The Dual Aspects of Accounting Transaction and the Assets-Claims on Assets Equality in Axiomatic Theory

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Abstract: - The purpose of this study is to analyze the structure of the financial statements´ balance sheet and the dual aspects of accounting transactions from the viewpoint of axiomatic theory showing the relationship between assets and claims on assets. The methodology is rationalistic and analytical; it uses a well-known axiomatic theory to analyzing the balance sheet. The procedure involves a definition of axioms, an application of axiomatic theory, and an analysis of the assets-claims on assets relationship Results show that assets and claims on assets have a set structure and can be analyzed with the axiomatic theory, leading to the conclusion that they are not equal, under the analysis of set equality.

Key-Words: - Dual aspect, accounting transactions, axiomatic method, assets, claims on assets, financial statements.

1 Introduction

This paper addresses the issue of identifying a set structure to the balance sheet and determining the relationship between assets and claims on assets using the axiomatic method.

The axiomatic method has been mainly used to create theories about the entire accounting system. The use of this method in accounting is significant (see [1], [2], [3], [4], [5]) and the analysis of financial statements can also include different types of logics, such as belief, circumscription, paraconsistent logics and dialogic, providing a different perspective on topics such as the accounting equation ([6], [7], [8]). The axiomatic method is appropriate in any science to analyze structures [9].

However, the emphasis on creating entire accounting axiomatic systems led to difficulties in understanding the complex applications of this method. Another approach is to fit an existing axiomatic theory to the accounting system and use the axioms and rules of that theory to test the trustiness of the accounting assumptions. This approach has the advantage of avoiding creating new theories, based on the author preferences; this is the approach used in this paper.

Otherwise, the dual aspects of accounting transactions determine the structure of accounting system. According to this principle, every accounting transaction is recorded in two accounts with different signs in a double classification system [10]. When it is extended to the assets-claims on assets relationship, it becomes a type of accounting assumption and, along with the double-entry bookkeeping system, is crucial to the organization of financial information.

However, other approaches criticize the accounting principles (see [11], [12], [13], [14]); among them, the fair value approach ([15], see [16] for a critic) provides a different view of the dual aspects.

2 Problem Formulation

The purpose of this research is to analyze the structure of the balance sheet and the dual aspects of accounting transactions, in its assumption form, under the axioms and principles of axiomatic theory.

The dual aspect of accounting transactions is a convention to register the credits and debits. This convention is also the foundation of the double-entry bookkeeping system that fully supports the balance sheet.

Justification exists to use the axiomatic method to analyzing an accounting principle; this method is one of the most important components of classical science [9] and provides a logical structure to a subject [17] and a scientific explanation of the basis of any field of knowledge.

The axiomatic method has been used in accounting on many occasions, usually to create a new axiomatic system for the accounting theory and practice. Nevertheless, this paper introduces a major difference to its use; instead of creating a new accounting-specific axiomatic system, as most of the authors do, it takes an existing, well-known, and not accounting-specific axiomatic method to analyzing the structure of the balance sheet and the dual aspects of accounting transaction. The purpose of doing so is to test the use of a solid axiomatic theory in analyzing accounting system structure.

Despite the accounting-specific axiomatic systems are well-defined, and they meet their goals of explaining the assumptions of accounting, they are quite diverse; no matter how good they are, no consensus exists about which one is the most appropriate to axiomatize the accounting principles ([10], [2]). Moreover, they are created to explain the assumptions and practice of accounting, and not to analyze critically them; that results in a variety of theories, depending on the author preferences, which pervades only for a short time. It is why this paper favors fitting a well-known axiomatic theory over creating a new one.

The well-known Zermelo-Fraenkel axiomatic method, used in this research, gives a solid logical structure to the analysis.

Another main difference regarding the utilization of the axiomatic method in this paper is that it neither is applied to mathematical expressions nor the double-entry bookkeeping, but only to one of the accounting assumptions. The double-entry bookkeeping is the practice associated with the dual aspects of accounting transactions, and the accounting equation is its ultimate mathematical expression. In a preliminary report this distinction was not made (see [18]); however, it is necessary to separate these topics of analysis, to properly identify their characteristics and reach more solid conclusions.

Finally, this paper analyzes the assets-claims on assets equality as an assumption of the accounting system, from the viewpoint of axiomatic theory.

2.1 Methodology

The axiomatic method is rationalistic and analytical; it uses axiomatic set theory along with predicate logic to develop rationales and conclusions. The method involves a set of axioms, and the logical rationale to apply them to any demonstration. The Zermelo-Fraenkel (ZF) axiomatic theory, used in this paper, comprises ten well-defined axioms that determine the possibility of applying logical operations to a predicate logic language. Initially, Zermelo created this system because advances in set theory did not involve a proper definition of sets; Fraenkel made some adjustments to the theory and added the replacement axiom [19]. This axiomatic theory remains as the most prevalent, and deals with infinite and finite sets.

3 Problem Solution

3.1 Primitives and Axioms of the Zermelo–Fraenkel theory

In the ZF theory (see [20]), the primitives are membership ∈ and set $\{x_i\}$. ZF theory deals only with sets; thus, the elements of a set are, in turn, sets; it does not accept elements not linked to any set (urelements).

The ZF theory comprises ten axioms; nevertheless, in this paper, only three of them will be used. They are: a) the axiom of specification that allows creating sets based on a formula, b) the axiom of union that gives a proper definition to group some sets into another set, and c) the axiom of extensionality that defines the equality of sets. The axioms will be explained all along the analysis.

The ZF theory also accepts the definition of subset as a set that is a member of another set.

3.2 Accounting axioms

According to the ZF theory, some sets exist, so in the accounting system some sets exist too.

Otherwise, the axiomatic method in accounting requires additional accounting primitives and axioms. The primitive in this system is the monetary unit u_i, which is the value unit used to valuate every asset or claim on assets.

The accounting axioms are as follows:

Accounting axiom 1. The elements of any set of assets and claims on assets are sets that contain sets of monetary units. This axiom means that the lowest level sets are always sets of monetary units. Therefore

$$\forall A \forall C \forall u_i [(\forall A_i \forall C_i \, (u_i \in A \mid u_i \in C) \rightarrow (u_i \in A_i \mid u_i \in C_i)] \tag{1}$$

with A = assets, C = claims on assets, A_i = element (subset) of assets, C_i = element (subset) of claims on assets, and u_i = monetary units. A special type of set is the single monetary unit $\{u_i\}$.

The monetary unit can be in the legal tender or any other unit; it does not make any difference to the analysis and does not need additional definition; once the monetary unit is chosen it is the same for all sets. To the purpose of this paper, the accounts in financial statements comprise a finite number of monetary units.

Accounting axiom 2. Every monetary unit $\{u_i\}$ is different to another monetary unit $\{u_j\}$.

$$\forall u_i \, \forall u_j [u_i \neq u_j] \tag{2}$$

This axiom is necessary, because if the monetary units were equal, a set containing ten monetary units would be equal to a set containing just one. Therefore, by this axiom, to any pair of monetary units $\{u_i\}$ and $\{u_j\}$

$$\forall u_i \, \forall u_j \, \forall x_i [(u_i \in x_i \wedge u_j \in x_i) \rightarrow u_i \neq u_j] \tag{3}$$
$$\forall u_i \, \forall u_j \, \forall x_i \, \forall y_i [(u_i \in x_i \wedge u_j \in y_i) \rightarrow u_i \neq u_j] \tag{4}$$

Accounting axiom 3. Every monetary unit has the property of being an asset and a claim on asset set, simultaneously. That is

$$\forall u_i \, \exists! C_i \, \exists! A_i \, \exists A \, \exists C \, [u_i \in A \wedge u_i \in C \rightarrow (u_i \in A_i \wedge u_i \in C_i)] \tag{5}$$

Therefore, a monetary unit $\{u_i\}$ can belong to two different sets A_i and C_i simultaneously. This axiom represents the dual aspect of the accounting transactions as an assumption, the duality assumption. However, it is not equal to the double-entry bookkeeping, the practice of the dual aspects of the accounting transactions.

3.3 The set structure of assets and claims on assets under the axiomatic method

In financial statements, and specifically in the balance sheet, assets (A) are equal to claims on assets (C).

All of the financial resources of an organization come from institutions, companies or individuals, and they have the right to make a claim on these resources. That is the rationale for this relationship. However, both groups refer to the only capital that exists.

From now on, the letters u, x, y, z, C, A, L, and E, are used to name sets, with no reference to elements not included in a set. Let us characterize the terms of the balance sheet, A and C, in the form of sets.

By the axiom 1, every monetary unit is allocated to some accounts (sets), and by the accounting axiom 3, these accounts are in both assets and claims on assets. Accordingly, every monetary unit is in an asset and a claim on assets accounts. A monetary unit is characterized as an asset, or claim on asset as follows:

u_A: monetary unit considered to be an asset under an accepted definition.
u_C: monetary unit considered to be a claim on assets under an accepted definition.

Then, the sets A and C, in any financial statements, need to be defined by formulae. The specification axiom allows the identification of subsets under certain conditions. This axiom states that

$$\forall z \, \forall w_1 \, \forall w_2 \ldots \forall w_n \, \exists y \, \exists x [x \in y \leftrightarrow (x \in z \wedge \phi)] \tag{6}$$

It means that a formula ϕ allows identification of subset y such that it contains every element x of the set z that has the property defined in the formula ϕ.

The sets A and C are subsets of the sets A_s and C_s, respectively. These sets A_s and C_s are also assets, and claims on assets respectively, but they are more comprehensive sets and comprise groups of companies, the industry, the country, or any other combination. In this sense, the sets A and C are subsets of other sets.

Then, applying the specification axiom to A and C

$$\forall A_s \, \exists A \, \exists u_A \, [u_A \in A \leftrightarrow (u_A \in A_s \wedge \phi_A)] \tag{7}$$

where ϕ_A: u_A is a monetary unit of the company´s assets. In the same form,

$$\forall C_s \, \exists C \, \exists u_C \, [u_C \in C \leftrightarrow (u_C \in C_s \wedge \phi_C)] \tag{8}$$

where ϕ_C: u_C is a monetary unit of the company´s claims on assets.

Claims on assets comprise the accounts (subsets) liabilities and stockholder´s equity. Then, applying this axiom to create the subsets L (liabilities) and E (stockholder´s equity) of C,

$$\forall C \, \exists L \, \exists u_L \, [u_L \in L \leftrightarrow (u_L \in C \wedge \phi_L)] \tag{9}$$

where ϕ_L: u_L is a monetary unit of the company´s liability, and

$$\forall C \exists E \exists u_E [u_E \in E \leftrightarrow (u_E \in C \wedge \phi_E)] \quad (10)$$

where ϕ_E: u_L is a monetary unit of the company´s stockholder´s equity.

It is important to note that this is not a partition of sets because a partition has different properties to that of subsets, which are the ones being defined here.

For the sake of clarity, the analysis will address only a few items of the financial statements. Therefore, by the specification axiom, one can create subsets, in such a way that the set A contains the subsets current assets A_c and non-current assets A_{nc}. Current assets A_c, in turn, comprises cash A_{cc} and accounts receivable A_{car}, whereas non-current assets A_{nc} contains long-term investments A_{nclti}, property, plant, and equipment $A_{ncppe,}$ and intangible assets A_{ncia}.

As already mentioned, the formula ϕ of the specification axiom allows the inclusion of monetary units in sets or subsets. This formula applies to any set or subset of financial statements.

3.3 The aggregated accounts as set and subsets

Financial statements allocate items to other items. Here, the ZF set theory assumes the definition of a subset as a set that is a member of another set. This definition is useful here; in predicate logic and set language, the definition of a subset is in the following form:

$$(x \subseteq y) \leftrightarrow (\forall z\, (z \in x \rightarrow z \in y)) \quad (11)$$

That means that if a set x contains a set z and y contains x, then y contains z, and x is a subset of y. Regarding monetary units, and keeping in mind that ZF theory includes only sets,

$$(u_i \subseteq u_j) \leftrightarrow (\forall u_n\, (u_n \in u_i \rightarrow u_n \in u_j)) \quad (12)$$

In the equation, u_i, u_j, and u_n are sets, and it means that u_i is a subset of u_j because every element u_n of u_i is contained in u_j.

Thus, total assets is a set A that consists of sets containing other sets:

$$A = \{\{A_c\}, \{A_{nc}\}\} \quad (13)$$
$$A_c = \{\{A_{cc}\}, \{A_{car}\}\} \quad (14)$$
$$A_{nc} = \{\{A_{nclti}\}, \{A_{ncppe}\}, \{A_{ncia}\}\} \quad (15)$$

The definition of the subset allows the following structure to be built:

$$(A_{cc} \subseteq A_c) \leftrightarrow (\forall u_i\, (u_i \in A_{cc} \rightarrow u_i \in A_c)) \quad (16)$$
$$(A_{car} \subseteq A_c) \leftrightarrow (\forall u_i\, (u_i \in A_{car} \rightarrow u_i \in A_c)) \quad (17)$$
$$(A_{ncia} \subseteq A_{nc}) \leftrightarrow (\forall u_i\, (u_i \in A_{ncia} \rightarrow u_i \in A_{nc})) \quad (18)$$
$$(A_{ncppe} \subseteq A_{nc}) \leftrightarrow (\forall u_i\, (u_i \in A_{ncppe} \rightarrow u_i \in A_{nc})) \quad (19)$$
$$(A_{nclti} \subseteq A_{nc}) \leftrightarrow (\forall u_i\, (u_i \in A_{nclti} \rightarrow u_i \in A_{nc})) \quad (20)$$
$$(A_c \subseteq A) \leftrightarrow (\forall A_i\, (A_i \in A_c \rightarrow A_i \in A)) \quad (21)$$
$$(A_{nc} \subseteq A) \leftrightarrow (\forall A_i\, (A_i \in A_{nc} \rightarrow A_i \in A)) \quad (22)$$

In this structure, A_i is any subset of A_c or A_{nc}. Likewise, the set L contains subsets, such as current liabilities L_c and non-current liabilities L_{nc}. Current liabilities L_c include, in turn, subsets such as accounts payable L_{cap} and unearned revenues L_{cur}, whereas non-current liabilities L_{nc} contains the set mortgage payable L_{ncmp} and notes payable L_{ncnp}. The set owners' equity E includes issued capital E_{ic}, common stocks E_{cs}, and retained earnings E_{re}. These sets, as in the total asset set, are in the form

$$L = \{\{L_c\}, \{L_{nc}\}\} \quad (23)$$
$$L_c = \{\{L_{cap}\}, \{L_{cur}\}\} \quad (24)$$
$$L_{nc} = \{\{L_{ncmp}\}, \{L_{ncnp}\}\} \quad (25)$$
$$E = \{\{E_{ic}\}, \{E_{cs}\}, \{E_{re}\}\} \quad (26)$$

According to the definition of subset, these sets are

$$(L_{cap} \subseteq L_c) \leftrightarrow (\forall u_i\, (u_i \in L_{cap} \rightarrow u_i \in L_c)) \quad (27)$$
$$(L_{cur} \subseteq L_c) \leftrightarrow (\forall u_i\, (u_i \in L_{cur} \rightarrow u_i \in L_c)) \quad (28)$$
$$(L_{ncmp} \subseteq L_{nc}) \leftrightarrow (\forall u_i\, (u_i \in L_{ncmp} \rightarrow u_i \in L_{nc})) \quad (29)$$
$$(L_{ncnp} \subseteq L_{nc}) \leftrightarrow (\forall u_i\, (u_i \in L_{ncnp} \rightarrow u_i \in L_{nc})) \quad (30)$$
$$(L_c \subseteq L) \leftrightarrow (\forall L_{ci}\, (L_{ci} \in L_c \rightarrow L_{ci} \in L)) \quad (31)$$
$$(L_{nc} \subseteq L) \leftrightarrow (\forall L_{nci}\, (L_{nci} \in L_{nc} \rightarrow L_{nci} \in L)) \quad (32)$$

Furthermore,

$$(E_i \subseteq E) \leftrightarrow (\forall u_i\, (u_i \in E_i \rightarrow u_i \in E)) \quad (33)$$

The sets and subsets $\{A\}$, $\{A_c\}$, $\{A_{nc}\}$, $\{A_{cc}\}$, $\{A_{cr}\}$, $\{A_{lti}\}$, $\{A_{ppe}\}$, $\{A_{ia}\}$, $\{L\}$, $\{L_c\}$, $\{L_{nc}\}$, $\{L_{ap}\}$, $\{L_{ur}\}$, $\{L_{mp}\}$, $\{L_{np}\}$, $\{E\}$, $\{E_{ic}\}$, $\{E_{cs}\}$, and $\{E_{re}\}$ are created by formulae; this grouping has three levels for assets and liabilities and two for equity.

Another application of the subset definition leads to define the set C as comprising the subsets L and E, in the form

$$(E \subseteq C) \leftrightarrow (\forall E_i\, (E_i \in E \rightarrow E_i \in C)) \quad (34)$$
$$(L \subseteq C) \leftrightarrow (\forall L_i\, (L_i \in L \rightarrow L_i \in C)) \quad (35)$$

Therefore,

$$C = \{\{L\}, \{E\}\} \tag{36}$$

3.4 The relationship between assets and claims on assets

In all of the previous analyzes, the lowest level sets contain the monetary unit sets $\{u_i\}$. However, these sets $\{u_i\}$ have no financial meaning because they lack proper identification in financial statements. They acquire financial meaning by their inclusion in the next higher category, such as A_{cc} …. E_{re}. The accounting axiom 1 states that sets in accounting system are sets that contain sets of monetary units. That allows aggregating sets of monetary units into higher order sets, which can be done by the union axiom of the ZF theory.

The axiom of union says that the union of sets is a set that contains the elements of the elements of another set. According to the axiom of union, the union of sets is

$$\forall X\ \exists Y\ \forall z\ \forall w\ [(w \in z \wedge z \in X) \rightarrow w \in Y] \tag{37}$$

It means that if a set X contains subsets z and these elements contain subsets w, the union of the elements w of the subsets z of the set X is another set Y. In the case of L (liabilities), current liabilities L_c and non-current liabilities L_{nc}, it is

$$\forall L\ \exists L_u\ \forall L_j\ \forall L_i\ [(L_i \in L_j \wedge L_j \in L) \rightarrow L_i \in L_u] \tag{38}$$

The set L contains the subsets L_j (L_c and L_{nc}); L_i is every element of the sets L_c and L_{nc}, and L_u is the union of the elements of the elements of all of L_j. That is, the set L_u includes all the subsets L_i of L_c and L_{nc}. With the definition of subset, L_u is included in set C:

$$(L_u \subseteq C) \leftrightarrow (\forall L_i\ (L_i \in L_u \rightarrow L_i \in C)) \tag{39}$$

where L_i is any subset of L_u.

Likewise, there are two sets on the claims on assets side: one is L_u and the other is E; E contains all its subsets defined above. The set C contains both sets. The union C_u of these sets is

$$\forall C\ \exists C_u\ \forall C_j\ \forall C_i\ [(C_i \in C_j \wedge C_j \in C) \rightarrow C_i \in C_u] \tag{40}$$

where C is the set that contains the sets C_j (L_u and E) and C_i any subset of L_u and E. Then, the set C_u comprises all C_i elements of L_u and E.

The union of the subsets of A is

$$\forall A\ \exists A_u\ \forall A_j\ \forall A_i\ [(A_i \in A_j \wedge A_j \in A) \rightarrow A_i \in A_u] \tag{41}$$

where A contains the subsets A_j (A_c and A_{nc}); A_i is any element of the sets A_c and A_{nc}; A_u is the union of the elements of the A_j subsets. That is, the set A_u includes all the subsets of A_c and A_{nc}.

As a result, there are two sets, A_u and C_u, which contain all the subsets of assets and all the subsets of claims on assets, respectively. These subsets are the lowest level sets with financial meaning because they have relevant item labels, such as cash, accounts receivable, accounts payable, mortgage payable, and so on. They contain all the subsets of monetary units $\{u_i\}$.

The accounting axiom 3 states that every monetary unit is simultaneously located in both assets and claims on assets. Therefore, one can look for the type of relationship between assets and claims on assets, taking into account the set structure they have. The test to be conducted is

$$A_u = C_u \tag{42}$$

According to the axiom of extensionality, the equality of sets is

$$\forall x\ \forall y\ [\forall z(z \in x \leftrightarrow z \in y) \rightarrow x = y)] \tag{43}$$

This formula means that set x is equal to set y if for every z, whenever z is a subset of x, z is a subset of y, and, conversely, whenever z is a subset of y, z is a subset of x. Then, for A_u and C_u,

$$\forall A_u\ \forall C_u[\forall x_i(x_i \in A_u \leftrightarrow x_i \in C_u) \rightarrow A_u = C_u)] \tag{44}$$

Accordingly, for A_u and C_u to be equal, they need to have the same subsets x_i. It means that the subsets A_i must be equal to the subsets C_i.

Also, for the subsets C_i and A_i to be equal, all of the monetary units $\{u_i\}$ in a set C_i should only be in a set A_i. Therefore, there must be a subset A_i for each C_i, such that both of them have the same elements $\{u_i\}$. Consequently, using the sets C_i and A_i of C_u and A_u respectively, for every C_i to be equal to an A_i,

$$\forall A_i\ \forall C_i\ [\forall u_i(u_i \in A_i \leftrightarrow u_i \in C_i) \rightarrow A_i = C_i)] \tag{45}$$

To assume that the C_i subsets are equal to the A_i subsets the $\{u_i\}$ elements should be the same in each set. That is, the monetary unit sets $\{u_i\}$ in a set C_i

are also in a single set A_i and both sets have to have the same elements.

However, it is not a requirement of accounting axiom 3 to have the subsets of monetary units $\{u_i\}$ of each set C_i located in a unique set A_i,. It can happen that some of the monetary units of C_i be allocated to another A_j. In general

$$\forall C_i \, \forall A_i \, \forall A_j \, [\forall u_i \forall u_j \, (u_i \in C_i) \rightarrow \exists A_i \, (\, u_i \in A_i \,) \rightarrow \exists u_j \, (u_j \in C_i \wedge u_j \in A_j)] \quad (46)$$

Therefore, the requirement that the C_i subsets are equal to the A_i would be in contradiction with accounting axiom 3.

Another mean to explain this, it is creating a new set by the axiom of specification. This axiom would be

$$\forall A_i \, \forall w_1 \, \forall w_2 \ldots \forall w_n \, \exists C_{ie} \, \exists u_i \, [u_i \in A_{ie} \leftrightarrow (u_i \in A_i \wedge \phi)] \quad (47)$$

where ϕ = elements $\{u_i\}$ of A_i that are also members of a particular C_i. That means that the application of the property ϕ to the elements of a set A_i will restrict the elements of a new set called A_{ie} to those that are also members of a given C_i.

Then, by the axiom of extension, in the case of $A_u = C_u$, it should be that

$$\forall A_i \, \forall A_{ie} \, [\forall u_i \, (u_i \in A_i \leftrightarrow u_i \in A_{ie}) \rightarrow A_i = A_{ie})] \quad (48)$$

where A_{ie} contains the elements of a particular A_i that are elements of a single C_i. However, by the accounting axiom 3, it might happens that $A_i \neq A_{ie}$, because some monetary units $\{u_i\}$ of the set A_i are in a different C_i. Therefore

$$\forall A_u \, \forall C_u \, \forall C_i \, [\forall A_i \, \forall A_j \, ((C_i \in C_u \wedge A_i \, A_j \in A_u) \rightarrow (\forall u_i \, (u_i \in C_i \wedge u_i \in A_i) \rightarrow \exists u_j \, (u_j \in C_i \wedge u_j \in A_j)))] \quad (49)$$

Then,

$$\forall C_i \, [\forall A_i \, (C_i \in C_u \wedge A_i \in A_u) \rightarrow C_i \neq A_i] \quad (50)$$

Consequently,

$$A_u \neq C_u \quad (51)$$

Assets and claims on assets are not equal when taking into account the different structures they have.

However, a new application of the axiom of union would produce a set with the union of all sets containing monetary unit sets $\{u_i\}$. The application of this axiom to the set of assets is

$$\forall A_u \, \exists A_{uu} \, \forall A_i \, \forall u_i \, [(u_i \in A_i \wedge A_i \in A_u) \rightarrow u_i \in A_{uu}] \quad (52)$$

The result is a set A_{uu} consisting of subsets of the type $\{u_i\}$. It is also possible to perform this operation on all financial obligations C_u in the following manner

$$\forall C_u \, \exists C_{uu} \, \forall C_i \, \forall u_i \, [(u_i \in C_i \wedge C_i \in C_u) \rightarrow u_i \in C_{uu}] \quad (53)$$

Again, the result is a set C_{uu} consisting of subsets of the type $\{u_i\}$. The sets A_{uu} and C_{uu} have all the monetary units $\{u_i\}$ because they are not included in any other item and, according to the axiom of extension,

$$\forall A_{uu} \, \forall C_{uu} \, [\forall u_i \, (u_i \in A_{uu} \leftrightarrow u_i \in C_{uu}) \rightarrow A_{uu} = C_{uu})] \quad (55)$$

All subsets $\{u_i\}$ are members of the sets A_{uu} and C_{uu}, and thus

$$A_{uu} = C_{uu} \quad (56)$$

Although these sets are equal, this is meaningless in financial accounting. An amount of monetary units is equal to the same amount of monetary units, removing their financial classification. Yet, this classification is the essence of financial accounting.

4 Conclusion

The application of axiomatic theory to the balance sheet leads to the conclusions that the assets and claims on assets can be analyzed with an existing axiomatic theory combined with a small number of accounting axioms, avoiding creating new theories. Also, it yielded the conclusion that assets and claims on assets are not equal considering their different set structures.

The results obtained needs to be understood within the framework of the axiomatic theory. Also, to achieve these results the analysis took only a few items on the balance sheet. However, the same results would have been reached with any number of items or levels.

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