Numerical analysis of free convection Casson fluid flow from a spinning cone in non-Darcy porous medium with partial slip and viscous dissipation effects

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Abstract: In the present study, a numerical analysis on free convection Casson fluid flow from a spinning cone in non-Darcy porous medium with radiation, partial slip, cross diffusion and viscous dissipation is considered. The surface of the cone is heated under linear surface heat flux (LSHF). The boundary layer partial differential equations were converted into a system of ordinary differential equations which were then solved using spectral relaxation method (SRM). In this study, we demonstrate the accuracy of the SRM as an alternative method in solving boundary value problems. The results obtained in this study were compared with others in the literature and found to be in excellent agreement. The boundary layer velocity, temperature and concentration profiles are computed for different values of the physical parameters. In particular, the effect of the Casson parameter, spin parameter, Eckert number, Soret number, velocity slip factor, thermal slip factor and concentration slip factor on velocity, temperature and concentration profiles was studied. It is shown that increasing the Casson parameter decrease velocity profiles. Increasing the velocity slip factor tend to assist the flow, while increasing the thermal and concentration slip factors tend to reduce temperature and concentration profiles respectively.

Key–Words: Casson fluid, spinning cone, partial slip, Spectral relaxation method

1 Introduction

The problem of heat transfer in spinning objects is important due to its application in industry. In particular, the design of automatic cooking machinery and movement of automotive parts in engines. These designs incorporate different solid shapes including spinning cones immersed in lubricants. It is important to study how the heat generated at these surfaces, the partial slip due these lubricants, and the presence of solid particles affects the ambient fluid. Other examples arise in engineering, where double diffusion is seen in the formation of microstructures, cooling of molten metals and fluid which flows close to shrouded fins [1]. Casson fluid flow can be buoyant driven and this situation is found in many practical applications such as soup simmering in a pot, effect of application of heat on blood and synovial fluid in humans, flow of sewage sludge on heated surfaces. Other applications are found in metallurgy, drilling operations, manufacturing of paints, manufacturing pharmaceutical products [2]. It is therefore necessary to investigate the flow of Casson fluid in different geometries under various conditions such as porous media and viscous dissipation. Partial slip conditions at the boundary are also necessary to investigate since this is mostly characteristic of Casson fluid flow due to their lubrication effects.

The study of free convection from a cone in a Newtonian fluid was studied by Ece [3] who considered flow about a cone under mixed boundary conditions and a magnetic field. This study investigated a flow outside the cone which gave an insight in such flows. Other similar studies of flows past cones were conducted by Cheng [4] -[5], who considered various situations such as Soret and Dufour effects, natural convection flows in viscoelastic fluid. Among others Alim [7] investigated pressure work effect and free convection from a cone in viscoelastic fluid, Narayana and Sibanda [8] investigated cross diffusion effects and free convection from a cone, they studied heat and mass transfer on this geometry, Awad [9] studied convection from an inverted cone in a porous medium with cross diffusion effects. Other cone studies were conducted by Agarwal [10] who studied flow past a cone at certain angles of attack. Other studies on cone geometry include the works of Anilkumar et al. [11],Takar et al.[12], Saleh et al [13] and Chamkha [14].

The study of Casson fluid flow was conducted by many authors, among others Mukhopadhyay and Vejrravelu [15], Mukhopadhyay et al. [16], Nadeem et al. [17], and Ramachandra et al. [18]. These studies considered the flow of Casson fluid on different geometries such as flow over a stretching sheet and circular surfaces, others considered partial slip conditions. They also considered blowing and suction situations on a circular geometry. These studies also considered diffusion of chemical species, porous media, MHD flows and unsteady flows. This shows the importance of studying this type of flow and most of the studies used the well-known Runge-Kutta numerical method solve their systems of equations. All of these studies considered a similar constitutive equation for Casson fluid.

Cross diffusion effects have been studied by among others Hayat et al. [19] investigated heat and mass transfer for Soret and Dufour effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with viscoelastic fluid. Cheng [6] studied Soret and Dufour effects on free convection boundary layer over a vertical cylinder in a saturated porous medium. None of these studies considered a spinning cone and partial slip on cone geometry. Each of these studies considered different boundary conditions and used either the cubic spline or the homotopy analysis methods.

In the present study we investigate the effects of radiation in natural convection from a spinning cone with partial slip in non-Darcy porous medium and cross diffusion effects in Casson fluid with viscous dissipation. The driving force in caused by temperature differences between the cone surface and the surroundings. The present work is also a further development of the work of Narayana et al. [1] in which the linear surface temperature (LST) and linear surface heat flux (LSHF) are considered. The study of Casson fluid has not been widely investigated for heat transfer past a rotating cone. The Darcian-drag force term and the viscous dissipation source terms are introduced in the governing equations. Similarity transformations are used to convert the governing equations into a system of partial differential equations which are then solved by using the Runge-Kutta-Felhberg integration scheme. The numerical method used is not only validated by comparison to previous work by other authors, but also by the use of the successive linearization method (SLM). In this work we investigate the effect of varying physical parameters on the velocity, temperature and concentration profiles with the presentation of graphical illustrations.

2 Mathematical formulation

The steady, laminar, viscous and buoyancy driven convection heat and mass transfer flow from a spinning vertical cone with viscous dissipation and radiation effects in a Casson fluid maintained at a non-uniform temperature T_a ($> T_\infty$). The solute concentration is considered to be $C_a (> C_\infty)$ at the surface of the cone and C_∞ in the ambient fluid. Ω is the angular velocity of the spinning cone, u, v and w are the velocity components in the x, y and z respectively. g is the acceleration due to gravity (see Figure 1).

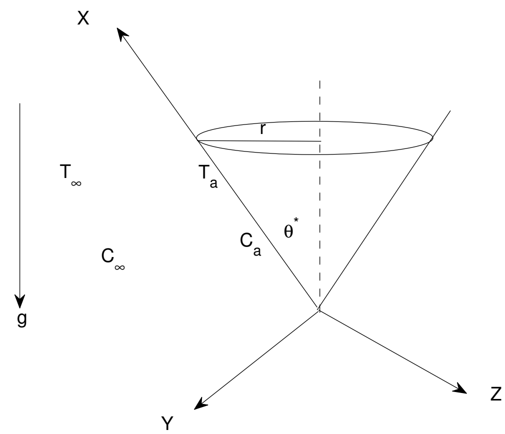

Figure 1: Schematic diagram of the spinning cone

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is given as

$$\tau_{ij} = \left(\mu_e^{\frac{1}{n}} + (\tau_y/\sqrt{2\pi})^{\frac{1}{n}}\right)^n e_{ij}, \quad |\tau_{ij}| > \tau_y \quad (1)$$

$$\text{if} \quad |\tau_{ij}| < \tau_y \quad \text{then} \quad \pi = 0, \quad \text{there is no flow}$$

where μ_e is plastic dynamic viscosity of the Casson fluid, τ_y is the yield stress of fluid, π is the product of the component of deformation rate with itself, namely, $\pi = e_{ij}e_{ij}$, e_{ij} is the (i, j)-th component of the deformation rate. For $n = 2$ we have the simple model for Casson fluid. In this paper we adopt the value $n = 1$ as used in [15],[16],[18]. The governing equations in this flow are given as;

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \quad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{w^2}{x} = \nu(1 + \frac{1}{\beta})\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K}u \quad (3)$$
$$+ g\beta_T(T - T_\infty)\cos\gamma + g\beta_C(C - C_\infty)\cos\theta^*$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + \frac{uw}{x} = \nu(1 + \frac{1}{\beta})\frac{\partial^2 w}{\partial y^2} - \frac{\nu}{K}w \quad (4)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \bar{D}\frac{\partial^2 C}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}$$
$$+ \frac{\nu}{\rho C_p}(1 + \frac{1}{\beta})\left(\frac{\partial u}{\partial y}\right)^2 \quad (5)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \bar{S}\frac{\partial^2 T}{\partial y^2} \quad (6)$$

Where the radius of the cone $r = x\sin\theta^*$, ν is kinematic viscosity of Casson fluid, $\beta = \mu_B\sqrt{2\pi_c}/P_y$ is the non-Newtonian Casson parameter. $\alpha = k/\rho C_p$ is the thermal diffusivity, k is thermal conductivity of the fluid, q_r is the radiative heat flux, C_p is the specific heat. g is the acceleration due to gravity, β_T and β_C are respectively the coefficients of thermal and concentration expansions, T is the temperature of the fluid, C is the solute concentration in the boundary layer, D is the mass diffusivity, $\bar{S}$ and $\bar{D}$ are Soret and Dufour coefficients respectively.

The Rosseland approximation for radiation may be written as follows;

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

where σ is the Stefan-Boltzmann constant and k^* is the absorption coefficient. If the temperature difference within the flow is such that T^4 may be expanded in Taylor series about T_∞ and neglecting higher powers we obtain $T^4 - 4T_\infty^3 - 3T_\infty^4$ and therefore the equation (5) can be written as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \bar{D}\frac{\partial^2 C}{\partial y^2} + \frac{16\sigma T_\infty^3}{3\rho C_p k^*}\frac{\partial^2 T}{\partial y^2}$$

The boundary conditions are given as

$$\begin{aligned} &u = N_0(1+\frac{1}{\beta})\frac{\partial u}{\partial y},\ v = -v_a,\\ &w = r\Omega + R_0(1+\frac{1}{\beta})\frac{\partial w}{\partial y}, T = T_a + K_0\frac{\partial T}{\partial Y},\\ &C = C_a + F_0\frac{\partial C}{\partial Y}, \text{at } y = 0, \end{aligned} \tag{8}$$

$$u, w \to 0,\ T \to T_\infty,\ C \to C_\infty \text{ as } y \to \infty. \tag{9}$$

where the subscripts a and ∞ refer to surface the free stream conditions respectively. We introduce the non-dimensional variables

$$\begin{aligned} &(X, Y, R) = \left(\frac{x, yGr^{\frac{1}{4}}, r}{L}\right),\ (U, V) = \left(\frac{u, vGr^{\frac{1}{4}}}{U_0}\right),\\ &W = \frac{w}{\Omega L},\ \bar{T} = \frac{T - T_\infty}{T_a - T_\infty}, \bar{C} = \frac{C - C_\infty}{C_a - C_\infty},\ Da = \frac{K}{L^2},\\ &U_0 = [g\beta_T(T_a - T_\infty)\cos\psi L]^{\frac{1}{2}},\ Gr = (\frac{U_0 L}{\nu})^2 \end{aligned} \tag{10}$$

The governing equations (2)-(6) reduce to

$$\frac{\partial}{\partial X}(RU) + \frac{\partial}{\partial Y}(RV) = 0, \tag{11}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} - \frac{Re^2}{Gr}\frac{W^2}{X} = \left(1+\frac{1}{\beta}\right)\frac{\partial^2 U}{\partial Y^2} + \bar{T} + N\bar{C} - \left(\frac{1}{DaGr^{\frac{1}{2}}}\right)U, \tag{12}$$

$$U\frac{\partial W}{\partial X} + V\frac{\partial W}{\partial Y} + \frac{UW}{X} = \left(1+\frac{1}{\beta}\right)\frac{\partial^2 W}{\partial Y^2} - \left(\frac{1}{DaGr^{\frac{1}{2}}}\right)W, \tag{13}$$

$$U\frac{\partial \bar{T}}{\partial X} + V\frac{\partial \bar{T}}{\partial Y} = \frac{1}{Pr}\left(\frac{\partial^2 \bar{T}}{\partial Y^2} + D_f\frac{\partial^2 \bar{C}}{\partial Y^2}\right) + \frac{4K}{3Pr}\frac{\partial^2 T}{\partial Y^2} + Ec\left(1+\frac{1}{\beta}\right)\left(\frac{\partial U}{\partial Y}\right)^2, \tag{14}$$

$$U\frac{\partial \bar{C}}{\partial X} + V\frac{\partial \bar{C}}{\partial Y} = \frac{1}{Sc}\left(\frac{\partial^2 \bar{C}}{\partial Y^2} + S_r\frac{\partial^2 \bar{T}}{\partial Y^2}\right). \tag{15}$$

The non-dimensional parameters in equations (11)-(15) are the rotational Reynolds number Re, Grashof number Gr, the Darcy number Da, the Prandtl number Pr, Dufour parameter D_f, the Eckert number Ec, the Schmidt number Sc and Soret parameter S_r. These parameters are defined as

$$\begin{aligned} &Re = \frac{\Omega L^2}{\nu}, \quad N = \frac{\beta_C}{\beta_T}\left(\frac{C_a - C_\infty}{T_a - T_\infty}\right), K = \frac{4\sigma^* T_\infty^3}{k^* Gr^{\frac{1}{4}}}\\ &Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D}, \quad Ec = \frac{U_0^2}{C_p(T_{wa} - T_\infty)},\\ &D_f = \frac{\bar{D}}{\alpha}\left(\frac{C_a - C_\infty}{T_a - T_\infty}\right), \quad S_r = \frac{\bar{S}}{D}\left(\frac{T_a - T_\infty}{C_a - C_\infty}\right). \end{aligned}$$

The boundary conditions of system (11)-(15) are given by

$$U = S_f\left(1+\frac{1}{\beta}\right)\frac{\partial U}{\partial Y}, \quad V = V_w, \tag{16}$$

$$\begin{aligned} &W = R + S_g\left(1+\frac{1}{\beta}\right)\frac{\partial W}{\partial Y},\ \bar{T} = 1 + S_T\frac{\partial \bar{T}}{\partial Y},\\ &\bar{C} = 1 + S_{co}\frac{\partial \bar{C}}{\partial Y} \quad \text{at} \quad Y = 0 \end{aligned} \tag{17}$$

$$U, W \to 0,\ \bar{T} \to 0, \bar{C} \to 0, \text{ as } Y \to \infty. \tag{18}$$

Introducing the stream function $\psi(X, Y)$ and similarity variables

$$U = \frac{1}{R}\frac{\partial \psi}{\partial Y} \quad \text{and} \quad V = -\frac{1}{R}\frac{\partial \psi}{\partial X} \tag{19}$$

$$\begin{aligned} &\psi(X, Y) = XRf(Y), \quad W(X, Y) = Rg(Y),\\ &\bar{T}(X, Y) = X\theta(Y), \quad \bar{C}(X, Y) = X\phi(Y). \end{aligned} \tag{20}$$

using the stream function defined in (19) and similarity variable in (20), Eqs. (11) together with boundary conditions (16) reduces to the following system of ordinary differential equations.

$$\left(1+\frac{1}{\beta}\right)f'''+2ff''-f'^2+\epsilon g^2+\theta+N\phi-k_pf'=0,\quad(21)$$

$$\left(1+\frac{1}{\beta}\right)g''+2fg'-2f'g-k_pg=0,\quad(22)$$

$$(1+\frac{4}{3}K)\theta''+Pr(2f\theta'-f'\theta)+D_f\phi''+EcPr\left(1+\frac{1}{\beta}\right)f''^2=0,\quad(23)$$

$$\phi''+Sc(2f\phi'-f'\phi)+S_r\theta''=0\quad(24)$$

with boundary conditions;

$$Y=0,\quad f=f_a,\quad f'=\left(1+\frac{1}{\beta}\right)S_ff'',\quad g=1+\left(1+\frac{1}{\beta}\right)S_gg',\theta=1+S_T\theta',\quad \phi=1+S_{co}\phi',\quad(25)$$

$$Y\to\infty,f'\to0,g\to0,\theta\to0.\phi\to0.\quad(26)$$

Where $k_p=1/DaGr^{\frac{1}{2}}$ is the Darcian-drag force term, β is the Casson parameter and ϵ is the spin parameter. In the above equations the primes refer to the derivative with respect to Y, $S_f=N_0Gr^{\frac{1}{4}}/L$, $S_g=M_0Gr^{\frac{1}{4}}/L$, $S_T=kGr^{\frac{1}{4}}/L$ and $S_{co}=F_0Gr^{\frac{1}{4}}/L$ are the non-dimensional velocity, rotational, thermal and solutal slip parameters respectively. $S_f=S_g=S_T=S_{co}=0$, corresponds to no-slip conditions. The parameter f_a is the blowing/suction parameter. The case $f_a<0$ represents blowing and $f_a>0$ represents suction. The engineering parameters of interest are the local skin friction coefficient and the local Nusselt number which are defined as follows.

The shear stress at the surface of the cone is given by

$$\tau_a=\frac{\mu\left(1+\frac{1}{\beta}\right)U_0}{LGr^{-\frac{1}{4}}}Xf''(0)\quad(27)$$

where μ is the coefficient of viscosity, the skin friction coefficient is given by

$$C_f=\frac{\tau_a}{\frac{1}{2}\rho U_0^2}\quad(28)$$

Using Eqs.(27) and (28) gives

$$C_fGr^{\frac{1}{4}}=2(1+\frac{1}{\beta})Xf''(0).\quad(29)$$

The heat transfer from the cone surface into the fluid is given by

$$q_a=\frac{-k(T_a-T_\infty)}{LGr^{-\frac{1}{4}}}X\theta'(0),\quad(30)$$

k is the thermal conductivity of the fluid, The Nusselt number under LST is given by

$$Nu=\frac{L}{k}\frac{q_a}{T_a-T_\infty}\quad(31)$$

Eqs.(30) and (31) together with Eqns. (19) and (20) give

$$NuGr^{-\frac{1}{4}}=-X\theta'(0).\quad(32)$$

The mass flux at the cone surface into the fluid is given by

$$J_a=\frac{-D(C_a-C_\infty)}{LGr^{-\frac{1}{4}}}X\phi'(0),\quad(33)$$

The Sherwood number is given by

$$Sh=\frac{L}{D}\frac{J_a}{C_a-C_\infty}\quad(34)$$

Eqs.(33) and (34) together with Eqns. (19) and (20) give

$$ShGr^{-\frac{1}{4}}=-X\phi'(0).\quad(35)$$

3 Method of solution

In this section we present the implementation of the spectral relaxation method for the problem of free convection from a spinning cone with partial slip in Casson fluid in non-Darcy porous medium with cross diffusion and viscous dissipation. The method is described in Motsa et al. [20] and implemented in the works of Shateyi [24], Kameswaran [22] and Motsa and Makukula [21]. The method is based on the Gauss-Seidel method normally used to solve a system of linear equations. The system of equations (21) can be written as a numerical scheme.

$$f'_{r+1}=p_r,\quad(36)$$

$$\left(1+\frac{1}{\beta}\right)p''_{r+1}+2f_{r+1}p'_{r+1}-k_pp_{r+1}=p_r^2-\epsilon g_r^2-\theta-N\phi_r,\quad(37)$$

$$\left(1+\frac{1}{\beta}\right)g''_{r+1}+2f_{r+1}g'_{r+1}-2p_{r+1}g_{r+1}-k_pg_{r+1}=0,\quad(38)$$

$$(1+\frac{4}{3}K)\theta''_{r+1}+Pr(2f_{r+1}\theta'-p_{r+1}\theta_{r+1})=-D_f\phi''_r-EcPr\left(1+\frac{1}{\beta}\right)(p'_{r+1})^2,\quad(39)$$

$$\phi''_{r+1}+2Scf_{r+1}\phi'_{r+1}-ScP_{r+1}\phi_{r+1}=-S_r\theta'(40)$$

with boundary conditions;

$$
\begin{aligned}
& f(0)_{r+1} = f_w,\ p(0)_{r+1} = \left(1+\frac{1}{\beta}\right) S_f p(0)'_{r+1}, \\
& g(0)_{r+1} = \left(1+\frac{1}{\beta}\right) S_g g(0)'_{r+1}, \\
& \theta(0)_{r+1} = 1 + S_T \theta(0)'_{r+1}, \\
& \phi(0)_{r+1} = 1 + S_{co} \phi(0)'_{r+1}, \qquad (41) \\
& p_{r+1}(\infty) \to 0,\ g(\infty)_{r+1} \to 0, \\
& \theta(\infty)_{r+1} \to 0,\ \phi(\infty)_{r+1} \to 0. \qquad (42)
\end{aligned}
$$

Applying the Chebyshev pseudo spectral method on (36) - (42) as in Shateyi and Marewo [23]

3.1 Improving the convergence of the spectral relaxation method

In this section we use the concept of successive over-relaxation (SOR) to accelerate the convergence rate of the spectral relaxation method (SRM). If the general SRM scheme is given by equations,

$$\mathbf{A}\mathbf{f}_{r+1} = \mathbf{B}. \qquad (43)$$

then the modified SRM scheme is defined as

$$\mathbf{A}\mathbf{f}_{r+1} = (1-\omega)\mathbf{A}\mathbf{f}_r + \omega\mathbf{B}. \qquad (44)$$

Where A_i and B_i are matrices and ω is the convergence controlling parameter. By applying this modified SRM in solving the system (21). Using the values of the controlling parameter $\omega = 0.9$ (accelerates convergence), $\omega = 1$(usual SRM scheme) and $\omega = 1.1$ (slows down convergence). Figure 2 shows the decoupling error E_d against iterations.

4 Results and discussion

In this section we discuss the physics of the problem by studying the effects of the physical parameters on velocity, temperature and concentration profiles. We also study the variation of both skin friction and local Nusselt number with the physical parameters. For validation of the numerical method used in this study, results for the skin friction coefficient $f''(0)$ and heat transfer coefficient $-\theta'(0)$ for the Newtonian fluid were compared to those of Narayana [1] and the Matlab bvp4c, for $1/\beta \to 0, \epsilon = N = K = D_f = S_r = Sc = Ec = 0$ and the Darcian drag force terms $-k_p f' = k_p g = 0$. The comparison is shown in Tables 1 -2 and it is found to be in excellent agreement to five decimal places.

Table 1: Comparison of the values of $f''(0)$ and $-\theta'(0)$ of Narayana [1] with the bvp4c.

Pr	Narayana et al. [1]		Present	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
1	0.68150212	0.63886614	0.68148625	0.63885897
10	0.43327726	1.27552680	0.43327848	1.27552816

Table 2: Comparison of the values of $f''(0)$ and $-\theta'(0)$ of Narayana [1] with the bvp4c

Pr	Narayana et al. [1]		bvp4c	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
1	0.68150212	0.63886614	0.68148334	0.63885473
10	0.43327726	1.27552680	0.43327820	1.27552877

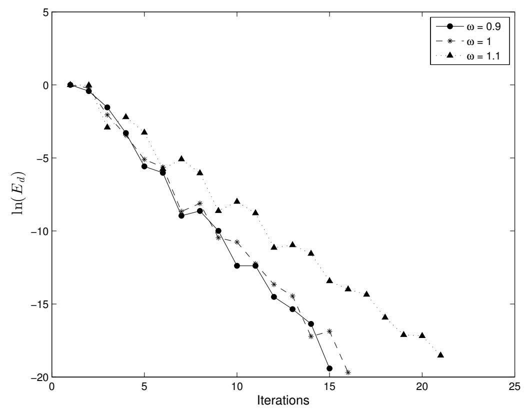

Figure 2: Effects of controlling parameter on decoupling error

In Figure 2, the SRM error reduces with the increasing numer of iterations, showing the accuracy of the spectral relaxation method (SRM). The decrease in the error shows that the method is convergent and give stale solutions. It is also shown that convergence can be controlled obtaining results in a fewer numer of iterations. The results obtained by this method are discussed in table 3 and 4 and are compared with the results obtained using the Matlab `bvp4c` and were found to e in excellent agreement.

The problem of free convection Casson fluid flow from a spinning cone in non-Darcy porous medium with radiation, partial slip, cross diffusion and viscous dissipation effects is solved numerically using the spectral relaxation method (SRM). The results depicted in Table 3 are the results obtained by SRM and `bvp4c`. A tolerance of 10^{-8} for both methods was used. Comparison of the asic SRM ($\omega = 1$) of the skin friction coefficient against those SRM with successive over relaxation (SOR) ($\omega = 0.9$) and ($\omega = 1.1$). The advantage of accelerating convergence is noted in all cases in which the results are obtained accurately in fewer iterations (see also Shateyi and Marewo [24]). The values are generated at selected values of the Darcian-drag force term k_p, the Prandtl number Pr, and the Casson parameter β. Increasing k_p and Pr decreases the skin friction coefficient while increasing the Casson parameter increase skin friction coefficient. In Table 4, the heat transfer coefficient decrease with decreasing Darcian-drag force term k_p, ut increases with increasing both Prandtl, Pr and Casson parameter β.

Table 3: Comparison of SRM solutions of the skin friction coefficient $f''(0)$ against those of bvp4c.

k_p Pr β	SRM(basic) $(\omega = 1) it$	it	SRM(SOR) $(\omega = 0.9) f''(0)$	bvp4c f"(0)
0 1 2	100	47	0.68148334	0.68148334
1 1 2	66	40	0.55974072	0.55974072
2 1 2	46	35	0.48675875	0.48675875
3 1 2	40	33	0.43677770	0.43677770
1 7 2	50	35	0.40562674	0.40562674
1 8 2	48	33	0.39565072	0.39565072
1 9 2	48	33	0.38695722	0.38695722
1 1 2	61	38	0.35629327	0.35629327
1 1 5	62	37	0.43037422	0.43037422
1 1 9	63	39	0.49760348	0.49760348

Table 4: Comparison of SRM solutions of the heat transfer coefficient $\theta'(0)$ against those of bvp4c..

k_p Pr β	SRM(basic) $(\omega = 1) it$	it	SRM(SOR) $(\omega = 0.9) f''(0)$	bvp4c f"(0)
0 1 2	91	36	0.59446782	0.59446782
1 1 2	62	37	0.52386360	0.52386360
2 1 2	48	36	0.47594764	0.47594764
3 1 2	42	33	0.44000560	0.44000560
1 7 2	50	33	0.96287011	0.96287011
1 8 2	50	33	1.00136750	1.00136750
1 9 2	49	32	1.03638459	1.03638459
1 1 2	61	37	0.52386360	0.52386360
1 1 5	62	39	0.54308263	0.54308263
1 1 9	63	40	0.54968947	0.54968947

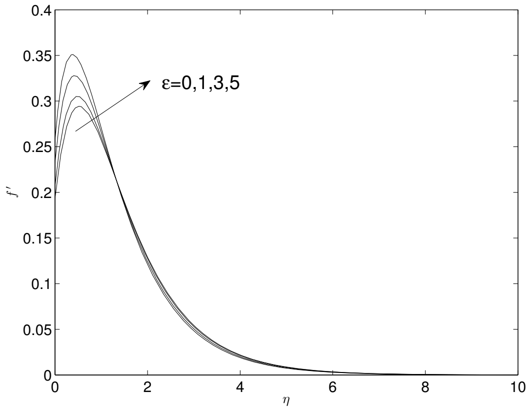

Figure 3: Effects of spin parameter ϵ on velocity profiles

We study the behaviour of velocity, temperature and concentration profiles of free convection Casson fluid from a spinning cone emedded in porous medium with viscous dissipation, a detailed numerical calculation is done for different parameter values that descrie the flow and the results are shown as graphs in Figures 3-12. In this study we do not study the effects of Darcy number, Grashof number, Prandtl number, Schmidt number and suction/blowing parameters on velocity, temperature and concentration profiles. These results are well-known and shown in among others Ramachandra et al. [18], Shateyi [24], Narayana et al. [1] and Ece [3]. Figure 3 shows the influence of the spin parameter ϵ on velocity profiles. Increasing the spin parameter assist the flow clse to the surface of the cone. The velocity profiles are situated close to the left side of the graph due to the presence of the suction effect. A reverse effect is noted further away from the surface, this is due to the coriolis effect created y rotation. This effect cause the shrinking of the oundary layer thickness.

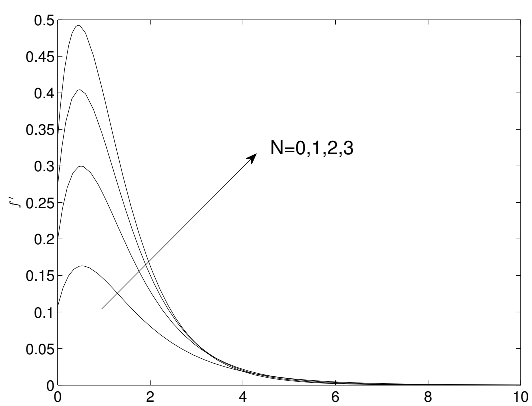

Figure 4: Effects of buoyancy parameter N on velocity profiles

Figure 4 shows the effect of buoyancy parameter N on velocity profiles. Increasing the buoyancy parameter assist the flow. The surface of the cone is heated under linear heat flux causing the fluid close to the surface to rise assisting motion. This effect is less enhanced further away from the surface of the cone.

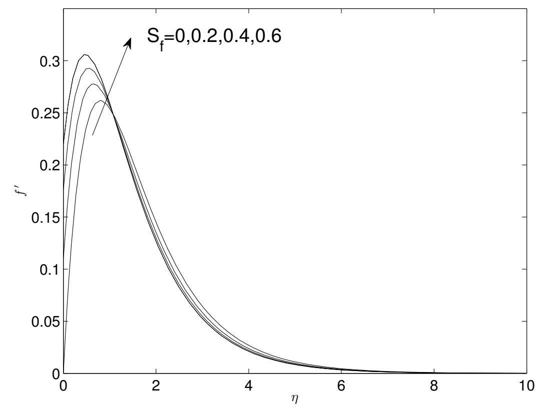

Figure 5: Effects of velocity slip factor S_f on velocity profiles

Figure 5 shows the effect of velocity slip factor S_f on velocity profiles. Increasing the velocity slip factor assist the flow, this is characteristic of a luricated surface. The case $S_f = 0$ referred to as the no-slip condition shows a velocity profile similar to that reported in Ece [3].

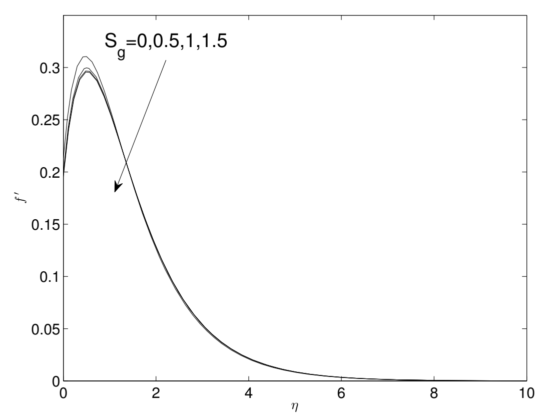

Figure 6: Effects of Rotational slip factor S_g on velocity profiles

Figure 6 shows the influence of rotational slip factor S_g on velocity profiles. Increasing the rotational slip factor reduce velocity profiles. A slip in the perpendicular direction of flow reduce motion in the direction of flow. This effect reduce the momentum oundary layer thickness.

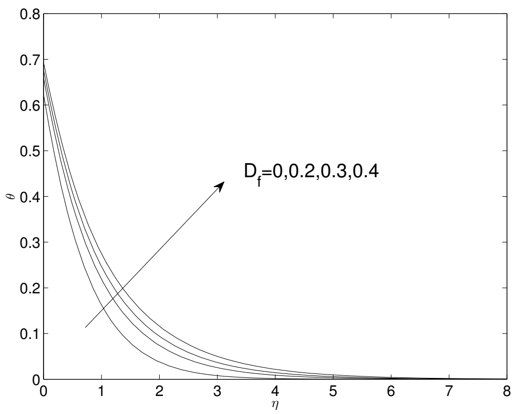

Figure 7: Effects of Dufour parameter D_f on temperature profiles

Figure 7 depict the effect of Dufour number D_f on temperature profiles. Increasing the Dufour numer increase temperature profiles. Increasing the Dufour number is interpreted as increasing the concentration gradient therey causing increase in thermal transport. This effect is called diffusion-thermo. Movement of soulte particles facilitate thermal transport.

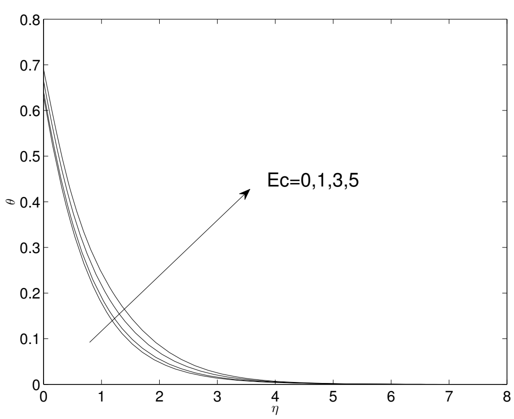

Figure 8: Effects of Eckert number Ec on temperature profiles

Figure 8 shows the effect of Eckert number Ec on temperature profiles. Increasing the Eckert number increase the temperature profiles, this caused by the heat generated within the fluid thereby increasing the temperature of the fluid.

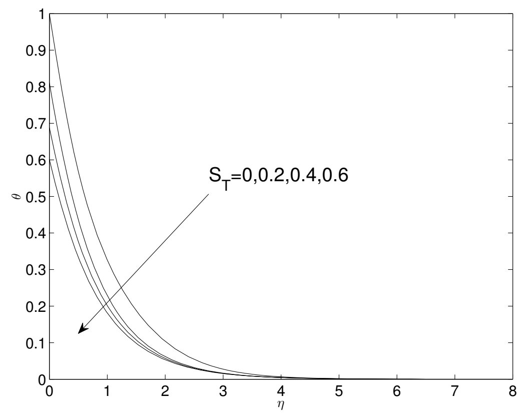

Figure 9: Effects of thermal slip factor S_T on temperature profiles

Figure 9 shows the effect of thermal slip factor S_T on temperature profiles. Increasing the thermal slip factor decrease temperature profiles. Thermal slip is associated with sudden temperature drop at the surface of the cone thereby decreasing the thermal boundary layer.

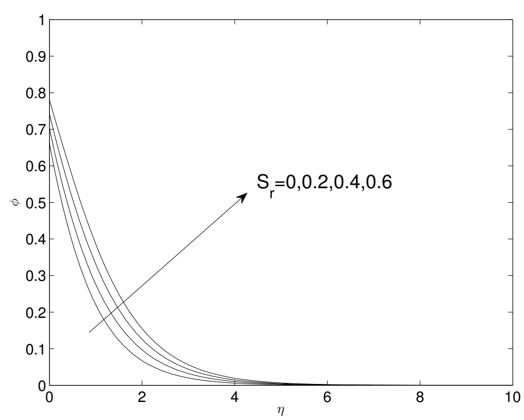

Figure 10: Effects of Soret parameter S_r on concentration profiles

Figure 10 shows the effect of Soret number Sr on concentration profiles. Increasing the Soret number in interpreted as increasing the temperature gradient thereby facilitating solutal transport. This effect is called thermal-diffusion or thermophoresis effect. The thermophoretic force drive soulteparticles into the oundary layer region causing the increase in the concentration profiles.

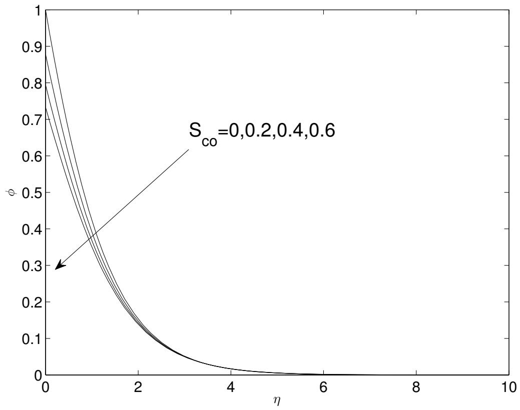

Figure 11: Effects of Solutal slip factor S_T on concentration profiles

Figure 11 shows the effect of solutal slip factor S_T on concentration profiles. Increasing the solutal slip factor decrease concentration profiles. Solutal slip is associated with siddent drop of solute at the surface of the cone.

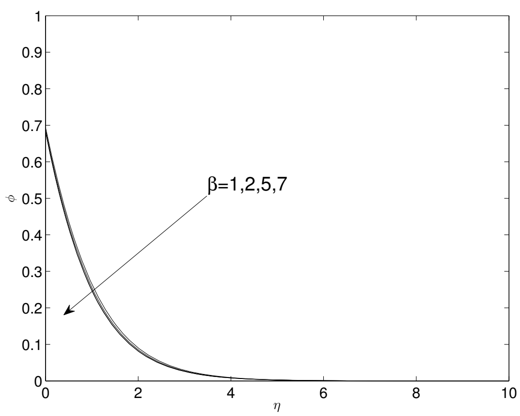

Figure 12: Effects of Solutal slip factor S_T on concentration profiles

Figure 12 shows the effect of Casson parameter β on concentration profiles. Increasing the Casson parameter decrease concentration profiles. The increase in the Casson parameter increase velocity profiles and reduce concentration in the oundary layer.

5 Conclusion

The investigation presented in this analysis of effects of radiation on free convection from a spinning cone with partial slip in Casson fluid in non-Darcy porous medium with cross diffusion and viscous dissipation provides numerical solutions for the boundary velocity, heat and mass transfer. The coupled nonlinear governing differential equations were solved using the spectral relaxation method (SRM). The interesting results in this work are the consideration of partial slip factores and the effect of spinning.

1. It is generally observed that increasing each of the slip factors S_f, S_T, S_{co} tend to assist velocity, temperature and concentration profiles respectively.

2. The reverse effects noted in the boundary layer for oth velocity and concentration profiles are caused y the presence of the combination of suction, slip factors and spinning effects.

3. The spectral relaxation method (SRM) can e used as an alternative method for solving oundary value problems.

4. The advantage of the SRM is that it can be controlled to otain accurate solutions in fewer iterations.

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