$$\Omega_{x_0}^N := \left\{ \begin{matrix} \boldsymbol{x_0} | \boldsymbol{A_c^k x_0} \in \boldsymbol{C_G}\ i.e.\ \boldsymbol{G A_c^k x_0} \geq \boldsymbol{0}, \\ \boldsymbol{k} = \boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{N} \end{matrix} \right\}$$

and proceed as in 4.2 or 4.3 depending on if $\boldsymbol{x_0} \in \Omega_{x_0}^N$ or not. The procedure stop at $\boldsymbol{k} = \boldsymbol{N}$ and after that we have $\boldsymbol{x}_{N+1} = \boldsymbol{x}_{N+2} = \ldots$. This case in described in [11] when the cone G_C coincide with nonnegative orthant, i.e. for $G = I$.

4.5. There is no solution to the problem

If the set $\Omega_{x_0}^N$ is empty, i.e. for all initial states $\boldsymbol{x_0} \in C_G$ but $\boldsymbol{A_c x_0} \notin C_G$, $\boldsymbol{A_c^2 x_0} \notin C_G$ ..., there is no solution of the problem.

5 Conclusion

In the paper, the solution of the infinite horizon LQR problem of positive linear discrete time systems with state restrictions, given as polyhedral cone belonging to nonnegative orthant, is studied. The existing of a solution strongly depends on the properties of the closed-loop matrix and matrix of constraints. It is investigated when *(a)* the solution of the unconstrained LQR problem coincide with the solution of LQR_+^{cone}problem, *(b)* there is no solution and *(c)* there is solution of LQR_+^{cone} problem, but it doesn't coincide with the solution of unconstrained problem.

Future research will focus on estimating the necessary and sufficient conditions on the system and the restriction matrices for the solution LQR_+^{cone} problem.

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