij} \in \mathbb{R}^+ \tag{7}$$

where the integers s, and p can be found for best fitting results.

The fitting coefficient c_{ij} are obtained and stored for that certain class of goods. After statistical analysis and for accurate results, these coefficients are found as many times as the portioning of the table. Usually, the transportation agencies limit their tables of prices by a maximum distance d_{max} and maximum cargo l_{max} per a truck.

Outside the cost tables, the calculation of the transportation costs for longer distances and or heavier cargo is carried out linearly by two scaling factors, c_d and c_l respectively, i.e.

$$c_d = \begin{cases} 1, \ d \le d_{max} \\ \frac{d}{d_{max}}, d > d_{max} \end{cases} \tag{8}$$

$$c_l = \begin{cases} 1, \ l \le l_{max} \\ \frac{l}{l_{max}}, l > l_{max} \end{cases} \tag{9}$$

Therefore, in such cases, the cost is calculated as

$$C = c_d c_l f(d_{max}, l_{max}) \tag{10}$$

III PROBLEM STATEMENT

The transportation of goods between sources and destinations often follows a road network. This means that there are several possible routes to reach a certain destination. Obviously, owners, drivers, and managers want to have the shortest route, which is defined by the smallest distance or the less time of travel. On the other hand, based on the carrying cargo, some of these roads may not be allowed to travel on. Therefore, the low price of transportation will be obtained whenever the right shortest roads are used.

Consider a certain road network of N nodes, of which a number of sources, m and destinations, n are there such that

$$m + n \le N \tag{11}$$

Therefore, the network nodes could be labeled as follows:

- The sources nodes are labeled 1, 2….. m
- The destinations nodes are labeled $m + 1$, $m + 2$….. $m + n$
- The other $(N - m - n)$ nodes, if any, are labeled $m + n + 1$, $m + n + 2$…. N

The roads connecting the N nodes are of different types, where for each type, a maximum specified cargo weight is allowed. This means that in spite of the node type, the trucks travel along those roads, which are suitable with their cargo, and hence; the minimum (shortest) distance and the transportation cost are changed accordingly. In other words, neither the transportation matrix Q nor the network distance matrix D, are available in advance. Therefore, the transportation via road network problem is unsolvable sequentially, and a certain type of iterative procedure must be taken in. These facts motivate us this work. To the authors' knowledge, results on such an issue are rarely found. The desired optimum solution is defined as the cargo distribution that satisfies the destination demands and delivered via the permissible roads with shortest distances such to achieve a minimum transportation cost.

IV PROPOSED ALGORITHM

The following steps represent the proposed iterative algorithm:

1. Select a range of values, λ_k (unit of weight) and a chosen positive integer value r such that

$$\lambda_k = k, k = 1,2 \dots r \qquad (12)$$

2. Select a certain assumed cargo weight vector L of length w, where w is equal to the integer number of the maximum supply divided by λ_1. The vector entries is

$$L = [\lambda_1 \quad 2\lambda_1 \quad 3\lambda_1 \quad \dots\dots \quad w\lambda_1] \qquad (13)$$

3. For each value of L, it is assumed that this value is transported from each source to each destination, then the network problem is solved (m x n) times, and the corresponding cost Q_i's matrices, are obtained, i = 1, 2 … w, by invoking the transportation cost model.
4. Determining an initial cost matrix, ${}^1Q_{initial}$ as an average of the Q matrices obtained in step 3, i.e.

$${}^1Q_{initial} = \frac{1}{w}\sum_{i=1}^{w} Q_i \qquad (14)$$

5. Using the matrix, ${}^1Q_{initial}$ to solve the transportation problem such to have the first cargo distribution ${}^1G_1$ between sources and destinations, thus the minimum linear programming cost function ${}^1Z_1$ is determined and storied.
6. For the obtained ${}^1G_1$, the network problem is solved again and a new cost matrix ${}^1Q_1$ is obtained.
7. In Iterative way, the matrices ${}^1G_2$, ${}^1Q_2$, ${}^1G_3$… are obtained and the minimum linear programming total costs ${}^1Z_2$, ${}^1Z_3$ … are storied.
8. The iterations are stopped, whenever the cargo distribution is no longer changed. This is indicated by the appearing of a repetitive sequence V_R of cost function value Z in a number of repeated zones.
9. The minimum ${}^1Q_{min}$, ${}^1G_{min}$, ${}^1Z_{min}$, with respect to the selected value λ_1, are obtained. The minimization of the cost function is indicated by

$${}^1Z_{min} < \{{}^1Z_1, {}^1Z_2 \dots {}^1Z_k\}, \;\; I_n \le k \le I_N] \qquad (15)$$

where, I_N is a fixed number of iterations, and I_n is given by

$$I_n = I_S + V_R N_Z - 1 \qquad (16)$$

where I_S is the start of repetitive values, and N_Z is the number of repeated zones. The corresponding optimum cargo distribution is ${}^1G_{min}$. Then after, the distribution ${}^1G_{min}$ and the minimum distances are used to obtain the minimum cost (in unit price) ${}^1C_{min}$ of the transportation; using equations (7 - 10).

10. Repeat steps 1-9 above for other values of λ, then the optimum solution is conducted from

$${}^sC_{opt} = \min\{{}^1C_{min}, {}^2C_{min}, \dots\dots {}^rC_{min}\},$$

$$s \in [1, r] \qquad (17)$$

The corresponding optimum cargo distribution ${}^sG_{opt}$ is assigned.

The above algorithm can be programmed to solve the transportation via road network problem. This requires merging the linear programming solution, the Floyd's algorithm, and the calculation of the transportation cost together with the proposed iterative algorithm. It is worth mentioning that all calculations will be fixed or rounded to integer values.

The convergence of the solution to the optimum value is indicated by two appearances. The first is that for each value of λ, the minimum values of the cost function (the solution of the transported problem) are repeated in a certain sequence. The second is that for a certain number of different values of λ, the same minimum value of the cost function is obtained.

The number of iterations of the proposed algorithm is controlled by three parameters. These are: a fixed number of iterations I_N, a number of values in the repetitive sequence V_R, and the number of repeated zones N_Z. For the certain case, these three parameters can be set and used to control the solution. The convergence to the solution can be also detected via the use of these parameters. The iterations continue up to I_N as long as the value of V_R is not reached. Otherwise, if V_R is reached, then iterations continue to achieve the value N_Z. In the next section, the manipulation of these three parameters will be illustrated.

V NUMERICAL EXAMPLE

Six warehouses of reinforcement steel bars have to supply five construction sites. The available supplier's quantities (from the first to the sixth) are 30, 40, 45, 60, 30, and 50 tons respectively. The sites demands (from the first to the fifth) are 27, 45, 58, 46, and 39 tons. Thus, the problem describes an unbalance one with 40 tones access to the supply side.

The road network consists of 25 nodes and 3 types of connected roads. These roads are classified for cargo weight up and equal to 25 tons, up to 50 tons, and for heavier than 50 tons. Obviously, lighter goods can be transported via the roads of large capacity, but not inversely. The sources and destinations are arbitrary distributed in the area that is covered by the assumed network. It assumed that all roads are used in two directions. Therefore, the three corresponding distance matrices are symmetrical with zero diagonal values. Numerically, for the unallowable roads and for the nonexistence direct roads between any pair of nodes, a very large value is assigned to the corresponding matrix entries. The network figure is given in the Appendix.

Based on the prices of domestic agencies, the obtained price tables of reinforcement steel are averaged. The averaged table is then normalized by the factor p_n, which is the first element in the table. Thereafter, the transportation cost is calculated from the following:

$$C = [l^3 \quad l^2 \quad l \quad 1]T \begin{bmatrix} d^3 \\ d^2 \\ d \\ 1 \end{bmatrix}; \; l \in [1,24], d \in [5,50]$$

where the matrix T is given for two intervals of cargo weight as:

For the cargo weight $1 \le l \le 12$

$$\begin{bmatrix} 1.16 \times 10^{-8} & 1.742 \times 10^{-7} & -1.48 \times 10^{-5} & -4.78 \times 10^{-4} \\ -1.67 \times 10^{-7} & -2.500 \times 10^{-6} & 2.127 \times 10^{-4} & 2.42 \times 10^{-3} \\ 6.576 \times 10^{-7} & 9.865 \times 10^{-6} & -8.3914 \times 10^{-4} & 0.45960 \\ -6.34 \times 10^{-7} & -9.509 \times 10^{-6} & 3.1578 \times 10^{-2} & 0.38462 \end{bmatrix}$$

For the cargo weight $13 \le l \le 24$

$$\begin{bmatrix} 1.36 \times 10^{-8} & 2.032 \times 10^{-7} & -8.7 \times 10^{-5} & -1.07 \times 10^{-3} \\ -7.7 \times 10^{-7} & -1.17 \times 10^{-5} & 4.997 \times 10^{-3} & 0.068 \\ 1.42 \times 10^{-5} & 2.1331 \times 10^{-4} & -0.091353 & -1.0144 \\ -8.06 \times 10^{-5} & -1.2084 \times 10^{-3} & 0.56366 & 9.8341 \end{bmatrix}$$

First, some detailed computation is given for $\lambda = 1$ unit of weight; therefore, w is equal to 60, which is the length of vector L. The three parameters I_N, V_R and N_Z, are set to 50, 5 and 2 respectively. Next, following the first three steps of the proposed algorithm, the initial cost matrix, ${}^1Q_{initial}$ is calculated as:

$${}^1Q_{initial} = \begin{bmatrix} 17 & 43 & 72 & 17 & 44 \\ 42 & 36 & 30 & 58 & 27 \\ 52 & 24 & 56 & 31 & 42 \\ 28 & 20 & 47 & 22 & 22 \\ 63 & 20 & 35 & 51 & 45 \\ 72 & 28 & 25 & 63 & 44 \end{bmatrix}$$

The first cargo distribution ${}^1G_1$ between sources and destinations is

$${}^1G_1 = \begin{bmatrix} 27 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 8 & 0 & 22 & 0 \\ 0 & 15 & 0 & 0 & 0 & 30 \\ 0 & 0 & 0 & 43 & 17 & 0 \\ 0 & 30 & 0 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 & 0 & 0 \end{bmatrix}$$

The minimum value of the cost function ${}^1Z_1$ is equal to 4874. Consequently, the solution of the transportation via road network problem with $\lambda = 1$ is shown in table 1. The demands are satisfied by all sources via the paths shown in the table. Using equations (7-10) and rescaled by the factor p_n, the minimum transportation cost ${}^1C_{min}$ is found to be 2072. Although the solution is obtained using optimized models, there are some impractical results; from source 2 to destination 3, a cargo of only one tone has to travel 90 kilometers! Therefore, the obtained minimum transportation cost could be not the optimum one. To remedy this problem, the proposed algorithm suggests solving for different values of λ. For this problem, $r = 15$, is selected, i.e. fifteen values of λ are used to solve and simultaneously to prove the convergence of the proposed algorithm.

Figure 1, shows the result of the variation of the minimum transportation cost with respect to λ in the selected rang [1, 15]. It can be noted the same value of the minimum transportation cost is obtained for different values of λ, for instance, ${}^2C_{min} = {}^7C_{min} = 2145$. However, the optimum transportation cost is obtained for $\lambda = 8$. The optimum value is ${}^8C_{opt} = 1768$. Near optimum values are there for λ equal to 12 and 13. Furthermore, simulation shows that the same optimum value of the transportation cost is also obtained with $\lambda = 18$. This locality result is important in the sense that one can add another constraint such as the minimum load to be transported or the traffic in a certain path, etc.

For the optimum value of λ, the iterative values of the cost function ${}^{8}Z_1$, ${}^{8}Z_2$…. ${}^{8}Z_{50}$ are shown in figure 2. As depicted, the convergence of the solution is indicated by the repetitive zone. In this zone, four values (in contrast to the five values in the case of λ = 1) are repeated starting from the iteration number I_S = 16. Therefore, the iterations continue up to the assigned value of I_N, which is more than enough value to prove the convergence existence. On the other hand, if the V_R is set to 4, then the iterations continue only up to 23 iterations as shown in figure 3-a, which illustrates two repetition zones. Figure 3-b, shows the same results for 4 repeated zones.

Over the interval of the used values of λ, the convergence is obtained similarly; however, after different values of the start of the repetition zone. For example, for the near optimum λ = 12, the repetition zone starts from the iteration number I_S = 7. The convergence can be shown utilizing the parameters, I_S, I_N, V_R, and N_Z.

For the optimum value of λ = 8, table 2, shows the transportation via road network solution. The 215 tons demands are fulfilled via similar and different paths as compared to the case with λ = 1. In fact, the significant reduction of the transportation cost is a result of the reduction of the total traveled distance. The total traveled distance is equal to 440 kilometers instead of 605 kilometers with λ = 1. In addition, the source number five does not participate in supplying the material. Finally, the total transportation cost distributes among the individual sources as in table 3.

Table 1 Transportation via road network solution for λ = 1

STEEL λ = 1 unit of weight				
Sou. No.	Des. No.	Paths	Cargo	Distance
1	1	1→7	27	55
1	4	1→10	3	28
2	3	2→21→9	1	90
2	5	2→25→11	39	85
3	4	3→13→12→10	43	97
4	2	4→15→8	22	64
5	2	5→8	23	63
5	3	5→22→6→9	7	89
6	3	6→9	50	34

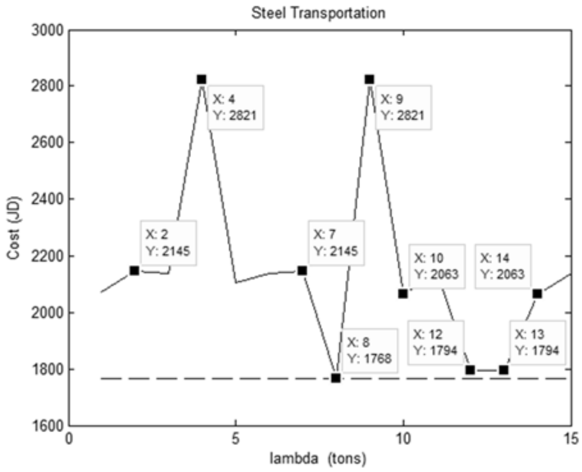


Figure 1 Minimum cost versus the λ parameter

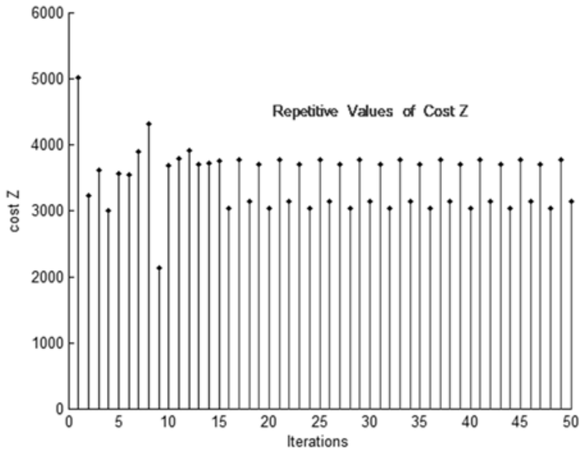
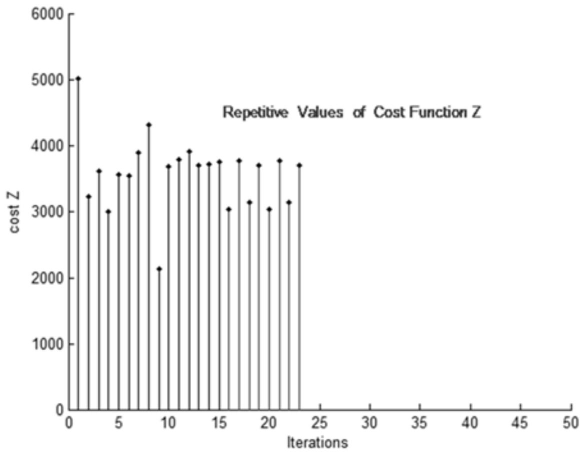
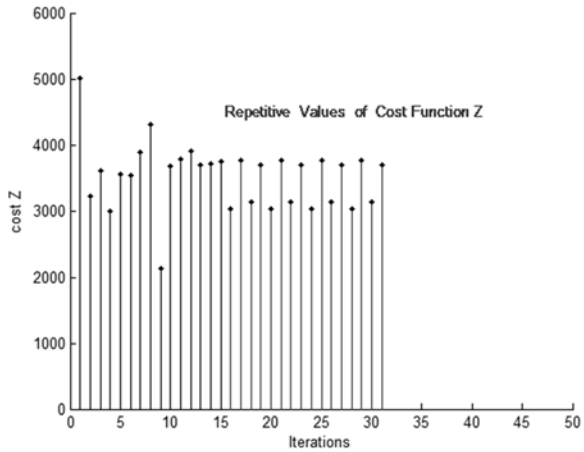


Figure 2 Convergence to minimum cost function Z



(a)



(b)

Figure 3 a) Minimum cost function Z with $V_R = 4$, and $N_Z = 2$, b) Minimum cost function Z with $V_R = 4$, and $N_Z = 4$

The developed software can satisfy the daily demands easily. For example, if the site demands are changed to 0, 0, 75, 95, and 85 tons, then the transportation problem is a balance one. For this situation, the algorithm finds that the minimum cost is obtained with $\lambda = 1$, and it is equal to 2569, and the corresponding transportation via road network solution is given in table 4. As it can be read, the demands are all satisfied through appreciated paths; the solution is optimum, nevertheless, only five tons traveled 152 kilometers. Finally, the developed software can be used for all types of goods provided that the tables of prices are available, Abdulla [2014].

VI CONCLUSIONS

The work herein presents an iterative solution of the transportation via road network problems for a minimum transportation cost. Such problems have the same independent variables, which are the cargo weight and the distance, and hence they cannot be solved in sequence. The problem is solved utilizing a proposed algorithm in one framework. The simulation results confirm the efficiency of the proposed algorithm to solve complex transportation via road network problems. The cargo distribution from sources to destinations, including the weights to be transported, the distances and traveled paths are all obtained. Furthermore, besides the obtained solution, the simulation shows clearly the convergence to the minimum cost function.

Table 2 Transportation via road network solution for $\lambda = 8$

STEEL λ = 8 unit of weight					
Sou No.	Des No.	Paths node → node	Cargo (tons)	Distance (Km)	Cost (unit of price)
1	1	1→7	27	55	180
1	4	1→10	3	28	446
2	3	2→21→9	8	90	573
2	5	2→11	22	49	148
3	2	3→14→8	45	76	455
4	4	4→16→10	43	71	292
4	5	4→11	17	37	167
6	3	6→9	50	34	308

Table 3 Transportation cost for individual sources

Source	1	2	3	4	5	6
Cost (unit of price)	229	275	449	507	0	308

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APPENDIX

Here, to show the road network figure that is used in the numerical example. The blue color is assumed for the roads that permit a cargo up and equal to 25 tons. The red color is assumed for the roads that permit a cargo heavier than 25 and up and equal to 50 tons. The black color is assumed for the roads that permit a cargo heavier than 50 tons. The brown, green and black colors are for the source, destination, and road nodes respectively.

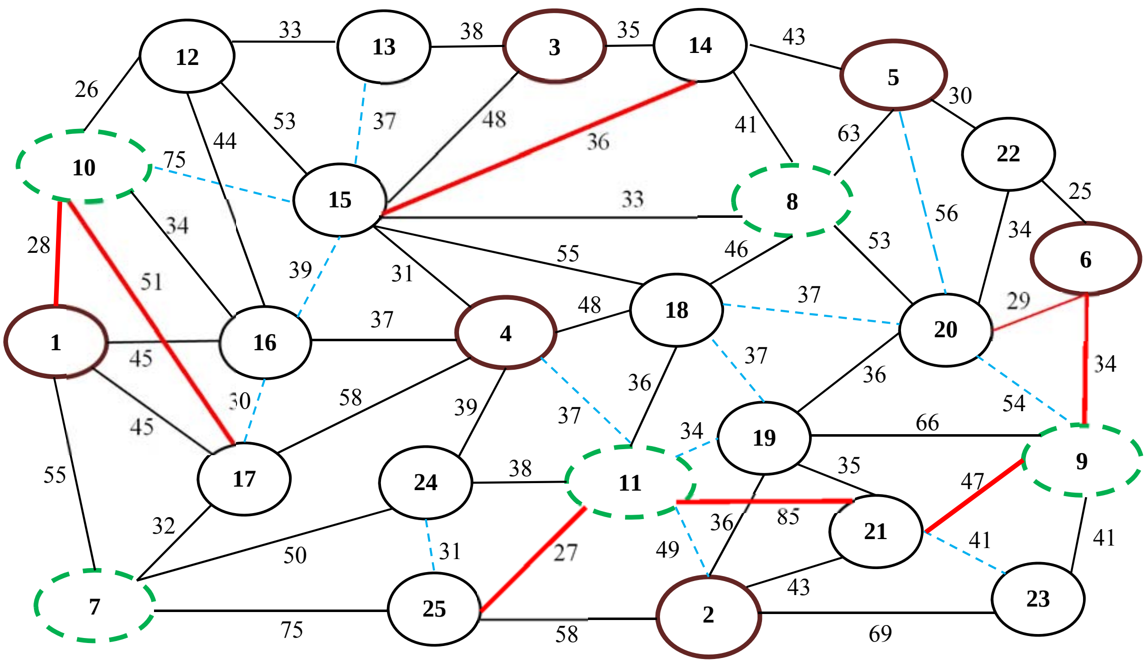

Figure 4 Road network used for the numerical example