$R_{re} \rightarrow {}^{j'}tl^iB'|reB'|ob^iB'|db^iP_{rela}$: Player i is tackled by player j, a conversion, stop the team, or pass the ball

$M_{re3} \rightarrow rb^iR_{re}\big|db^iP_{rela}\big|{}^{j'}tl^iB'|odB'$: Run, pass the ball, or player i is tackled by player j

$P_{rela} \rightarrow cb^iM_{re3}|obB'|cb^{i'}M_2'$: Catch the ball or stop the team

$M_6 \rightarrow rb^iR\big|db^iM_7\big|{}^{j'}tl^iA_x|odA_x|tdT$: Run, player i is tackled by player j, or a touchdown

$P_{la} \rightarrow rb^iR\big|db^iM_7\big|{}^{j'}tl^iA_x|odA_x$: Run, pass the ball, or player i is tackled by player j

$M_7 \rightarrow cb^iP_{la}|obA_x|cb^{i'}M_2'|{}^{j'}tl^iA_x$: Actions after kick off following a touchdown

$P \rightarrow cb^iM_6\big|obD_y^{o_{i+1}}\big|cb^{i'}M_2' \quad M_2'$: Defensive team changes to offensive team

$R \rightarrow db^iM_7|{}^{j'}tl^iA_x\big|td^iT\big|ob^iA_x$: Pass, player i is tackled by player j, touchdown, or stop the team

$A_x \rightarrow adb_xD_{y=y-x}^{o_{i+1}}\ |beo_xD_{y=y+x}^{o_{i+1}}$: Sum or subtraction of yards

2.2 Finite State Machine

The CFG language that formal describes AF is read by the corresponding finite state machine (FSM). Let $(Ó, S, s_0, \delta, H)$ be a push-down automata, where

- Ó is the alphabet;
- $S = \{B, M, M_1, M_2, M_3, D_y^{o_i}, M_5, M_6, M_7, T, PA, RE, M_{re}, P_{re}, M_{re}, M_{re2}, R_{re}, M_{re3}, P_{rela},\ P, P_{la}, R, A_x, M_1', M', B'\}$ is the set of states;
- δ: $S \times Ó \rightarrow$ S is the transition function;
- $B \in S$ is the initial state; and
- $H = \{M_1', M', B'\} \subseteq S$ is the set of halt states.

The FSMs for (a) the game start, (b) the touchdown annotation, and (c) the execution of the plays in the field are illustrated in **Figs. 1 (a)**, **(b)** and **(c)**, respectively.

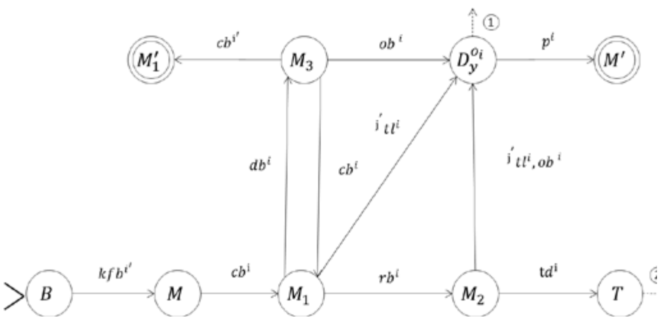

Fig. 1 (a) FSM for the game start

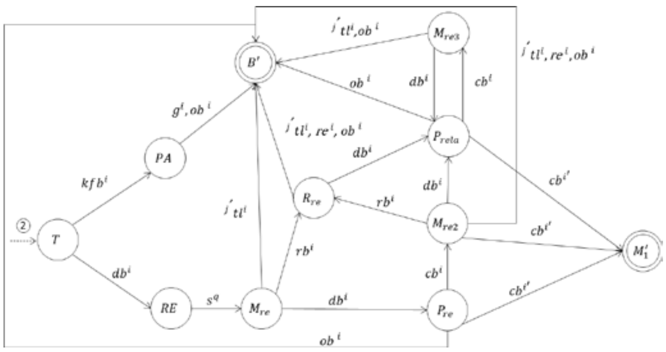

Fig. 1 (b) FSM for a *touchdown*

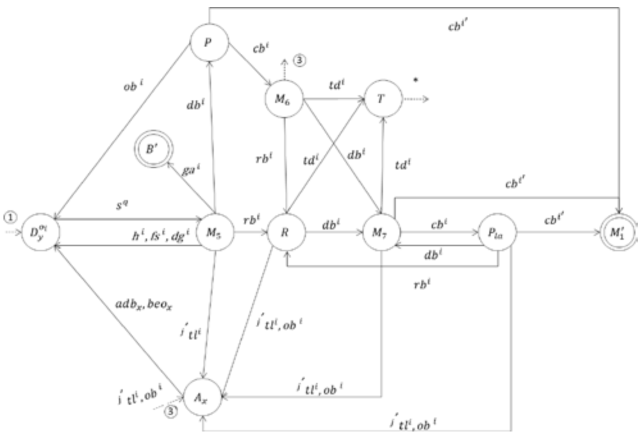

Fig. 1 (c) FSM to describe plays in the field

The set of FSMs is the mathematical device to read the language describing AF game. The parsing of strings starts at the FSM initial state then following through intermediate states to end in one FSM halt state [28], similar to the baseball algorithmic scheme in [26].

2.3 Statistics and Probability Function to Real Simulations

We add the statistical frequency of real plays occurrence to get real simulations. We use statistics of empirical human plays to define the distribution function of probability to games simulation. Hence, a play with higher statistical frequency is higher probability of occurrence, so more likely included in the simulation that the plays with low frequency or probability of occurrence. Realism to computer simulations is warranted this way.

As happens in modeling under uncertain circumstances likewise in AF, in order to learn on the parameter values about the uncertainties, the statistics confidence or credible intervals allow dealing with. Confidence interval regards frequentist statistics [29] whereas credible intervals regard Bayesian statistics approach. The interval is constructed on the base of data samples and can vary from sample to sample. To learn about win option for teams in AF games, we define both confidence and credible interval to 95% of confidence so the right value can be in the interval up to this chance. We construct statistics confidence and credible intervals on the base of one thousand game simulations in Section 5.

3 Payoff Functions by Roles and Plays

To selection of strategies NE or PE is applied by introducing payoff matrices that incorporates the AF quantitative analysis by including the game conditions for the third and fourth downs. For each down, the strategies are to kick the ball to the other team (punt), play the ball (either pass or run), or attempt a field goal. The matrices for the respective representation of the strategies for the downs are given below.

3.1 PE and NE Matrices

The payoff matrices consist of the payoff function valuations of the strategy profiles. Each matrix entry contains a player's strategy profile valuation. The M payoff matrix for the n players is built from the set of M^i payoff matrices for each player i. The M entries are the strategy profiles that are joint to the profile payoff value r_z: hence, $((s_1, \dots, s_i, \dots, s_n), r_z)$. The profile $(s_1, \dots, s_i, \dots, s_n)$ represents the strategies that the players can perform under specific AF game conditions, and r_z is the payoff value that the player i receives for this profile. The payoff matrix data can support the coach's decision-making over the course of a game. The payoff matrix represents the quantitative analysis for an entire AF game, considering the AF game conditions described above.

As we mentioned, the values of the payoff matrices are given from the payoff functions valuations on the strategy profiles. In order to illustrate this process, we introduce in the normal form game description of American football, general payoff functions that evaluate the strategy profiles and return payoff values. These payoff function values are the players' payoffs to the strategy profiles. To define the payoff functions we characterize the AF plays by each player-role. We classify AF plays according to defensive /offensive roles and then state the payoff functions of the players. AF plays and the general payoff functions follow.

3.2 Prayer-Roles

The AF player-roles are the base to define the utility function for valuing the strategy profiles. We present some AF player-roles according to the offensive or defensive team's position during the game. For offensive the player-roles are: offensive linemen, quarterback, backfield and receivers. For defensive the player-roles are: defensive linemen, linebacker and defensive backfields. For the special team, the player-roles are: kicker and kicker return, punter and punter-return. Each player-role or role for short, mostly use a set of plays, see **Table 4**.

Table 4: Offensive and defensive plays

Offensive plays		**Defensive plays**	
Abb.	Description	Abb.	Description
kb	Kick the ball	tl	Tackling
cb	Catch the ball by product of a pass	sf	Safety
rb	Run with the ball	sb	Stop the ball
pb	Pass the ball	in	Interception
fd	Scoring yards	qs	Tackling the quarterback
td	Touchdown	yb	Roll back the contraries
p	Extra point (1 point by product of a kick)	fb	Fumble the ball
re	Conversion (2 points)	fr	Turnover the ball
fg	Field goal	tb	Touchback

Offensive roles

- Offensive linemen players OL have two major tasks: 1) block the defensive team members which try to tackle quarterback (QB), and 2) open ways in order to runners can pass the ball. The OL players are, the center, left guard, right guard, left tackle, and right tackle. We defined these players as OL and the plays to consider are $OL_{\text{plays}} = \{tl, yb\}$. The OL main function is tackling the adversary to allow QB send pass; as well, open space for receiver runs with the ball, or, in some cases, push back the opposing team.
- The quarterback (QB) is the offensive leader, whose set of plays is $QB_{\text{plays}} = \{rb, pb, fd, td, re, tb\}$. QB's major action is quite pass the ball to receivers, to score so many yards and touchdown.

- The backfield players BF are: the halfback, tailback the fullback. The BF plays follow, $BF_{\text{plays}} = \{rb, fd, td, re, tb, tl\}$. The BF preferred score is touchdown or conversion, and should run to get there. As well, get a first down, or tackling an adversary player.
- Receiver's role RC is to catch the ball passed by the QB; RC players are the tight end and wide. The RC plays follow, $RC_{\text{plays}} = \{cb, rb, fd, td, re\}$. The basic action of RC is to receive the ball and run to try to reach to the touchdown line.

Defensive roles

- The defensive linemen players DL are: the defensive end, defensive tackle and nose tackle, their main task is to stop running plays on the inside and outside, respectively, to pressure the QB on passing plays. The DL plays follow, $DL_{\text{plays}} = \{tl, sf, sb, qs, yb, fb, fr\}$. The DL's major actions is to try to tack the opposing QB, roll back yards to the opposing team or get a safety; in descent order of importance the next is to stop the ball, tackling and cause fumbles and try to recover it by the opponent.
- The linebacker players LB's tasks are: defend passes in shortest paths, stop races that have passed the defensive line or on the same line and attack the QB plays penetration; they can be three or four. The LB plays follow, $LB_{\text{plays}} = \{tl, sf, sb, qs, fb, fr\}$. The main function of LB is to recover a lost ball and then could be to generate a safety.
- The defensive backfield players DS are: the cornerbacks and safeties, which major task is to cover the receivers. The DS plays follow, $DS_{\text{plays}} = \{tl, in, fb, fr\}$. For DS is important to intercept a pass or get the other team loses control of the ball.

Special team roles

- Kicker player K kicks off the ball and do field goals and extra points. The kicker's plays follow, $K_{\text{plays}} = \{kb, p, fg\}$. For K, the most important is to make a field goal, followed by an extra point and typically perform the corresponding kicks.
- The kickoff returner R is the player on the receiving team who catches the ball. The plays are $R_{\text{plays}} = \{rb, td, tb\}$. For R, the best choice is to score a touchdown with the return of the kick, but usually just run until stopped, or perform touchback for time.

3.3 Payoff Functions

The payoff function for each of the roles mentioned above value the strategy profiles considering relevant skills of the role. Each role is qualified on the base of its performance on certain plays, and the statistics of the role resumes these qualifications. Let $(x_1, \ldots, x_i, \ldots, x_n)$ the strategy profile such that x_i is one play of role i; let $V_i(x_i)$ be the role i's preference on x_i, and $\rho(x_i)$ be the average statistics of occurrence of x_i from role i regarding the statistics (may be NFL). The payoff functions by the role i is:

$$u_i(x_1, \ldots, x_i, \ldots, x_n) = V_1(x_1) \times \rho(x_1) + \cdots + V_i(x_i) \times \rho(x_i) + \cdots + V_n(x_n) \times \rho(x_n).$$

The payoff function should consider as well the contributions of the other roles that are directly involved with the execution of x_i.

Offensive team

We define the strategy profile for the offensive roles. Let (w, x, y, z) be a strategy profile with $w \in QB_{\text{plays}}, x \in RC_{\text{plays}}, y \in OL_{\text{plays}}, z \in BF_{\text{plays}}$.

- For the QB payoff function, the QB and the OL plays are considered, so the payoff function follows:

$$u_{QB}(w, x, y, z) = V_{QB}(w) \times \rho(w) + V_{OL}(y) \times \rho(y).$$

- For the RC payoff function, the RC, QB and OL plays should be considered, so the payoff function follows:

$$u_{RC}(w, x, y, z) = V_{RC}(x) \times \rho(x) + V_{QB}(w) \times \rho(w) + V_{OL}(y) \times \rho(y).$$

- For the BF payoff function, the BF, QB and OL plays should be considered, so the payoff function follows:

$$u_{BF}(w, x, y, z) = V_{BF}(z) \times \rho(z) + V_{QB}(w) \times \rho(w) + V_{OL}(y) \times \rho(y).$$

- For the OL payoff function, the OL plays are the only considered, so the payoff function follows:

$$u_{OL}(w, x, y, z) = V_{OL}(y) \times \rho(y).$$

Defensive team

We define the strategy profile for defensive roles. Let (x, y, z) be a strategy profile with $x \in DL_{\text{plays}}, y \in LB_{\text{plays}}, z \in DS_{\text{plays}}$.

- For the DL and LB payoff function, the DL and LB plays should be considered, so the payoff function follows:

$$u_{DL|LB}(x, y, z) = V_{DL}(x) \times \rho(x) + V_{LB}(y) \times \rho(y).$$

- For the DS payoff function, the DS plays are the only considered, so the payoff function follows:

$u_{DS}(x, y, z) = V_{DS}(z) \times \rho(z)$.

Special team

- For K, the payoff function follows:

$u_K(x) = V_K(x) \times \rho(x)$ where $x \in K_{\text{plays}}$.

- For R, the payoff function follows:

$u_R(x) = V_R(x) \times \rho(x)$ where $x \in R_{\text{plays}}$.

4 Experiments: Statistics and Strategic Choices

The benefit of strategic choices is measured on the base of game simulations' results regarding the next circumstances in experiments:

- When a team sole use NFL statistics compared with the same team that uses NFL statistics and the NE or the PE.
- When statistics of a team are used alone and the other team simulations are with using its statistics and some strategic choices by the NE or the PE method.

To simulate the players' actions regarding their history, we use NFL statistics for the Denver (DEN) team and the Oakland (OAK) team in the 2012 season. Each play's frequency of occurrence in the NFL statistics is used in the computer simulation. We compare the results in AF games simulation: by one hand games that use NFL statistics without any strategic analysis, comparing them to games that use NFL statistics combined with strategic choices by PE and or NE. Let Team 1 (T_1) and Team 2 (T_2) be. One thousand computer simulations are conducted for each of the next conditions.

1) T_1 with DEN' statistics versus T_2 with OAK' statistics.
2) T_1 with DEN' statistics versus T_2 with OAK' statistics and using the NE to the strategic analysis.
3) T_1 with DEN' statistics versus T_2 with OAK' statistics and using the PE to the strategic analysis.

Results from games between Oakland and Denver AF teams, under circumstance described there, are reported in **Table 5. Fig. 2** shows the simulation results when only DEN statistics are used for T_1 and only OAK statistics are used for T_2 and when either the PE or the NE are also used for T_2. Statistically, when DEN statistics are used for T_1, and OAK statistics are used for T_2, the performance of T_1 is superior to that of T_2, because the DEN team performed better than the OAK team in the NFL 2012 season. However, using either the PE or the NE to select strategies for T_2 improves the performance of T_2, and T_2 outperforms T_1. Using either strategy selection approach increases the team's level of play and enables the team to select the most appropriate strategies under the given AF circumstances, even when the team is statistically inferior to its opponents.

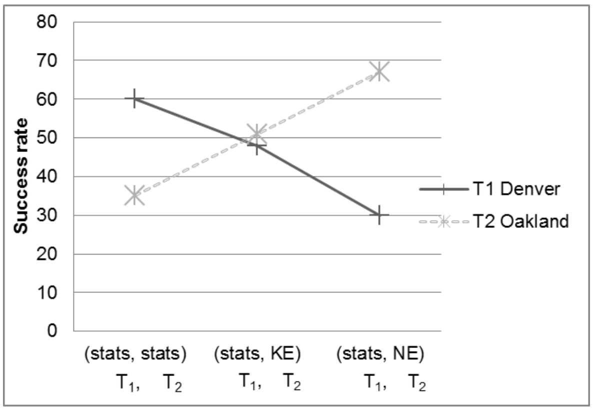

Fig. 2 Using only statistics for T_1 and statistics, PE or NE for T_2

Now, to measure the impact of the different strategic choices, NE or alternatively PE, on equally behave AF teams, we experiment on teams that use the same statistics, so equal characteristics to playing, but different strategic choices to observe the impact on their proficiency. One thousand computer simulations are performed for games in which DEN statistics are used for both T_1 and T_2 under the following conditions:

4) using the NE for T_1 and only using DEN statistics for T_2,
5) using the PE for T_1 and only using DEN statistics for T_2.

Rows 4-5 in **Table 5** (items 4-5), show that using the NE or the PE to select strategies for T_1 and only statistics for T_2 gives T_1 an advantage over T_2.

Figs. 3 to **5** compare the results for simulations of teams with the same playing characteristics but different strategy selection methods. **Fig. 3** shows the simulation results when only DEN statistics are used for T_1 and DEN statistics are used for T_2 in addition to the PE or the NE. The results show that using only DEN statistics for T_1 and changing the strategic approach for T_2 improves the performance of T_2.

Fig. 4 shows the simulation results when only the PE is used for T_1 and DEN statistics are used for T_2 along with the PE or the NE. In these simulations, the performance of T_1 is inferior to that of T_2 when only the NE is used to select strategies for T_2.

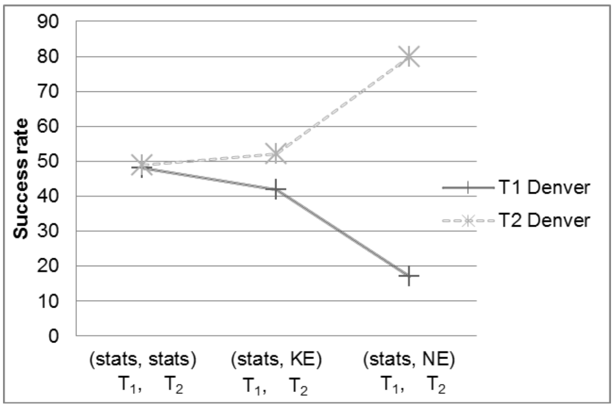

Fig. 3 Using only statistics for T_1 and statistics, PE or NE for T_2

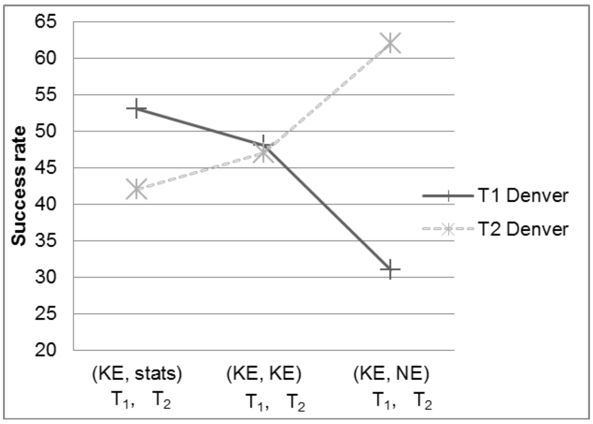

Fig. 4 Using PE for T_1 and statistics, PE or NE for T_2

The last set of experiments is without use of statistic but sole strategic choices, NE or PE, as follows:

6) using the NE for T_1 and using the NE for T_2,
7) using the NE for T_1 and using the PE for T_2,
8) using the PE for T_1 and using the NE for T_2, and
9) using the PE for T_1 and using the PE for T_2.

The results in Table 5 row 6 (item 3) show that T_1 and T_2 are equally balanced (477 wins to 472 wins) when the NE is used for both teams. Row 7 (item 4) shows that using the NE for T_1 results in superior abilities to using the PE for T_2 by 681 wins to 259 wins. Row 8 (item 5) shows that using the PE for T_1 produces an inferior performance to using the NE for T_2 by 302 wins to 622 wins. Row 9 (item 6) shows that when the PE is used for both teams, their performances are balanced at 481 wins to 471 wins. Without loss of generality, this set of experiments confirms that a team can positively impact its own abilities by using the NE or the PE to choose strategies in playing an AF game.

Fig. 5 shows the simulation results when only the NE is used for T_1 and DEN statistics are used for T_2 along with the PE or the NE. In this case, the performances of T_1 and T_2 are balanced, except when the NE is used for both teams.

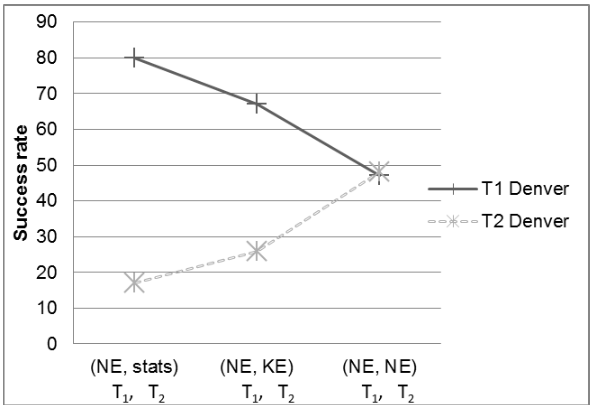

Fig. 5 Using only NE for T_1 and statistics, PE or NE for T_2

Data in the last two columns in **Table 5** is the base to the statistical analysis in 5.3. Theoretically, the Pareto-efficient profiles are the most profitable; however, we find that, these profiles in a real match are unlikely to occur than others, therefore impractical. PE profiles are Pareto-efficient which means, PE profiles even be efficient but they are unlikely to occur than NE profiles, as we showed through the set of computer simulations.

5 Results and Statistical Analysis

In classic parametric statistics the confidence interval is the range of values statistically consistent with the current observed value in the study [29]. The effect of the used strategy is meaningful described using the ratio of proportions that is an alternative in Bayesian statistics to the difference in the sample proportions.

5.1 Confidence intervals

We use the confidence interval at 95% regarding the difference of probability that each team wins in each scenario following the formula in (3) [30]:

$$p_{T_1} - p_{T_2} \in \hat{p}_{T_1} - \hat{p}_{T_2} \pm 1.96\sqrt{\frac{\hat{p}_{T1}+\hat{p}_{T2}-(\hat{p}_{T1}-\hat{p}_{T2})^2}{n}} \quad (3)$$

with $\hat{p}_{T_x} = \frac{\text{Number of Winnings for } T_x}{n}$, $x \in \{1, 2\}$ and n the number of played games. For positive confidence interval, $(a, b) \subseteq \mathbb{R}^+$, $p_{T_1} - p_{T_2} > 0$, so $p_{T_1} > p_{T_2}$, with significance level 0.05. Symmetrically, by $(a, b) \subseteq \mathbb{R}^-$, $p_{T_1} < p_{T_2}$. In the equation (3), the confidence interval depends on the number of observations: with few observations the interval $p_{T_1} - p_{T_2} \in (-0.1,0,2)$ could be got, then cannot reject the hypothesis $p_{T_1} - p_{T_2} \leq 0$ nor

$p_{T_1} - p_{T_2} \geq 0$. In our problem, 1000 simulation times are enough to get confidence intervals not having the cero, so reject the hypothesis $p_{T_1} - p_{T_2} \geq 0$ when the confidence interval $(a,b) \subset R^-$, or reject the hypothesis $p_{T_1} - p_{T_2} \leq 0$ when the confidence interval $(a,b) \subset R^+$, both with 0.05 significance level. Henceforth, $p_{T_1} < p_{T_2}$ with 95% confidence in the first case, and $p_{T_1} > p_{T_2}$ in the second one, are given. These are the cases for conclusive results.

Table 5: Confidence interval from computer simulation results

	strategic choices method used					**Confidence Interval at 95% for**	
	T1	**T2**	**Winning games T1**	**Winning games T2**	**Tied Games**	$p_{T_1}^{y-z} - p_{T_2}^{y-z}$	**Conclusion**
1	DEN stat.	OAK stat.	610	355	35	(0.19,0.31)	$p_{T_1}^{stat-stat} > p_{T_2}^{stat-stat}$
2	DEN stat.	OAK stat. and NE	305	668	27	(-0.41,-0.3)	$p_{T_1}^{stat-NE} < p_{T_2}^{stat-NE}$
3	DEN stat.	OAK stat. and PE	469	511	20	(-0.1,0.01)	$p_{T_1}^{stat-PE} \approx p_{T_2}^{stat-PE}$
DEN' statistics for both teams, so equally behave gaming but use of different strategic choices							
4	DEN stat. and NE	OAK stat.	812	166	22	(0.59,0.69)	$p_{T_1}^{NE-stat} > p_{T_2}^{NE-stat}$
5	DEN stat. and PE	OAK stat.	536	422	42	(0.05,0.17)	$p_{T_1}^{PE-stat} > p_{T_2}^{PE-stat}$
6	DEN stat. and NE	OAK stat. and NE	477	472	51	(-0.05,0.06)	$p_{T_1}^{NE-NE} \approx p_{T_2}^{NE-NE}$
7	DEN stat. and NE	OAK stat. and PE	681	259	60	(0.36,0.47)	$p_{T_1}^{NE-PE} > p_{T_2}^{NE-PE}$
8	DEN stat. and PE	OAK stat. and NE	302	622	76	(-0.37,-0.26)	$p_{T_1}^{PE-NE} < p_{T_2}^{PE-NE}$
9	DEN stat. and PE	OAK stat. and PE	481	471	48	(-0.05,0.07)	$p_{T_1}^{PE-PE} \approx p_{T_2}^{PE-PE}$

Let $x \in \{1,2\}$ be the team and $z, y \in \{stat, NE, PE\}$; $stat$ means team uses only statistics, NE or PE team uses statistics and NE or PE as strategic choice method. Let $p_{T_x}^{y-z}$ be the probability of team x wins, when T_1 uses y and T_2 uses z. In **Table 5**, we summarize the computer simulation results illustrated since **Fig. 2** to **Fig. 10**, describing the winning games for each team, the confidence interval and the probability of winning for each team. When both teams use statistics, the probability of winning T_1 is greater than the one of T_2 (rows 1 in **Table 5**). Main conclusion is that up to the confidence interval at 95%, when OAK statistics are used for T_2 uses with NE, and DEN statistics are sole used for T_1, the probability of winning T_2 is greater than the one of T_1. So, in this strategic choice T_2 performance is better (rows 2 from **Table 5**) regardless that is statistically inferior than T_1.

5.2 Credible Intervals

We calculate the Bayesian credible interval of this quantity with non-informative priors since its expressive properties [31].

For decision-making to know that the probability P_{T_1} of winning with strategy *A* is better than the probability P_{T_2} of winning with strategy *B*, is useful. Even more useful is to know the proportion $\frac{P_{T_1}}{P_{T_2}}$, so, as an instance, if $\frac{P_{T_1}}{P_{T_2}} = 2$, P_{T_1} is twice the probability of winning with P_{T_2}.

A credibility interval using Bayesian statistics is required for this kind of analysis. We define a non-informative prior distribution for random vector (P_{T_1}, P_{T_2}), assuming independence for the two random variables. Then, using Bayes Theorem, the posterior distribution for (P_{T_1}, P_{T_2}), it combines the prior distribution with the got simulations results. Using the posterior distribution we transform $(P_{T_1}, P_{T_2}) \to \frac{P_{T_1}}{P_{T_2}}$ in order to obtain a credible interval for $\frac{P_{T_1}}{P_{T_2}}$. 1000 times simulation is enough to get credible intervals without the number one, so rightly quantify $\frac{P_{T_1}}{P_{T_2}}$.

For instance, if $\frac{P_{T_1}}{P_{T_2}} \in (2,3)$ with credibility 95%, then, P_{T_1} is at least twice than P_{T_2} because $\frac{P_{T_1}}{P_{T_2}} \in (2,3) \Leftrightarrow 2 < \frac{P_{T_1}}{P_{T_2}} < 3 \Leftrightarrow 2P_{T_2} < P_{T_1} < 3P_{T_2}$ with 0.95 probability. Conversely, if $\frac{P_{T_1}}{P_{T_2}} \in (0,0.5)$, with credibility 95%, then $\frac{P_{T_1}}{P_{T_2}} \in (0,0.5) \Leftrightarrow 0 < \frac{P_{T_1}}{P_{T_2}} < 0.5 \Leftrightarrow 0 < P_{T_1} < 0.5P_{T_2} \Leftrightarrow 0 < 2P_{T_1} < P_{T_2}$ with 95% probability, then P_{T_2} is at least twice greater than P_{T_1} with probability 0.95. Notice that if the credible interval contains number one then we are not able to obtain such conclusions.

See Appendix A Table A.1 is related to T_1 (DEN) and **Table A.2** is related to T_2 (OAK) provide credible intervals of the ratio of the probability of winning of each team for all the possible combinations of strategies used by each team.

Let $p_{T_2}^{stat-NE}$ be the probability of winning T_2 when uses OAK statistics and NE, and T_1 uses only DEN statistics. Let $p_{T_2}^{stat-PE}$ be the probability of winning from team T_2 when uses OAK statistics and PE, and T_1 uses only DEN statistics. Let $p_{T_2}^{stat-stat}$ be the probability of winning T_2 when uses only OAK statistics and T_1 uses only DEN statistics. The probability of winning T_2 versus T_1 in this circumstance follows.

- Using the confidence interval reported in cell (2,1) **Table A.2** in Appendix A, the percentage of wining comparing $p_{T_2}^{stat-NE}$ versus $p_{T_2}^{stat-stat}$ follows:
  - $1.71 < p_{T_2}^{stat-NE} / p_{T_2}^{stat-stat} < 2.07$
  - $p_{T_2}^{stat-stat} \times 1.71 < p_{T_2}^{stat-NE} < 2.07 \times p_{T_2}^{stat-stat}$
  - $p_{T_2}^{stat-stat} \times (1 + 0.71) < p_{T_2}^{stat-NE} < (1 + 1.07) \times p_{T_2}^{stat-stat}$

The percentage of wining from $p_{T_2}^{stat-NE}$ versus $p_{T_2}^{stat-st}$ is from 71 % to 107 %.

- Using the confidence interval reported in cell (3,1) **Table A.2** in Appendix A, the percentage of wining comparing $p_{T_2}^{stat-PE}$ versus $p_{T_2}^{stat-stat}$ follows:
  - $1.3 < p_{T_2}^{stat-PE} / p_{T_2}^{stat-stat} < 1.6$
  - $p_{T_2}^{stat-stat} \times 1.3 < p_{T_2}^{stat-PE} < 1.6 \times p_{T_2}^{stat-stat}$
  - $p_{T_2}^{stat-stat} \times 1.3 < p_{T_2}^{stat-PE} < 1.6 \times p_{T_2}^{stat-stat}$
  - $p_{T_2}^{stat-stat} \times (1 + 0.3) < p_{T_2}^{stat-PE} < (1 + 0.6) \times p_{T_2}^{stat-stat}$

The percentage of wining from $p_{T_2}^{stat-PE}$ versus $p_{T_2}^{stat-stat}$ is from 30 % to 60 %.

- Using the confidence interval reported in cell (2,3) **Table A.2** in Appendix A, the percentage of wining comparing $p_{T_2}^{stat-NE}$ versus $p_{T_2}^{stat-PE}$ follows:
  - $1.21 < p_{T_2}^{stat-NE} / p_{T_2}^{stat-PE} < 1.41$
  - $p_{T_2}^{stat-PE} \times 1.21 < p_{T_2}^{stat-NE} < 1.41 \times p_{T_2}^{stat-PE}$
  - $p_{T_2}^{stat-PE} \times 1.21 < p_{T_2}^{stat-NE} < 1.41 \times p_{T_2}^{stat-PE}$
  - $p_{T_2}^{stat-PE} \times (1 + 0.21) < p_{T_2}^{stat-NE} < (1 + 0.41) \times p_{T_2}^{stat-PE}$

The percentage of wining from $p_{T_2}^{stat-NE}$ versus $p_{T_2}^{stat-PE}$ is from 21 % to 41 %.

From the previous analysis the conclusion is that when T_2 uses own statistics with NE or PE, while T_1 sole uses own statistics, the probabilities that win T_2 are greater than the ones of T_1, in spite of T_2 is inferior statically to T_1. The team score of T_2 is still improved by using NE than PE, which percentage of profit is from 21 % to 41 %. Analyses like this may be by using Tables reported in Appendix A.

Theoretically, the Pareto-efficient profiles are the most profitable. However, these profiles in a real game are low likely to occur than others. On the other hand, more likely to occur are NE profiles as results from the results of computer simulations.

In our analysis, a meaningful fact is that Nash equilibrium is used to identify relevant circumstance of cooperation in an AF game. When some players should sacrifice their ambitious to ensure a better team result: theoretical best actions, touchdown by long ball pass, is low probably to occur so give a major chance to more probably play, step by step ball carrying, is need. In the context of an AF game, Pareto efficiency identifies the best actions for the whole team, beyond their plausibility of occurrence. Nash equilibrium can be used to identify team actions with more realistic plausibility of occurrence. Cooperation passes by the players' ambitious sacrifice to practice a more probably play.

5.3 Score Forecasting

Former investigations in forecasting AF games are resuming in **Table 6.**

Table 6: Forecasting methods in AF games

Team	Description
Song et al. [9]	Forecast the winners but not the score on games from National Football League (NFL) in the season 2000 – 2001 and the accuracy of the predictions is compared by experts and the statistical systems.
Baker and McHale [10]	Forecast the exact scores of games in NFL games, using a set of covariates based on past game statistics.
Gonzalez and Gross [5]	Developed a program that learned to play a game based on information that was obtained by observing historical database of human opponent's plays.
Deutsch and Bradburn [11]	Developed a simulation model for AF plays in which the individual football players' positions and velocities were represented as functions of time in a Monte Carlo model.
Janssen and Daniel [12]	Developed decision criteria using the maximum expected utility, based on a *von Neumann-Morgenstern* utility function, with stochastic dominance as an alternative criterion.

Due to lack of information of methods described above, we could not make fair comparisons among them versus our proposal. With our approach the results forecasting allows predict the exact scores. Reliable and realistic results are obtained from the computer simulations of AF games using a formal language, a FSM and a generator for American football plays (see Section 2). Within our approach, all of the possible ways to play AF are considered from the start to the end of a game: real games among NFL teams are simulated by basing all of the players' actions on their own NFL statistics. The complex scoring plays presented by Baker and McHale [10] is a functions-based approach so included in our formal language model that use transition functions for modeling AF. Next examples illustrate the forecasting general approach:

- A touchdown with kickoff return: T_2 kicks the ball, and the kick returner from T_1 scores a touchdown:
$$kfb^4 team2\ cb^9 rb^9 td^9.$$
- A touchdown with a one-point conversion: the quarterback makes two passes to score a touchdown, followed by a one-point conversion:
$$s^{qb} db^1\ cb^1\ {}^1tl^{12}\ s^{qb} db^4\ cb^4\ rb^4 td^4\ s^{10} re^1.$$
- A touchdown with a two-point conversion: the quarterback makes two passes to score a touchdown, followed by a two-point conversion:
$$s^{qb} db^1\ cb^1\ {}^1tl^{12}\ s^{qb} db^4\ cb^4\ rb^4 td^4\ s^{qb} db^2\ cb^2\ re^2$$
.
- A safety, i.e., a ball carrier is tackled in his own end zone: T_2 kicks the ball, and the kick returner 1 of T_1 is tackled in his own end zone by player 6:
$$kfb^3 team2\ cb^1\ {}^1tl^6\ sf^1.$$
- A field goal: after three plays towards the opponent's end zone, the team decides to kick a field goal:
$$s^{qb} fs^1\ s^{qb} db^4\ cb^4\ {}^4tl^{12} \ldots\ s^{10} ga^2.$$

The aforementioned strings describe particular routes to score points, although other routes are possible. Recall that to perform one part of the experiments described in the previous section, one thousand computer simulations are conducted on games between the Denver team and the Oakland team using only the NFL statistics for the 2012 season and without making any strategic choices. The winning percentage and the average points that are obtained in one thousand computer simulations are reported and compared with the real scores for the games in the 2012 season (**Table 7**). The results show a high degree of accuracy for the forecasting of the exact scores with a difference of ± 1.21 points between the actual and predicted scores.

Table 7: Forecasting game results using computer simulations

Team	Winning percentage	Average points	Actual score
Denver	62%	27.21	26
Oakland	38%	12.25	13

Song et al. [9] have stated that statistical models may yield more accurate forecasts than human judgment because objective criteria are employed in models to guard against bias and the non-rational interpretation of data. However, statistical models sometimes cannot capture non-quantitative factors; hence, forecasts are not completely accurate. Our model produces a high precision for forecasting winning teams and exact scores.

6 Discussion

Our approach results in correct algorithmic simulations of AF games, so the possible ways to play an AF game. Reliable games are obtained for computer simulations of AF games using the context-free-language and the state machine, and realistic results by the distribution of statistics-probabilities of plays.

For decades Pareto efficiency has been a benchmark to select from a population of solutions, the optimal solutions for problem in economic, scientific and engineering fields. In evolutionary algorithms allows selecting the next best generation of individuals. PE formalism supports the design of models to identify theoretical optimal strategy profiles. We use PE to choice the theoretical optimum cooperative profiles for team collaboration, by assuming that a team cooperation mindset is operational, i.e., mutual confidence is an assumed condition for a successful team. The abilities of each group member are considered in a collective procedure for the deployment of a complex task, i.e., a theoretical Pareto-efficient design of collective strategies is used to plan a complex task. However, this theoretical perspective on each member's best strategies may not always be realized in a real (non-theoretical) game. A NE strategy profile can be or not Pareto optimal. In both cases it can be applied to get a best decision for a team. When NE strategy profile is not optimal it can be useful to identify highly frequent combinations of plays. On the other hand, the NE optimal strategy profiles can be useful to identify combinations of plays of low frequency but of high benefit to the team.

Games among NFL teams are reproduced using NFL statistics describing the players' history of actions. The use of NFL statistics to feed the players' actions in thousands of computer simulations set to accuracy forecast the futures scores of game results. Baker and McHale [10] used a forecasting model with a continuous-time Markov birth process to analyze the ways in which points could be scored in NFL games. The authors focused on an unconverted touchdown (6 points), a touchdown with a one-point conversion (7 points), a touchdown with a two-point conversion (8 points), a safety (2 points), and a field goal (3 points). For each type of score, various hazard functions were used for each team, home and away, that depended on the state of play. As previously described, our developed approach can be used to formally score these particular circumstances by substituting a probabilistic generator for the hazard functions and finite state automata for the Markov process developed for scoring plays, So this approach is generalized in our model within an elegant algorithmic setting.

Although baseball and AF differ considerably in terms of respective game rules and play methods, there are considerable similarities between these games formal account. Both games are multi-player sport games in which each player has a specific role to perform in strategies during the offensive and defensive plays that are directed by the coach [26, 32, 33]. CFG is also used in the formal modeling and the algorithmic setting for both games to simulate an entire game. In both sport games, strategic analysis using statistics is a determining factor in making correct decisions [26, 32], [7].

The design and use of collective strategies has an impact far beyond the field of multi-player sports or multi-agent systems. Coen [34] studied the multiple-team social dilemma by integrating empirical studies of actual human behavior with behavioral predictions from simulations. Coen examined the findings from each approach to the single-team social dilemma and then combined elements of each approach for application to the multiple-team social dilemma. These empirical studies were used to reveal the decision-making process, and computer simulations were used to determine the most effective decisions. Our approach is similar to Coen's study in the use of computer simulations for social interactions. In addition, we consider the circumstances of a multi-player game in the application of mathematical methods to explore different players' actions. Our strategic analysis is quantitatively accurate because the NE and PE are used to choose an appropriate strategy to increase team performance.

7 Conclusion

In AF as a collective sport game wherein strategic analysis is essential for success, strategic decision based on NE or PE analytical methods strengthens the team performance, thereby increasing the expectations of winning. The results of computer simulations showed that using the NE for strategy selection improved the team performance over using the PE, even though the PE fits the theoretical Pareto-efficient selection of the strategy profiles, thereby incorporating each member's best strategies. However, in a real (non-theoretical) game, these strategies are low likely to occur and are therefore low practical.

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Appendix A

Let define $p_{T_x}^{y-z}$ as the probability of wining of team x when T_1 (DEN) uses y and T_2 (OAK) uses z, $x \in \{1,2\}$ and $y, z \in \{\text{stat}, \text{NE}, \text{PE}\}$, where stat means team uses only statistics, NE or PE team uses statistics and NE or PE as strategic choice method.

Table A.1: Credible interval for $p_{T_1}^{a-b} / p_{T_1}^{c-d}$ at 95%.

		1	2	3	4	5	6	7	8	9
	a-b *c-d*	**stat-stat**	**stat- NE**	**stat- PE**	**NE-stat**	**PE-stat**	**NE- NE**	**NE- PE**	**PE- NE**	**PE- PE**
1	**stat.-stat**	-	(1.8,2.22)	(1.2,1.41)	(0.71,0.8)	(1.05,1.23)	(1.18,1.39)	(0.84,0.96)	(1.82,2.25)	(1.17,1.38)
2	**stat- NE**	(0.45,0.56)	-	(0.58,0.73)	(0.34,0.41)	(0.51,0.63)	(0.57,0.72)	(0.4,0.5)	(0.88,1.15)	(0.57,0.71)
3	**stat- PE**	(0.71,0.83)	(1.37,1.72)	-	(0.54,0.62)	(0.8,0.95)	(0.9,1.08)	(0.64,0.74)	(1.38,1.74)	(0.89,1.07)
4	**NE-stat**	(1.26,1.41)	(2.42,2.94)	(1.61,1.86)	-	(1.42,1.62)	(1.59,1.83)	(1.13,1.26)	(2.44,2.97)	(1.57,1.81)
5	**PE-stat**	(0.81,0.95)	(1.58,1.96)	(1.05,1.25)	(0.62,0.7)	-	(1.03,1.23)	(0.73,0.85)	(1.59,1.98)	(1.02,1.22)
6	**NE- NE**	(0.72,0.85)	(1.4,1.75)	(0.93,1.12)	(0.55,0.63)	(0.82,0.97)	-	(0.65,0.76)	(1.41,1.77)	(0.91,1.09)
7	**NE- PE**	(1.05,1.19)	(2.02,2.48)	(1.34,1.57)	(0.8,0.88)	(1.18,1.37)	(1.32,1.54)	-	(2.03,2.5)	(1.31,1.53)
8	**PE- NE**	(0.44,0.55)	(0.87,1.13)	(0.57,0.72)	(0.34,0.41)	(0.5,0.63)	(0.56,0.71)	(0.4,0.49)	-	(0.56,0.7)
9	**PE- PE**	(0.73,0.86)	(1.41,1.77)	(0.94,1.12)	(0.55,0.64)	(0.82,0.98)	(0.92,1.1)	(0.65,0.76)	(1.42,1.79)	-

Table A.2: Credible interval for $p_{T_2}^{a-b} / p_{T_2}^{c-d}$ at 95%.

		1	2	3	4	5	6	7	8	9
	a-b *c-d*	**stat-stat**	**stat- NE**	**stat- PE**	**NE-stat**	**PE-stat**	**NE- NE**	**NE- PE**	**PE- NE**	**PE- PE**
1	**stat.-stat**	-	(0.48,0.58)	(0.63,0.77)	(1.82,2.52)	(0.75,0.94)	(0.68,0.84)	(1.2,1.57)	(0.52,0.63)	(0.68,0.84)
2	**stat- NE**	(1.71,2.07)	-	(1.21,1.41)	(3.48,4.66)	(1.46,1.72)	(1.31,1.53)	(2.3,2.89)	(1.01,1.15)	(1.31,1.54)
3	**stat- PE**	(1.3,1.6)	(0.71,0.82)	-	(2.65,3.58)	(1.1,1.33)	(0.99,1.18)	(1.75,2.23)	(0.76,0.89)	(0.99,1.19)
4	**NE-stat**	(0.4,0.55)	(0.21,0.29)	(0.28,0.38)	-	(0.34,0.46)	(0.3,0.41)	(0.54,0.76)	(0.23,0.31)	(0.3,0.41)
5	**PE-stat**	(1.06,1.33)	(0.58,0.69)	(0.75,0.91)	(2.17,2.97)	-	(0.81,0.99)	(1.43,1.85)	(0.62,0.74)	(0.81,0.99)
6	**NE- NE**	(1.2,1.48)	(0.65,0.76)	(0.84,1.01)	(2.44,3.32)	(1.01,1.23)	-	(1.61,2.06)	(0.7,0.82)	(0.91,1.1)
7	**NE- PE**	(0.64,0.83)	(0.35,0.43)	(0.45,0.57)	(1.31,1.86)	(0.54,0.7)	(0.48,0.62)	-	(0.37,0.47)	(0.49,0.62)
8	**PE- NE**	(1.59,1.93)	(0.87,0.99)	(1.13,1.32)	(3.24,4.34)	(1.35,1.61)	(1.22,1.43)	(2.14,2.7)	-	(1.22,1.43)
9	**PE- PE**	(1.19,1.48)	(0.65,0.76)	(0.84,1.01)	(2.43,3.31)	(1.01,1.23)	(0.91,1.09)	(1.61,2.06)	(0.7,0.82)	.

In the perspective of multi-agent systems, Capraro et al proposed an Iterated Cooperative Equilibrium (ICE) [35]. In each round the players forecast how the game would be played if they form coalitions, and select their actions accordingly; up to the reward to be obtained the participants' behavior change and the Nash equilibrium convergence is not mandatory, but cooperation behavior can be observed.

Dual equilibrium (DE) with respect to NE for two players is studied in the so called prescriptive games, Corley et al. [36]. In DE each player acts motivated by the others' best interest and non-selfish behavior influence the outcomes. The concept in DE formalizes an optimal team collaboration and is a particular instance of the cooperation in PE. The altruism and envy behavior in contests for two players is formally analyzed by Kai Konrad [37]. Share in outcomes, at which altruists and envious players have identical payoffs in the games are observed; Konrad claims that the presence of altruism and envy behavior provide stability to the whole population dynamic. We emphasize the relevance of both, cooperation and non-cooperation behavior in human relationships. In our AF analysis both attitudes cooperation and non-cooperation, result in a complementary advantage for the team.

In Roy [38], collective strategies in businesses define the conditions under which this type of strategy can emerge and stabilize and demonstrated the endogenous nature of the dissolution of the strategy. Viguier et al. [39] deal with the modeling of the strategic allocation of greenhouse gases emission allowances in the EU-wide trading. Flåm [40] studies balanced environmental games, on coalitional games among economic agents plagued by aggregate pollutions of diverse sorts. Dornhaus [41] analyzed the behavior of social insects, such as ants and bees, and showed that individual-based models can be used to identify non-intuitive benefits of different mechanisms of communication and division of labor. Dornhaus also found that these benefits may depend on the external environment and concluded that individual-based models are useful for testing hypotheses about the benefits of different collective strategies under varying ecological conditions.