Modelling of Diffusion Process in Porous Bricks

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Abstract: - We consider drying process of bricks made of soil and cellulose mixture. The particle sizes of the soil are about 0.005 μm, which are considered to be clay. We apply a diffusion type equation to model the drying process. The model is based on macro modeling, where the particle and pore sizes are considered much smaller compared to the size of nails to measured the moisture content of the area inside the bricks. The model is a nonlinear diffusion equation where the diffusion rate depends on the moisture content. The solution is computed analytically applying a finite difference method for one and two dimensional cases. The simulation shows that the drying process of two dimensional case is slightly faster than of the one dimensional case. The cause is the diffusion to the side wall of two dimensional case where this wall is does not exist in one dimensional case.

Key-Words: - Diffusion equation, drying process, macro modeling, moisture content, pore size, diffusion rate.

1 Introduction

This paper examines diffusion process in bricks produced traditionally in home industry in Kendari, Indonesia. It is a continuation of [1]. The bricks production is divided into 4 (four) simple stages i.e. molding, drying, burning, and unloading. 500 bricks can be produced in a day, and it requires approximately 3 (three) weeks to accomplish the process. Based on sieve analysis and *atterberg limit* test, the soil is classified as *silty clay with sand.*

In molding process, clay is mixed with water at specific composition, then molded in standardized shape like box. This is followed by drying process in open air with direct sunlight for 2 (two) weeks. After that, those bricks are burned by arranging them with a hole in the middle for the firewoods. This process takes 5 to 6 hours for each burning process that can reach more or less a thousand bricks. Then, the burned bricks will be unloaded after 4 to 5 days. At this stage, most of the bricks could be broken, especially those on the outer parts, as a result of patchy heat transfer from inside to outside of those bricks. Recent modeling on heat transfer is reported in [2, 3, 4, 5]. Hence there is consequence for economic value that can be obtained from the bricks market. Bricks are most preferred building materials due to its cheap and reasonable price, good thermal insulator properties, and that they are exposed and can be produced locally through home industry. Nevertheless, bricks production in home industry had encountered classic situation where the level of productivity is akin to the previous period and tends to decrease as a consequence of modest technique of production process. In addition, environmental impacts e.g flood and erosion might ensue resulted from the excavation of the soil and logging.

There are some requirements for brick to be used as material construction in [6]. How heat transfer works on brick surface will determine the quality of the bricks. Some innovative fillers and design e.g. sawdust, reed, corn, coconut waste, and hollow brick design [1, 7, 8] demonstrates good performance since the compressive strength is higher than the conventional model.

Typically, 20% of bricks are considered broken, i.e. not meeting the compressive strength requirement (> 6 MPa), following the burning

process using firewood, since there is patchy heat transfer [1]. In terms of shrinkage, about 4% size reduction in length, width and thickness from the beginning until the last stage [9]. Furthermore, the density of bricks is varied between 1.6 and 1.8 kg/cm^3. By knowing the diffusion process in porous bricks, it is expected that efficiency and effectiveness might be measured, where it might generate essential impact on production cost.
This study is essential to understand the diffusion of fluid inside the brick, not only during the drying process, but also after the brick is utilized in the building construction. It is because some dangerous fluid may infiltrate into the brick and porous materials as into concrete studied in [10]. Diffusion in porous material is presented in [11].

2 Problem Formulation

The brick sizes are 21 cm in length, 11 cm in width and 6 cm in thickness. At first, it is assumed that the moisture content (MC) in the whole body is similar (right after molding process). Then, the moisture content at brick's surface will be similar with equlibrium moisture content (EMC), and inside the bricks, the moisture content will decrease gradually. Moreover, at infinite time, the moisture content in the bricks is identical with the air moisture content.

To address the problem, firstly, it is presumed that the thickness of bricks is relatively small compared to its length and width. Therefore, the one dimensional analysis is performed to examine the diffusion process. Thenceforth, two dimensional analysis is conducted based on conjecture that the thickness and width are relatively small compare to the length. These two analysis are then evaluated to compare their moisture content. Should they share almost similar values, it may be determined that three dimensional analysis is not required and the complete behavior is precisely clear. On the other hand, further analysis on three dimensional basis is necessitated to be carried out.

3 Mathematical Model

For writing the mathematical model, we will use symbols presented the Table 1. The symbols are in the left colomn and the physical meaning are in the right.

Table 1. Symbols and the physical meaning used in modeling

Symbol		Meaning
x	:	Spatial variable
y	:	Spatial variable
z	:	Spatial variable
t	:	Temporal variable
u	:	Sate variable representing moisture content
K	:	Diffusion rate
∂_x	:	Partial derivative with respect to x
u_x	:	Partial derivative of u with respect to x
∇	:	Gradient

The model is based on macro modeling, where the pore size, particle size are assumed much less significants compared to the size of dot under consideration. Hence, our mathematical model has the form

$$u_t = \nabla \bullet \left(K(u)\nabla u\right), \left\{(x, y, z): \begin{array}{r} 0 < x < d_1, \\ 0 < y < d_2, \\ 0 < z < d_3, \end{array}\right\}, \ t > 0. \quad (1)$$

For constant diffusion rate, (1) is nothing else but the heat equation, where the derivation of can be found in many classical standard textbooks. In [13] the derivation for more general K has been discussed. This allows the applications for nonlinear diffusion such as on wood drying. Some materials such as wood show that the diffusion rate depends on the moisture content. Some types of concrete are almost impermiable when they are completely dry. It means that the diffusion rate become close to zero.

Often, in some application we may neglect the y and z component of the spatial variables. This may happen if the thickness of the domain is much smaller than the width and the length. For one dimensional case, (1) has the form

$$u_t = \partial_x\left(K(u)\,u_x\right), \quad 0 < x < d_1, \quad t > 0. \qquad (2)$$

During the drying process of bricks, the EMC of the surounding air acts as the boundary condition. EMC depend on the humidity of the air. If the process happens in an oven or in a laboratory, the EMC may be set at certained value. On the other hand, the initial condition is the MC of the bricks for example 100%.
and Tables should be numbered as follows: Fig.1, Fig.2, … etc Table 1, Table 2, ….etc.
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4 Numerical Method

While we do not solve the model explicitly, we solve it numerically. We apply a finite difference method for (2). Eqn (2) can be written in the form

$$u_t = F(u)(u_x)^2 + K(u)\,u_{xx}. \qquad (3)$$

where $F(u) = \dfrac{dK(u)}{du}$.

Suppose $\Delta x \cdot N_x = d_1$, $x_i = (i-1)\cdot\Delta x$, and $t_j = (j-1)\cdot\Delta t$. We write $u(x_i, t_j) \approx U_i^j$. A finite difference for (3) is

$$U_i^{j+1} = U_i^j + \frac{\Delta t}{(\Delta x)^2}\begin{pmatrix} F(U_i^j)\left(U_{i+1}^j - U_{i-1}^j\right)/2 \\ + U_{i+1}^j - 2U_i^j + U_{i-1}^j \end{pmatrix}. \qquad (4)$$

For computation, the boundary condition is set $U_1^j = U_{N_x}^j = 0.2$ for any *j*, and the initial condition $U_i^1 = 1$ for $2 \le j \le N_x$.

The two dimensional case, (1) becomes

$$u_t = F(u)(u_x)^2 + K(u)\,u_{xx} + F(u)(u_y)^2 + K(u)\,u_{yy}. \qquad (5)$$

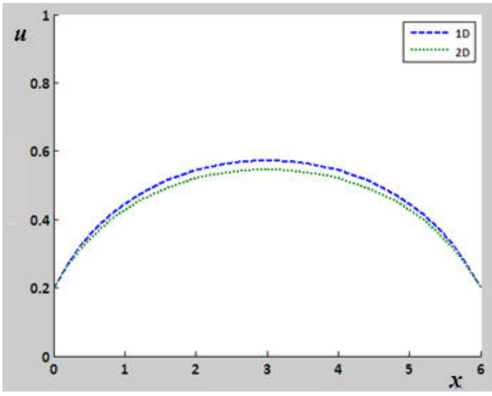

Fig. 1 Moisture content at $t = 10$ for one dimensional and two dimensional case

Writing $y_k = (k-1)\cdot\Delta x$, $u(x_i, y_k, t_j) \approx U_{i,k}^j$. A finite difference for (5) is

$$U_{i,k}^{j+1} = U_{i,k}^j + \left(\begin{array}{l} \dfrac{\Delta t}{(\Delta x)^2}\begin{pmatrix} F(U_{i,k}^j)\left(U_{i+1,k}^j - U_{i-1,k}^j\right)/2 \\ + U_{i+1,k}^j - 2U_{i,k}^j + U_{i-1,k}^j \end{pmatrix} \\ + \dfrac{\Delta t}{(\Delta y)^2}\begin{pmatrix} F(U_{i,k}^j)\left(U_{i,k+1}^j - U_{i,k-1}^j\right)/2 \\ + U_{i,k+1}^j - 2U_{i,k}^j + U_{i,k-1}^j \end{pmatrix} \end{array}\right). \qquad (6)$$

For computation, the boundary condition is set for any *j* that is $U_{1,k}^j = U_{N_x,k}^j = 0.2$ for $1 \le k \le N_y$, and $U_{i,1}^j = U_{i,N_y}^j = 0.2$ for $1 \le i \le N_x$. The initial condition $U_{i,k}^1 = 1$ for $2 \le j \le N_x - 1$, and $2 \le k \le N_y - 1$.

5 Result and Discussion

Implementing a finite difference (4) in a program, we get U_i^j for any *i* and *j*. In this computation, $d_1 = 6$, $\Delta x = 0.2$, $\Delta t = 0.005$, and $N_x = 31$. Fig 1 shows the value of *u* in the interval of $0 < x < d_1 = 6$ for several value of *t*. The curve at the top represents the value of *u* at $t = 0.05$. Observe that at points close to the boundary the value of *u* decrease much faster. The curve at the bottom is the value of *u* at $t = 10$. The decrease of the value u becomes slower than at $t = 0.05$. Other curves are orderly in time from the top to the bottom.

The similar data are used for the computaion of two dimensional case. Moreover, as additional $d_2 = 11$, $\Delta x = 0.2$, $\Delta t = 0.005$, and $N_y = 56$. Fig 2 shows the value of u in the region of $0 < x < d_1 = 6$ and $0 < y < d_2 = 11$ for several value of t. The curve at the top represents the value of u at $y = 5.5$ and $t = 0.05$. The curve at the bottom is the value of u at $y = 5.5$ and $t = 10$. Similar phenomenon is observed as for the case one dimensional case. However, comparing the curves at $t = 10$ for one dimensional case and two dimensional case we observe the difference. The value u of the two dimensional case decrease slightly faster compared to the one dimensional case. The diffusion process two the side wall in two dimensional case is responsible for this.

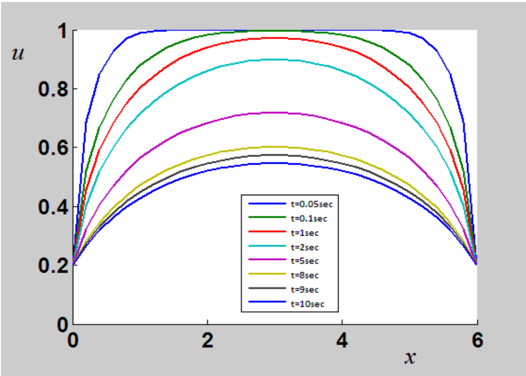

Fig. 2 Moisture content at several time for two dimensional case

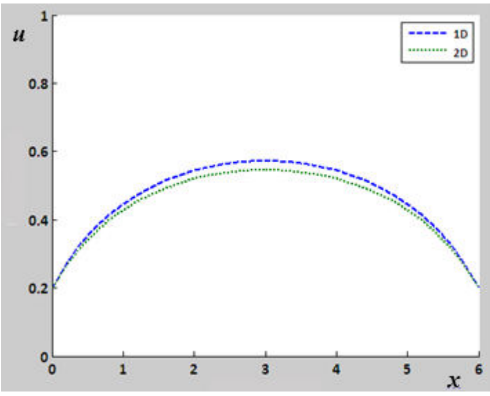

Fig. 3 Moisture content at $t = 10$ for one dimensional and two dimensional case

6 Conclusion

We have discussed drying process of bricks made of soil and cellulose mixture. We apply a diffusion type equation to model the drying process. The model is based on macro modeling, where the particle and pore sizes are considered much smaller compared to the size of nails to measured the moisture content of the area inside the bricks. The model is a nonlinear diffusion equation where the diffusion rate depends on the moisture content.

While the analytical solution of the nonlinear model is still diffficult to obtain, the solution is computed analytically applying a finite difference method for one and two dimensional cases. The simulation shows that the drying process of two dimensional case is slightly faster than of the one dimensional case. The cause is the diffusion to the side wall of two dimensional case where this wall is does not exist in one dimensional case.

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