2-D FIR and IIR Filters' design: New Methodologies and New General Transformations

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Abstract — New general transformations for designing 2-D (Two-Dimensional) FIR and IIR filters will presented in this paper. The present methodology can be viewed as an extension of the McClellan Transformations and can be applied in several cases of 2-D FIR and IIR filter design. Numerical examples illustrate the validity and the efficiency of the method.

Keywords — 2-D Filters, FIR Filters, IIR Filters, Multidimensional Systems, Multidimensional Filters, Filter Design, McClellan Transformations

I. Introduction

2-D (Two-Dimensional) filter design is not a simple task due to the heavy computational load and to the non-existence of stability conditions in an explicit form. Roughly speaking, the design of 2-D filters FIR includes *a Fourier method* that uses Fourier analysis, where appropriate window Functions can also eliminate the so called Gibbs' oscillations as in 1-D case, *a Transformations' method* which is based on McClellan Transformations from appropriate 1-D filters [1],[2] and *an optimization method* i.e. the minimization of an appropriate norm, [1],[2].

On the other hand, the design of 2-D filters IIR includes also transformations, Mirror Image Polynomials, SVD (Singular Value Decomposition) and Optimization, [1],[2]. Several Authors have published works on optimization-based 2-D filter design while a great number of papers are dedicated to transformations and mainly to McClellan Transforms.

McClellan Transformations were introduced in [3] and have been used for the last forty years. A brief overview with the various extension of McClellan Transformations can be found in [1],[2],[4],[8],[9]. In general, a McClellan Transformation is described by

$$\cos(\omega) = \sum_{k=1}^{N} \sum_{l=1}^{M} C_{kl} \cos(\omega_1) \cos(\omega_2)$$

ω is the frequency of the original 1-D filter, whereas ω_1, ω_2 the frequencies of the 2-D filter in design.

As Harn and Shenoi pointed out in [5] and as Nguyen and Swamy reported in [6], till now a transformation for IIR filter design analogous to McClellan transformation does not exist due to the requirements of 2-D stability.
Nguyen and Swamy in [7] use the usual McClellan transformation in the special case of separable denominator. Fundamental results on McClellan transformation can be found in [8] and [9] while remarkable studies are given [10]÷[18]. Various useful results for 2-D IIR Filters' design are presented in [19]÷[25]. In [26], we proposed some transformations for the first-order IIR 2-D and second-order IIR 2-D notch filters. In [26], we propose the transformation

$$z^{-1} = \frac{\lambda_1 z_1^{-1} + \lambda_2 z_2^{-1}}{\lambda_1 + \lambda_2}$$ with λ_1, λ_2 real numbers or simply

$$z^{-1} = \lambda z_1^{-1} + (1-\lambda) z_2^{-1} \quad \text{with } 0 < \lambda < 1$$

For first-order and second-order IIR 2-D notch filters' design.

This paper examines this transformation as well as its generalization to the general 2-D filters design.

II. The Transformation and its generalizations

Instead of the classic McClellan Transformation $\cos(\omega) = \sum_{k=1}^{N} \sum_{l=1}^{M} C_{kl} \cos(\omega_1) \cos(\omega_2)$ where ω is the frequency of the original 1-D filter, whereas ω_1, ω_2 the frequencies of the 2-D filter in design, we propose here the transformation of [26]

$z^{-1} = \frac{\lambda_1 z_1^{-1} + \lambda_2 z_2^{-1}}{\lambda_1 + \lambda_2}$ with λ_1, λ_2 real numbers or simply

or simply $z^{-1} = C_1 z_1^{-1} + C_2 z_2^{-1}$ where in [26], for the 2-D notch filters, we demanded $C_1 + C_2 = 1$

As a simple generalization of this transformation we propose the following transformation not only for the design of Notch Filters, but for every 2-D filter (either IIR or FIR):

$$z^{-1} = C_1 z_1^{-1} + C_2 z_2^{-1}$$

where C_1, C_2 are real numbers with $C_1 + C_2 = 1$ and $C_1 C_2 > 0$

Unlike the original McClellan Transform $\cos\omega = C_1 \cos\omega_1 + C_2 \cos\omega_2$ where we demand only $C_1 + C_2 = 1$, in our transformation $z^{-1} = C_1 z_1^{-1} + C_2 z_2^{-1}$ we demand not only $C_1 + C_2 = 1$,but also $C_1 C_2 > 0$. The disadvantage of McClellan Transform is that it can be applied in FIR filters i.e. in a filter with transfer function $H(z^{-1}) = \frac{A(z^{-1})}{B(z^{-1})}$ with $B(z^{-1})$=1. In this paper with the new proposed transformation we can apply it in every 1-D prototype filter with $B(z^{-1})$in general polynomial of z^{-1}. We are ready now to prove the Theorem.

Theorem 1. *Consider a prototype 1-D BIBO stable filter a filter with transfer function*

$$H(z^{-1}) = \frac{A(z^{-1})}{B(z^{-1})} \quad (1)$$

Under the transformation

$$z^{-1} = C_1 z_1^{-1} + C_2 z_2^{-1} \quad (2)$$

with $C_1 + C_2 = 1$ *and* $C_1 C_2 > 0$, *the prototype 1-D BIBO of (1) gives*

$$H_2(z_1^{-1}, z_2^{-1}) = \frac{A_1(z_1^{-1}, z_2^{-1})}{B_2(z_1^{-1}, z_2^{-1})} \quad (3)$$

where the new transfer function $H_2(z_1^{-1}, z_2^{-1})$ *is also stable and the origin of the axes* $(\omega = 0)$ *is depicted to the point* $(\omega_1, \omega_2) = (0,0)$

Proof . Start first to prove that the origin of the axes $(\omega = 0)$ is depicted to the point $(\omega_1, \omega_2) = (0,0)$ which is obvious because from (2) one has $e^{j\omega} = C_1 e^{j\omega_1} + C_2 e^{j\omega_2}$ or equivalently $\cos\omega = C_1 \cos\omega_1 + C_2 \cos\omega_2$. Therefore because $C_1 + C_2 = 1$ the solution of the equation $1 = C_1 \cos\omega_1 + C_2 \cos\omega_2$ (i.e. $(\omega = 0)$ must be $(\omega_1, \omega_2) = (0,0)$. Hence the origin of the axes $(\omega = 0)$ is depicted to the point $(\omega_1, \omega_2) = (0,0)$.

For Stability we have to prove that $B_2(z_1^{-1}, z_2^{-1}) \neq 0$ for every z_1^{-1} and z_2^{-1} inside the unit bi-disk i.e. for every z_1^{-1} and z_2^{-1} with $|z_1^{-1}| < 1$ and $|z_2^{-1}| < 1$.

Assume first that there are some ζ_1^{-1} and ζ_2^{-1} with $|\zeta_1^{-1}| < 1$ and $|\zeta_2^{-1}| < 1$ such that $B_2(\zeta_1^{-1}, \zeta_2^{-1}) = 0$. However, in this case we have a ζ^{-1}

$\zeta^{-1} = C_1 \zeta_1^{-1} + C_2 \zeta_2^{-1}$ such that $B(\zeta^{-1}) = 0$, on the other hand, since $|\zeta_1^{-1}| < 1$ and $|\zeta_2^{-1}| < 1$, we have

and

$$|\zeta^{-1}| = |C_1 \zeta_1^{-1} + C_2 \zeta_2^{-1}| \le |C_1||\zeta_1^{-1}| + |C_2||\zeta_2^{-1}| < |C_1| + |C_2| =$$
$$= |C_1 + C_2| = 1$$

(since $C_1 C_2 > 0$)

that makes our 1-D filter with transfer function $H(z^{-1}) = \frac{A(z^{-1})}{B(z^{-1})}$ non-stable (in BIBO sense), but this contradicts to the assumption. So, this completes the Proof. ∎

A very interesting extension of this transformation can be the following $z^{-1} = f(z_1^{-1}, z_2^{-1}) = \sum_{k=0}^{N} \sum_{l=0}^{M} C_{kl} z_1^{-k} z_2^{-l}$

with $\sum_{k=0}^{N} \sum_{l=0}^{M} C_{kl} = 1$ and $C_{k_1 l_2} C_{k_2 l_2} > 0$ for all $k_1, k_2 = 0,1,\ldots,N$ and $l_1, l_2 = 0,1,\ldots,M$ and the following theorem can be proved.

Theorem 2. *Consider a prototype 1-D BIBO stable filter a filter with transfer function*

$$H(z^{-1}) = \frac{A(z^{-1})}{B(z^{-1})} \quad (1)$$

Under the transformation

$$z^{-1} = f(z_1^{-1}, z_2^{-1}) = \sum_{k=0}^{N} \sum_{l=0}^{M} C_{kl} z_1^{-k} z_2^{-l} \quad (4)$$

with $\sum_{k=0}^{N}\sum_{l=0}^{M} C_{kl} = 1$ *and* $C_{k_1 l_2} C_{k_2 l_2} > 0$, *for all* $k_1, k_2 = 0,1,...,N$ *and* $l_1, l_2 = 0,1,...,M$ *the prototype 1-D BIBO of (1) gives*

$$H_2\left(z_1^{-1}, z_2^{-1}\right) = \frac{A_1\left(z_1^{-1}, z_2^{-1}\right)}{B_2\left(z_1^{-1}, z_2^{-1}\right)} \quad (3)$$

where the new transfer function $H_2\left(z_1^{-1}, z_2^{-1}\right)$ *is also stable and the origin of the axes* $(\omega = 0)$ *is depicted to the point* $(\omega_1, \omega_2) = (0,0)$

Proof . It is easy to prove that necessary and sufficient condition for the depiction of the origin of the axes $(\omega = 0)$ to the point $(\omega_1, \omega_2) = (0,0)$ is $\sum_{k=0}^{N}\sum_{l=0}^{M} C_{kl} = 1$

For Stability we have also to prove that $B_2\left(z_1^{-1}, z_2^{-1}\right) \neq 0$ for every z_1^{-1} and z_2^{-1} inside the unit bi-disk i.e. for every z_1^{-1} and z_2^{-1} with $\left|z_1^{-1}\right| < 1$ and $\left|z_2^{-1}\right| < 1$. Assuming that there are some ζ_1^{-1} and ζ_2^{-1} with $\left|\zeta_1^{-1}\right| < 1$ and $\left|\zeta_2^{-1}\right| < 1$ such that $B_2\left(\zeta_1^{-1}, \zeta_2^{-1}\right) = 0$, in this case we would have a ζ^{-1} with $\zeta^{-1} = \sum_{k=0}^{N}\sum_{l=0}^{M} C_{kl} \zeta_1^{-k} \zeta_2^{-l}$ such that $B\left(\zeta^{-1}\right) = 0$, on the other hand, since $\left|\zeta_1^{-1}\right| < 1$ and $\left|\zeta_2^{-1}\right| < 1$, we would have

$$\left|\zeta^{-1}\right| = \left|\sum_{k=0}^{N}\sum_{l=0}^{M} C_{kl} \zeta_1^{-k} \zeta_2^{-l}\right| \leq \sum_{k=0}^{N}\sum_{l=0}^{M} \left|C_{kl}\right| \left|\zeta_1^{-k}\right| \left|\zeta_2^{-l}\right| < \sum_{k=0}^{N}\sum_{l=0}^{M} \left|C_{kl}\right|$$

$$= \left|\sum_{k=0}^{N}\sum_{l=0}^{M} C_{kl}\right| = 1$$

(all the C_{kl} have the same sign) that makes our 1-D filter with transfer function $H\left(z^{-1}\right) = \frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}$ non-stable (in BIBO sense), but this contradicts to the assumption. So, this completes the Proof. ∎

Consider now the most general transformation

$$z^{-1} = \frac{f(z_1^{-1}, z_2^{-1})}{g(z_1^{-1}, z_2^{-1})} = \frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1} C_{kl} z_1^{-k} z_2^{-l}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2} D_{kl} z_1^{-k} z_2^{-l}}$$

under what circumstances this transformation would transform the prototype 1-D BIBO stable filter of (1) to a stable 2-D filter?

The 2-D rational function

$$z^{-1} = \frac{f(z_1^{-1}, z_2^{-1})}{g(z_1^{-1}, z_2^{-1})} = \frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1} C_{kl} z_1^{-k} z_2^{-l}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2} D_{kl} z_1^{-k} z_2^{-l}} = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} h(k,l) z_1^{-k} z_2^{-l}$$

It is easy to verify that a necessary and sufficient condition can be $\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} h(k,l) = 1$ and all the $h(k,l)$ to have the same sign. It is also known that the 2-D system $\frac{f(z_1^{-1}, z_2^{-1})}{g(z_1^{-1}, z_2^{-1})} = \frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1} C_{kl} z_1^{-k} z_2^{-l}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2} D_{kl} z_1^{-k} z_2^{-l}}$ is BIBO stable if and only if $\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \left|h(k,l)\right| \leq K < \infty$

After these preparations we are ready to prove the following theorem

Theorem 3. *Consider a prototype 1-D BIBO stable filter a filter with transfer function*

$$H\left(z^{-1}\right) = \frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)} \quad (1)$$

Under the transformation

$$z^{-1} = \frac{f(z_1^{-1}, z_2^{-1})}{g(z_1^{-1}, z_2^{-1})} = \frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1} C_{kl} z_1^{-k} z_2^{-l}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2} D_{kl} z_1^{-k} z_2^{-l}} = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} h(k,l) z_1^{-k} z_2^{-l} \quad (5)$$

where

with $\frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1} C_{kl}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2} D_{kl}} = 1$ *and* $\frac{f(z_1^{-1}, z_2^{-1})}{g(z_1^{-1}, z_2^{-1})} = \frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1} C_{kl} z_1^{-k} z_2^{-l}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2} D_{kl} z_1^{-k} z_2^{-l}}$

a stable 2-D system with $\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \left|h(k,l)\right| = 1$, *the prototype 1-D BIBO of (1) gives*

$$H_2\left(z_1^{-1}, z_2^{-1}\right) = \frac{A_1\left(z_1^{-1}, z_2^{-1}\right)}{B_2\left(z_1^{-1}, z_2^{-1}\right)} \quad (3)$$

where the new transfer function $H_2\left(z_1^{-1}, z_2^{-1}\right)$ *is also stable and the origin of the axes* $(\omega = 0)$ *is depicted to the point* $(\omega_1, \omega_2) = (0,0)$

Proof . It is easy to prove that necessary and sufficient condition for the depiction of the origin of the axes $(\omega = 0)$ to the point $(\omega_1, \omega_2) = (0,0)$ is also

$\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}h(k,l)=1$ which is equivalent from (5) that

$$\frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1}C_{kl}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2}D_{kl}}=1$$

For Stability we have also to prove that $B_2\left(z_1^{-1},z_2^{-1}\right)\neq 0$ for every z_1^{-1} and z_2^{-1} inside the unit bi-disk i.e. for every z_1^{-1} and z_2^{-1} with $\left|z_1^{-1}\right|<1$ and $\left|z_2^{-1}\right|<1$. This is true, because if one assumes that there are some ζ_1^{-1} and ζ_2^{-1} with $\left|\zeta_1^{-1}\right|<1$ and $\left|\zeta_2^{-1}\right|<1$ such that $B_2\left(\zeta_1^{-1},\zeta_2^{-1}\right)=0$, we would have a ζ^{-1} with $\zeta^{-1}=\sum_{k=0}^{N}\sum_{l=0}^{M}C_{kl}\zeta_1^{-k}\zeta_2^{-l}$ such that $B\left(\zeta^{-1}\right)=0$. On the other hand, since $\left|\zeta_1^{-1}\right|<1$ and $\left|\zeta_2^{-1}\right|<1$, we would have

$$\left|\zeta^{-1}\right|=\left|\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}h(k,l)\zeta_1^{-k}\zeta_2^{-l}\right|\leq\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left|h(k,l)\right|\left|\zeta_1^{-k}\right|\left|\zeta_2^{-l}\right|<1$$

all the C_{kl} have the same sign) that makes our 1-D filter with transfer function $H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}$ non-stable which is this contradicts to the assumption. ■

III. NUMERICAL EXAMPLES

Example 1. Consider the example of 6.4 of [27]. A 1-D (digital) IIR three-pole Butterworth filter is described as follows

$$H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}=K\frac{(1+z^{-1})^3}{(1-0.9047z^{-1})(1-1.9925z^{-1}+0.9065z^{-2})}$$

$$K=1/9000 \tag{6}$$

with magnitude response in Fig.1

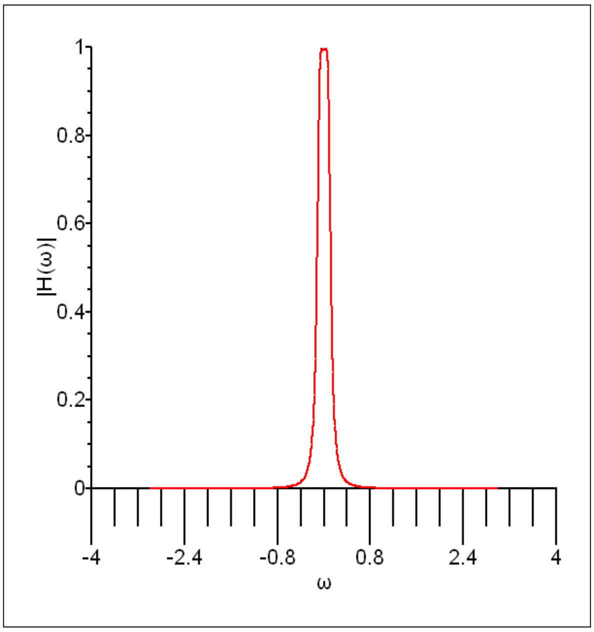

Fig 1. Magnitude response of the filter of (6)

Consider now the transformation

$$z^{-1}=C_1z_1^{-1}+C_2z_2^{-1}$$

where C_1,C_2 are real numbers with $C_1+C_2=1$ and $C_1C_2>0$ for example $C_1=C_2=1/2$

$$H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}=\frac{(1+z^{-1})^3}{(1-0.9047z^{-1})(1-1.9925z^{-1}+0.9065z^{-2})}=$$

$$=\frac{(1+z^{-1})^3}{(1-0.9047z^{-1})(1-0.9521e^{j0.08635}z^{-1})(1-0.9521e^{-j0.08635}z^{-1})}$$

that gives the 2-D (digital) IIR filter

$$H_2\left(z_1^{-1},z_2^{-1}\right)=\frac{A_1\left(z_1^{-1},z_2^{-1}\right)}{B_2\left(z_1^{-1},z_2^{-1}\right)}=$$

$$=\frac{(2+z_1^{-1}+z_2^{-1})^3}{(2-0.9047(z_1^{-1}+z_2^{-1}))(4-3.985(z_1^{-1}+z_2^{-1})+0,9065(z_1^{-1}+z_2^{-1})^2)}$$

$$=\frac{(2+(z_1^{-1}+z_2^{-1}))^3}{(2-0.9047(z_1^{-1}+z_2^{-1}))(2-0.9521e^{j0.08635}(z_1^{-1}+z_2^{-1}))(2-0.9521e^{-j0.08635}(z_1^{-1}+z_2^{-1}))} \tag{7}$$

with the 2-D magnitude response in Fig.2.

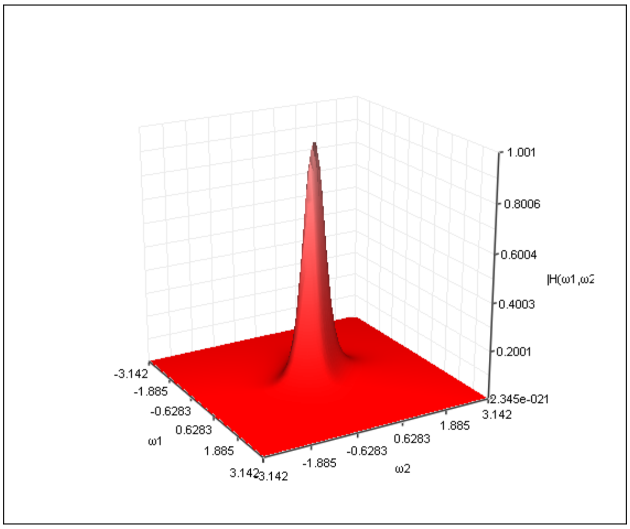

Fig.2 Magnitude Response of the 2-D filter of (7)

Example 2. Chebyshev filters ([28]) have the property that the magnitude of the frequency response is either equiripple in the passband and monotonic in the stopband or monotonic in the passband and equiripple in the stopband. The digital filter for this 4th-order Chebyshev I digital lowpass filter is expressed as follows:

$$H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}=\frac{0.001836(z^{-1}+1)^4}{(1-1.4996z^{-1}+0.84z^{-2})(1-1.5548z^{-1}+0.6493z}$$

$$=\frac{0.001836(z^{-1}+1)^4}{0.84(z^{-1}-1.0911e^{j0.6133})(z^{-1}-1.0911e^{-j0.6133})}\cdot$$

$$\cdot\frac{1}{0.6493(z^{-1}-1.2410e^{j0.2662})(z^{-1}-1.2410e^{-j0.2662})}$$

(8)

with magnitude response in Fig.3

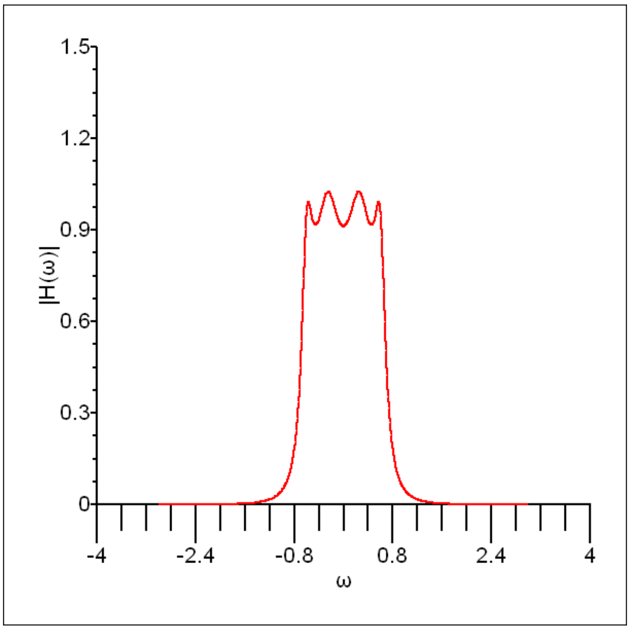

Fig 3. Magnitude response of the filter of (8)

Consider again the transformation $z^{-1}=(z_1^{-1}+z_2^{-1})/2$ one takes $H_2\left(z_1^{-1},z_2^{-1}\right)=\frac{A_1\left(z_1^{-1},z_2^{-1}\right)}{B_2\left(z_1^{-1},z_2^{-1}\right)}=$

$$=\frac{0.001836((z_1^{-1}+z_2^{-1}+2)^4}{0.84(z_1^{-1}+z_2^{-1}-2\cdot1.0911e^{j0.6133})(z_1^{-1}+z_2^{-1}-2\cdot1.0911e^{-j0.6133})}\cdot$$

$$\cdot\frac{1}{0.6493(z_1^{-1}+z_2^{-1}-2\cdot1.2410e^{j0.2662})(z_1^{-1}+z_2^{-1}-2\cdot1.2410e^{-j0.2662})}$$

(8)

with the 2-D magnitude response in Fig.4.

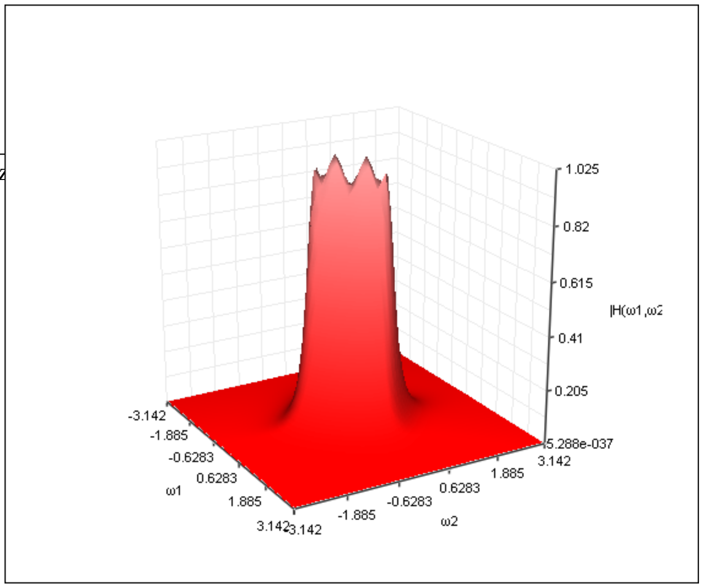

Fig.2 Magnitude Response of the 2-D filter of (8)

IV. Conclusion

New general transformations have been introduced for the design of 2-D (Two-Dimensional) FIR and IIR filters. It seems that this methodology can be viewed as an extension of the McClellan Transformations and can be applied in several cases of 2-D FIR and IIR filter design, while the McClellan Transformations are applied only for the design of 2-D FIR filters. Two Numerical examples illustrated the validity and the efficiency of the method. The proposed methods ensure stability in all the cases due to Theorems 1, 2, 3.

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