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Dynamic IS-LM-AS Model Augmented by the Inflation Expectations

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Abstract: The paper investigates a dynamic IS-LM-AS model augmented by the expectations. The money and the potential product grow by the exogenous rates. In general, the inflation is adjusted to the production gap and the inflation expectations are adaptive or rational. We will find the stability conditions. There is a possibility of the cyclical movement of the economic variables. The results are similar to ones generated by the Sidrauski model of the monetary growth – money is super-neutral in the steady state. Using the model, it is possible to demonstrate the various views of the different economic schools as the Keynesian school, the Classical School, the Monetary School, the Neo-Classical Synthesis, the New Classical School and the New Keynesian School.

Key-Words: adaptive and rational expectations, inflation adjustment, stability conditions, economic schools.

1 Introduction

There have been conflicts of the economic views for the decades. We tried to create a scheme on the basis of the IS-LM-AS model that could help to better understand the different economic schools using the similar logic as Heijdra and van der Ploeg (2002). The scheme provides a possibility to analyze the closed economy under the different assumptions. The economic school may be created by certain combination of such assumptions. Our model helps to focus on different views in the monetary policy.

The brief literature overview summarizes the baseline papers of the different economic schools in the second part. We define and solve the model in the third and the fourth parts. In the fifth part we analyze the influence of the different assumptions on the solution. Mostly we use the adaptive expectations approach. Special assumptions are new classical, where we remove this approach by the rational expectations. The interpretations of the different economic views are in the conclusions.

2 Brief Literature Overview

Hicks (1937) found the IS-LM model that expresses the main relationships of Keynes's theory (1936). The model has been used by a lot of economists till today. The model failed, however, in explanation of the inflation or the supply shocks. This led to some modifications of the model by appending Phillips Curve developed by Phillips (1958) and Lipsey (1960) and later expectations-augmented Phillips Curve developed by Phelps (1968) to define IS-LM-AS model augmented by expectations. The aggregate goods and services demand is function of real interest rate while the aggregate money demand is function of nominal interest rate. We can therefore augment the LM curve by expectations to define IS-ALM as show Baily and Friedman (1994) in their book.

We can form the expectations variously. The simplest ways are exogenous or adaptive expectations. Muth (1961) introduced the conception of the rational expectations. This way modelling expectations became influential when it was used by Lucas (1976) developing his critique. Lucas argued that functions such as IS and LM curves are not in principle invariant to changes in economic policy rules and therefore cannot be used for policy evaluation.

The single model based on Sargent and Wallace (1975) can represent the basic idea. Since our model is continuous, we need to form the rational expectations in continuous time. Thus, we need to introduce jump variables. As Turnovsky (1997, p. 69) states, the ordinary economic dynamic model represents traditional approach. The economic dynamic variables evolve from the past and move continuously (sluggish variables). However, some economic variables are free to move discontinuously at appropriate points (jump variables). Sargent and Wallace (1973) first introduced the idea that the price level (and inflation) need not to be sluggish variable.

Romer (2000) suggested the IS-MP model by removing the LM curve by the monetary policy rule according to the approach of Taylor (1993). Central banks can control the real interest rates and their movement depends on inflation and national income. Another modification comes from the efforts to add in the IS-LM model micro-foundations that are in line with the New Classical School. An example of such modification made McCallum and Nelson (1999). The history and the evolution of the IS-LM model are in detail discussed in Vercelli (1999). Various dynamic IS-LM models are presented in books focused on economic dynamics as Shone (2003). The dynamics of the model is mostly given by market adjustment or expectations. Blanchard (1981) extended the IS-LM analysis to allow stock market behavior employing the Tobin-Blanchard model. Our model uses IS-LM relations, the Lucas aggregate supply (augmented Phillips Curve) and adaptive or rational expectations formulas.

3 Model

Consider the model, all variables are in logarithm:

$$m = p + l\left[y(r), r + \pi^e\right] \tag{1}$$

$$\pi = \pi^e + \alpha\left[y(r) - y^p\right] \tag{2}$$

$$\dot{\pi}^e = \beta\left(\pi - \pi^e\right) \tag{3}$$

The relation (1) expresses the money market equilibrium. The real money supply, $(m - p)$ has to equal to the money demand, l. The money demand is given by a continuous differentiable function of the real output and the nominal interest rate, $r + \pi^e$. The real output is given by a decreasing continuous differentiable IS function of the real interest rate, r. Nominal interest rate is sum of the real interest rate and the inflation expectations. To keep algebra simple, we assume that the elasticities are constant and we will denote the elasticity of the influence of the change of the real interest rate on the real output as $y_r < 0$, the elasticity of the influence of the change of the real output on the transaction money demand as $l_y > 0$, and the elasticity of the influence of the change of the nominal interest rate on the speculative money demand as $l_r < 0$.

The relation (2) expresses the Lucas aggregate supply (augmented Phillips Curve) with the parameter $\alpha > 0$. We denote the potential product as y^p. Finally, the equation (3) is the adaptive expectation formula with the parameter $\beta > 0$.

We assume that the real output and the real money stock grow by the constant rates n, and μ:

$$y^p = y_0^p + nt \tag{4}$$

$$m = m_0 + \mu t \tag{5}$$

where y_0^p, $m_0 > 0$ are the constant initial values of the real output and real money stock and symbol t denotes the continuous time. We will refer the equations (4) and (5) as the real and the monetary growth. The relation between the inflation and the price level is given by:

$$\pi = \dot{p} \tag{6}$$

3.1 Real Growth

Let us express the inflation expectations from the aggregate supply (2) and substitute the real growth (4):

$$\pi^e = \pi - \alpha\left[y(r) - y_0^p - nt\right] \tag{7}$$

Substituting of (7) and its first derivation by time to the adaptive expectations formula (3) we will get:

$$\dot{\pi} - \alpha\left(y_r\dot{r} - n\right) = \beta\left\{\pi - \pi + \alpha\left[y(r) - y_0^p - nt\right]\right\}$$

After modifications we will get the linear differential equation with constant coefficients expressing the dynamics caused by the real growth in the form:

$$\dot{\pi} - \alpha y_r\dot{r} - \alpha\beta y(r) = -\alpha\left[n + \beta\left(y_0^p + nt\right)\right] \tag{8}$$

3.2 Monetary Growth

Let us substitute the monetary growth (5) and inflation (6) into the money market relation (1) and express the inflation expectations in the form:

$$\dot{\pi}^e = \frac{\mu - \pi - \left(l_y y_r + l_r\right)\dot{r}}{l_r} \tag{9}$$

Substituting the inflation expectations (9) and its first derivation by time into the first derivation of the adaptive expectations (3) by time and multiplying by the speculative money demand elasticity l_r, we will get:

$$-\dot{\pi} - \left(l_y y_r + l_r\right)\ddot{r} = \beta\left[l_r\dot{\pi} - \mu + \pi + \left(l_y y_r + l_r\right)\dot{r}\right]$$

Rearranging terms we will get another linear differential equation with constant coefficients expressing the dynamics caused by the monetary growth in the form:

$$-\left(1 + \beta l_r\right)\dot{\pi} - \beta\pi - \left(l_y y_r + l_r\right)\ddot{r} - \beta\left(l_y y_r + l_r\right)\dot{r} = -\beta\mu \tag{10}$$

4 Solution

The equations (8) and (10) form the simultaneous system of the linear differential equations with the constant coefficients. We will solve the system by expressing the stability and the oscillatory conditions, and the steady state. In order to find the stability and oscillatory conditions we need to express the homogeneous system.

4.1 Homogenous System

We can write the homogeneous system to the system (8) and (10) as:

$$\left(1+\beta l_r\right)\dot{\pi}+\beta\pi+\left(l_y y_r+l_r\right)\ddot{r}+\beta\left(l_y y_r+l_r\right)\dot{r}=0$$

$$\ddot{\pi}-\alpha y_r\ddot{r}-\alpha\beta y_r\dot{r}=0$$

The solution of the homogeneous system is in the form: $\pi = a_1e^{\lambda t}$ and $r = a_2e^{\lambda t}$, where a_1 and a_2 are non-zero constants. Substituting the solution into the homogeneous system we will get:

$$\left(1+\beta l_r\right)\lambda a_1e^{\lambda t}+\beta a_1e^{\lambda t}+\left(l_y y_r+l_r\right)\lambda^2 a_2e^{\lambda t}+$$
$$+\beta\left(l_y y_r+l_r\right)\lambda a_2e^{\lambda t}=0$$

$$\lambda^2 a_1e^{\lambda t}-\alpha y_r\lambda^2 a_2e^{\lambda t}-\alpha\beta y_r\lambda a_2e^{\lambda t}=0$$

after modifications in the matrix form:

$$\mathbf{A}\mathbf{a}e^{\lambda t}=\mathbf{0} \tag{11}$$

where

$$\mathbf{A}=\begin{bmatrix}\lambda\left(1+\beta l_r\right)+\beta & \lambda\left(l_y y_r+l_r\right)\left(\lambda+\beta\right)\\ \lambda & -\alpha y_r\left(\lambda+\beta\right)\end{bmatrix}\mathbf{a}=\begin{pmatrix}a_1\\a_2\end{pmatrix}\mathbf{0}=\begin{pmatrix}0\\0\end{pmatrix}$$

In general $e^{\lambda t}$ is non-zero and so we can write the system (11) in the form:

$$\mathbf{A}\mathbf{a}=\mathbf{0} \tag{12}$$

The system (12) has a non-trivial solution (i.e. $\mathbf{a} \neq \mathbf{0}$) if and only if:

$$\left|\mathbf{A}\right|=0 \tag{13}$$

The equation (13) is the characteristic equation of the homogeneous system corresponding to the (8) and (10). We can rewrite it in the form:

$$-\left(\lambda+\beta\right)\left[\left(l_y y_r+l_r\right)\lambda^2+\left(1+\beta l_r\right)\alpha y_r\lambda+\alpha\beta y_r\right]=0 \tag{14}$$

4.2 Stability Condition

The general solution of the homogeneous system converges to zero if all roots of the characteristic equation (14) are negative. One of the roots of the characteristic equation (14) is $\lambda_1 = -\beta < 0$, the others we can get by putting the term in the square brackets of the (14) equal to zero and solving for λ. Since $l_yy_r + l_r < 0$ and $\alpha\beta y_r < 0$, applying the Descarte's theorem, the stability condition is:

$$\beta<-\frac{1}{l_r} \tag{15}$$

The system is therefore stable, if the adaptive expectations parameter is less than the negative inverted value of the speculative money demand elasticity.

4.3 Oscillatory Movement

Both the inflation and the real interest rate oscillate if there are complex conjugate roots of the characteristic equation (14). One of the root of the characteristic equation (14) is real $\lambda_1 = -\beta < 0$, the others we can get by putting the term in the square brackets of (14). The roots are complex conjugate if and only if the discriminant of this equation is negative:

$$\left[\left(1+\beta l_r\right)\alpha y_r\right]^2-4\left(l_y y_r+l_r\right)\alpha\beta y_r<0 \tag{16}$$

Solving the inequality (16), if $\alpha > 0$, we will get the oscillatory conditions in the form:

$$\left[\alpha<\frac{4\left(l_y y_r+l_r\right)\beta}{\left(1+\beta l_r\right)^2 y_r}\wedge\beta\neq-\frac{1}{l_r}\right]\vee\left(\beta=-\frac{1}{l_r}\right) \tag{17}$$

Note that if $\beta = -1/l_r$, the system oscillates with constant amplitudes.

4.4 Steady State

We define the steady state by the condition:

$$\dot{\pi}^{e,SS}=0 \tag{18}$$

The index *SS* denotes the equilibrium value of the variable. Substituting the steady state condition (18) into the adaptive expectations formula (3) we will get the perfect expectations condition:

$$\pi^{SS}=\pi^{e,SS} \tag{19}$$

It follows from (18) and (19) that both the steady state inflation and its expectations do not vary. Combining the steady state, perfect expectations (19), Lucas aggregate supply (2) and the real growth (4) we will get the condition:

$$y^{SS}=y^{p,SS}=y_0^p+nt \tag{20}$$

The real output equals to the potential product, or $\dot{y}^{SS}=n$, the real output grows by the real rate n in the steady state. The real output in the steady state influences the real interest rate according to the negative sloped IS curve $\dot{y}^{SS}=y_r\dot{r}^{SS}$. We will get the real interest rate movement in the steady state by substituting the real growth into the IS curve:

$$\dot{r}^{SS}=\frac{n}{y_r} \tag{21}$$

The real interest rate decreases as real output grows in the steady state. The nominal interest rate decreases as well as real:

$$\dot{r}^{SS}+\dot{\pi}^{e,SS}=\frac{n}{y_r} \tag{22}$$

as inflation expectations are constant in the steady state.

Relations (18), (19), (20) (or $\dot{y}^{SS}=n$), (21) and (22) express the steady state solution of the system

(8) and (10). We can get the same solution by substituting of the conditions $\ddot{\pi} = \dot{\pi} = \ddot{r} = 0$ into the system (8) (its first derivation by time) and (10). It is clear from the steady state solution (20), (21) and (22) that the money super-neutrality theorem holds. The real variables are in the steady state independent on the monetary growth. The hypothesis of the super-neutrality of money presented Walsh (2003) using Sidrauski (1967) model.

5 Sensitivity Analysis

Modifying some assumptions, our model can serve as a demonstration of the different economic schools in the similar fashion as Heijdra and Ploeg (2002, sect. 1.3) provide, using the static IS-LM model. In this part we express the influence of changing some assumptions on the steady state, stability and oscillations. We focus on the most controversial prerequisites as the price elasticity, expectations and the speculative money demand.

5.1 Prices of the Goods and Services

If the aggregate supply parameter vanish, $\alpha = 0$, we assume that inflation is constant. It is clear from the relation (8) that $\dot{\pi} = 0$. The system dynamics is then caused only by the monetary growth. If we substitute this condition into (10), we will get the linear first order differential equation with the constant coefficients expressing the dynamics given by the monetary growth:

$$\ddot{r} + \beta\dot{r} = \frac{\beta}{l_y y_r + l_r}(\mu - \pi) \tag{23}$$

The solution of the equation (23) is:

$$\dot{r} = (\dot{r}_0 - \dot{r}^{SS})e^{-\beta t} + \dot{r}^{SS} \tag{24}$$

The inferior index 0 denotes the initial value and superior *SS* denotes the steady state value of the variable. The steady state is given by:

$$\dot{r}^{SS} = \frac{\mu - \pi}{l_y y_r + l_r} \tag{25}$$

We can express the movement of the real output by substituting the steady state (25) into the IS curve:

$$\dot{y} = y_r(\dot{r}_0 - \dot{r}^{SS})e^{-\beta t} + y_r\dot{r}^{SS} \tag{26}$$

The steady state movement of the real output is:

$$\dot{y}^{SS} = \frac{y_r}{l_y y_r + l_r}(\mu - \pi) \tag{27}$$

We can express the *sustainable monetary rule* by substituting the real growth rate *n* by the steady state output growth into the left side of the relation (27) and expressing the monetary growth rate:

$$\mu = \frac{\left(l_y y_r + l_r\right)n}{y_r} + \pi \tag{28}$$

The expression (28) represents the monetary growth rule that a central banker should keep to sustain the real output growth with rate *n*. If we substitute the rule (28) into the relation (25) we will get the sustainable monetary rule in respect of the real interest rate which corresponds to Taylor's (1993) approach. The Taylor rule coincides with the movement of the real interest rate in the steady state (21).

On the contrary, if the prices of the goods and services are perfectly elastic, the aggregate supply parameter is very high, $\alpha \to \infty$. If $\beta \neq -1/l_r$, it is clear from (17), that system moves monotonically.

5.2 Expectations

If households do not vary their expectations, $\beta = 0$. Substituting the constant expectations condition into the relation of the dynamics caused by the real growth (8) we will get:

$$\dot{\pi} - \alpha y_r\dot{r} = -\alpha n \tag{29}$$

Substituting the condition of the constant expectations into the adaptive expectations formula (3) we will get, $\dot{\pi}^e = 0$. Substituting (29) into the first derivation of the money market relation (1) by time we will get:

$$\pi + \left(l_y y_r + l_r\right)\dot{r} = \mu \tag{30}$$

The equation (30) is not differential; if households do not vary their expectations, the monetary growth causes no dynamics. Expressing the change of the real interest rate from the relation (30) and its substituting into the (29), we will get the linear first order differential equation with the constant coefficients expressing the dynamics of the inflation caused by the real growth:

$$\dot{\pi} + \frac{\alpha y_r}{l_y y_r + l_r}\pi = -\frac{\alpha\left[y_r\mu - \left(l_y y_r + l_r\right)n\right]}{l_y y_r + l_r} \tag{31}$$

The solution of the equation (31) is:

$$\pi = \left(\pi_0 - \pi^{SS}\right)e^{-\frac{\alpha y_r}{l_y y_r + l_r}t} + \pi^{SS} \tag{32}$$

and the steady state inflation rate is:

$$\pi^{SS} = \frac{\left(l_y y_r + l_r\right)}{y_r}n - \mu \tag{33}$$

Substituting the relations (33) and (4) into the aggregate supply (2) we will get the steady state output level:

$$y^{SS} = \frac{\left(l_y y_r + l_r\right)n - y_r\mu - y_r\pi^e + \alpha y_r\left(y_0^p + nt\right)}{y_r\alpha} \quad (34)$$

Applying the first derivation of the (34) by time we will get the real output grows by rate n, $\dot{y}^{SS} = n$ which coincides with the steady state defined in the previous part.

On the contrary, if we assume the *perfect expectations*, $\beta \to \infty$, the stability condition (15) holds only if $l_r = 0$; the oscillatory condition (17) in general does not hold. We can form the perfect expectations case by removing adaptive expectations formula (3) by:

$$\pi^e = \pi \quad (35)$$

Substituting perfect expectations (35) into the Phillips Curve, (2) we see that under perfect expectations hypothesis, the output equals to the potential output:

$$y(r) = y^p \quad (36)$$

Respectively:

$$y_r\dot{r} = n \quad (37)$$

By expressing the change of the real interest rate from the (37) and expected inflation rate from the (35), and by substituting them into the first derivative of the money market relation (1) we reduce the model to the Cagan (1956) model of hyperinflation with perfect expectations:

$$\dot{\pi} + \frac{1}{l_r}\pi = \frac{\mu}{l_r} - \frac{\left(l_y y_r + l_r\right)n}{l_r y_r} \quad (38)$$

The steady state is unstable:

$$\pi^{SS} = \mu - \frac{l_y y_r + l_r}{y_r}n \quad (39)$$

To form the *rational expectations* we consider the price level to be *jump variable* following Sargent and Wallace (1973). We follow the logic of Turnovsky (1997, sect. 3.4).

Consider equation of the Cagan model (38), which we rewrite in the form:

$$\dot{\pi}(t) + \frac{1}{l_r}\pi(t) = \frac{\mu(t)}{l_r} - \frac{\left(l_y y_r + l_r\right)n(t)}{l_r y_r} \quad (40)$$

We relax the assumption of constant monetary and real growth and consider that both $\mu(t)$ and $n(t)$ are generic functions of time for now. Using the calculus of variation procedure (Gandolfo; 1997, sect. 12.2.6) the general solution of (40) is:

$$\begin{aligned}\pi(t) = Ae^{-\frac{1}{l_r}t} &+ \frac{1}{l_r}e^{-\frac{1}{l_r}t}\int_0^t \mu(s)e^{\frac{1}{l_r}s}\,\mathrm{d}t - \\ &- \frac{\left(l_y y_r + l_r\right)}{l_r y_r}e^{-\frac{1}{l_r}s}\int_0^t n(s)e^{\frac{1}{l_r}s}\,\mathrm{d}s\end{aligned} \quad (41)$$

Sargent and Wallace suggested determining the arbitrary constant A by a terminal condition at $t \to \infty$. Let the temporary condition be:

$$\lim_{t\to\infty}\pi(t) = 0 \quad (42)$$

Using this information, we determine the arbitrary constant as:

$$A = \frac{\left(l_y y_r + l_r\right)}{l_r y_r}\int_0^\infty n(s)e^{\frac{1}{l_r}s}\,\mathrm{d}t - \frac{1}{l_r}\int_0^\infty \mu(s)e^{\frac{1}{l_r}s}\,\mathrm{d}s$$

Substituting this value for A into (41) yields the solution:

$$\begin{aligned}\pi(t) = \frac{\left(l_y y_r + l_r\right)}{l_r y_r}e^{-\frac{1}{l_r}t}\int_t^\infty n(s)e^{\frac{1}{l_r}s}\,\mathrm{d}s - \\ -\frac{1}{l_r}e^{-\frac{1}{l_r}t}\int_t^\infty \mu(s)e^{\frac{1}{l_r}s}\,\mathrm{d}s\end{aligned} \quad (43)$$

The crucial thing to observe is that this solution for the current inflation rate is entirely *forward looking*. Substituting constant values for monetary and real growth: $\mu(t) = \mu$ and $n(t) = n$ which formally means that agents correctly (rationally) forecast constant real and monetary growth we obtain the solution in the form:

$$\pi = \mu - \frac{\left(l_y y_r + l_r\right)}{y_r}n \quad (44)$$

The solution coincides with the steady state (39).

5.3 Speculative Money Demand

The dynamics of the system is under assumption of $l_r = 0$ given only by the real growth. We can express it from the second order differential equation with the constant coefficients (8). If $l_r = 0$, we interpret the money market relation (1) as the quantitative theory of the money with constant money velocity, l_y. Let us express from the quantitative theory of the money the first and the second order derivations of the real production by time and substitute them into the first derivation of the dynamics caused by the real growth (8):

$$\ddot{\pi} + \frac{\alpha}{l_y}\dot{\pi} + \frac{\alpha\beta}{l_y}\pi = \alpha\beta\left(\frac{\mu}{l_y} - n\right) \quad (45)$$

The equation (45) is second order linear differential with constant coefficients. The discriminant of its characteristic equation is negative if $\alpha < 4\beta l_y$. This inequality coincides with the oscillatory condition (17) with $l_r = 0$. All coefficients of the characteristic equation are positive – the system is stable. This outcome coincides with the stability condition (15) with $l_r \to 0$. The particular solution of the equation (35) is the steady state $\pi^{SS} = \mu - l_y n$, and coincides with the solution (39) with $l_r = 0$.

If $l_r \to -\infty$, the stability condition (15) does not hold and the oscillatory condition (17) in general does not hold.

6 Conclusion

We demonstrated the model of the closed economy. The model with all its assumption represents the Neo-Classical Synthesis. It follows from the stability condition of our model (15) that the system needs not to be stable. If the oscillatory condition (17) holds, the dynamics of the system given by the real and the monetary growth causes cyclical movement of the economic variables. The unique steady state states that monetary policy has no long-run influence on the real economy in line with the money super-neutrality theorem. Such outcome corresponds to the New Classical Economic School. In the short run, however, it is possible to affect the real variables by the monetary policy. The short run aspect of the monetary policy is the subject of the Keynesian and New Keynesian Economic School.

If we relax or modify any assumption, we will demonstrate the different economic views. It follows from the solution of the equation (23) that if we assume constant inflation, the system monotonically converges towards the steady state. The stability condition (15) holds and the oscillatory condition (17) does not hold, since $\alpha = 0$. The system converges faster the higher is the value of the adaptive expectation parameter β. It follows from the relations (25) and (27) that the real interest rate and real output movements depend on the difference of the monetary growth and the inflation. If the monetary growth exceeds (is less than) the inflation, the real interest rate increases (decreases). The money super-neutrality hypothesis does not hold, since monetary growth affects the real output and the interest rate change. The constant inflation hypothesis is in line with New Keynesian School.

The solution of the equation (31) demonstrates that if population does not vary its expectations the system converges monotonically towards the steady state. The stability condition (15) holds and the oscillatory condition (17) does not hold, since $\beta = 0$. Both the aggregate supply parameter α and the elasticity $l_y y_r + l_r$, affect positively the convergence speed. On the contrary, the elasticity y_r affects the convergence speed negatively. The influence of the inflation caused by an unexpected monetary intervention in the steady state demonstrates the relation (34). The monetary growth affects positively the real output, since the expectations are constant. This statement is crucial in the time-consistency theory of Kydland and Prescott (1977) or Barro and Gordon (1983). The monetary authority has an incentive to unexpectedly increase the inflation. However as the private sector disposes with the perfect information, it knows the problem of a central banker and correctly expects the inflation. The outcome is higher inflation with the constant output on the potential production level. If the monetary growth was non-linear, the first derivation of the money according to the time would not be constant (μ), but it would be a function of the time. In general the first derivation of the term (34) would depend on the monetary policy parameters. The money super-neutrality hypothesis therefore does not hold by the constant expectations assumption.

We form the continuous rational expectations hypothesis by assuming both perfect expectations (35) and terminal condition (42). Turnovsky (1997, p. 70) provides two rationales that may be given for imposing the terminal condition. First, it seems reasonable to suppose that the kind of time paths for monetary growth observed in practice is unlikely to lead an exploding price level. Second, the transversality conditions at infinity are derived from optimizing models. The solution (43) is entirely *forward looking*. The inflation at any time *t* depends upon the discounted future monetary and real growth. Implicit in the solution (43) is the assumption that economic agents know time path for future growths. Using the solution (43) one can demonstrate different monetary and real disturbances in similar way as Turnovsky (1997, p. 71-76) does. In our case (constant monetary and real growth), (44), change in any growth rate leads to immediate change in the inflation rate. The inflation jumps instantaneously to its steady-state level and there are no transitional dynamics. The super-neutrality of the money hypothesis holds.

If we assume the constant inflation, $\alpha = 0$, and the constant expectations, $\beta = 0$, the model is static IS-ALM augmented by the expectations. If we assume, moreover, $\pi^e = 0$, the model is ordinary Hicks IS-LM. The solution of both models is generally known.

The Classical and the Monetarist Economic School come from the quantitative theory of the money. We can specify the Classical School by the conditions, $l_r = 0$, $\alpha \to \infty$ and $\beta \to \infty$. If we remove in our model the conditions $\alpha \to \infty$ and $\beta \to \infty$ by the conditions $y = y^p$ and $\pi = \pi^e$, we would be able to explain the Classical School by the static IS-LM model in the sense of Heijdra of Ploeg (2002). We can demonstrate the Keynesian School using the liquidity trap example by Modigliani (1944). If the interest rate is too low, the speculative demand is highly elastic, $l_r \to \infty$. The potential product exceeds the real product, the prices are therefore fixed and the expectations are constant, $\alpha = \beta = 0$. The New Classical School assumes that expectations are rational.

Completing the discrete version of the model and its comparing with the continuous one could be interesting. Moreover the discrete version of the model should be augmented by the stochastic variables. This approach allows verifying empirically the different economic views using the SVAR techniques. The study of the open economy should be another improvement.

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