Multi-objective planning method for flexible adjustment to the aspiration levels of water supply system designers

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Abstract: - We propose a hybrid method using interactive multi-criteria programming and minimum cost flow optimization to make a useful schedule for waterworks. In case of constructing a mathematical model, it is difficult to adjust the method parameters in the actual application, and to make the user satisfied with the solution under the predetermined parameters. In our research, the parameters are generated automatically based on the abstract planning, in which interactive multi-criteria programming is employed, and then the minimum cost flow is calculated under the generated parameters. As a result, the proposed method can make an applicable quickly, reflecting the user’s aspiration flexibly.

Key-Words: - electric vehicle, energy resurrection, limitation of battery capacity, maximization of traveling point, round cost, monotonic increase

1 Introduction

Water supply system management involves planning the daily water intake, purified water quantity, and reservoir storage quantities in a water supply network. This kind of network transportation problem, having holders, can be reduced to a minimum cost flow problem in a multilayer extended network model (hereinafter “multilayer network model”) (1). Here, “cost” refers to a concept for the purpose of mathematical modeling; it does not need to be the actual cost of water transportation (2)-(4).

In optimization that is based on this kind of single objective function, the solution at the moment of formulation is determined implicitly, and skillful modeling results merely in the preconditions for obtaining a satisfactory solution. One approach to overcome this limitation is to formulate the problem as a multi-objective planning problem having multiple objective functions (5). In this case too, if we apply a scalarizing method that appropriately weights and combines the various vector-valued objective functions, it becomes effectively impossible to solve the problem of the single objective function (6)-(8). We suggest that this issue can be resolved by means of an interactive multi-objective planning method, in which information about the preferences of the decision makers is extracted interactively using the concept of “aspiration level” and used to determine the Pareto solution that most closely agrees with the aspiration levels.

However, if we formulate water supply as a multi-objective planning problem in a multilayer network model, as is, the scale of the problem becomes extremely large. As a result, it becomes impossible to employ a basis factorization method (1) (the technique that is usually applied to solve such a problem) and the required computational effort becomes enormous. A further difficulty is that as the model of the operation rules becomes larger, the problem becomes more complex and difficult.

In this paper, we present a method for resolving the above issues. The method works by applying the constraint that the water level of water reservoirs must be restored, which is generally considered the main aim of water supply system management, and it utilizes the fact that it is possible to develop separate and independent solutions for the problem at an abstract level, for planning of daily volumes, and at a detailed level, for planning water flows and water storage quantities on an hourly basis. The
detailed-level plan for achieving the planned volumes of the abstract level focuses on ensuring that the water reservoir level is restored. The proposed method produces a detailed plan by means of a minimum cost flow computation as usual, after automatically generating costs in a multilayer network model, based on the results of the plan at the abstract level.

2 Outline of water supply planning problems

A water supply system is made up of a network of pipeline or other conduits for distributing water to end users according to their demand by means of a series of connected processes involving the intake of raw water from water sources, transport of raw water to water purification plants, purification of raw water at the purification plants, transport of the purified water to water reservoirs, and distribution of water from reservoirs to end users on demand.

In order to ensure a plentiful supply of water in the event of an emergency, such as an earthquake disaster, it is also desirable that the rate of water storage at the reservoirs is maintained as high as possible at all times. Thus, it is also necessary for water volumes to be adjusted within the range of operational capacity; sometimes it may be necessary to connect and transfer water between water reservoirs. The more extensive and complex a water supply network is, the more difficult this problem is to solve. This is referred to as “water management”. In the event of a crisis situation, typified by drought, or when water supply is interrupted due to pump inspections or pipeline work, it is necessary to make quick and flexible decisions about water collection fees, water distribution adjustments, and reservoir operation, in accordance with the conditions of facilities. The guidelines usually applied in dealing with the operation of this kind of water transportation system are outlined below.

1) Quickly restore the water level of each reservoir
2) Minimize flow fluctuations in pipelines used for flow smoothing

Below, we discuss a method for formulating a plan to satisfy these requirements.

3 Proposed method

3.1 Multilayering a water system planning problem

If the condition for restoring the water reservoir storage volume is expressed such that the unit of time \( t \) in Eq. (1) is taken to be 1 day, then \( v_i(t - 1) = v_i(t) \). Equation (1) then becomes the following:

\[
\sum_{j \in N_i} x_j(t) = \sum_{j \in N_i} x_j(t) + d_i(t) \quad (1)
\]

When formulating the daily volume plan, treating the water reservoirs the same as other branch points enables the condition of reservoir water level restoration to be embedded in the constraint formula. However, because the supply of water from the water intake points must satisfy the demand for water, nodes that are considered to be large reservoirs connect between source and sink. Thus, the daily volume plan can be formulated under a multilayer network model with only a single layer, allowing the number of design variables to be greatly reduced.

By setting the intake volume as an objective here, the objective can be expressed in simple form as in the following equation:

\[
\sum_{i=1}^{P} \sum_{t=1}^{T} x_i(t) = \sum_{i=1}^{P} \sum_{j=1}^{m} d_i(t) = A \text{ constant} \quad (2)
\]

If one of the intake volumes is varied, at least one other intake volume changes. Hence, this setup has the advantage of making trade-off analysis relatively simple.

However, if we do not take into account this kind of change over time, it becomes impossible to plan the flow smoothing, such as to minimize change over time in reservoir management (transfer of storage volume between reservoirs) and in pipeline flow. Therefore, we propose here a method for automatically generating the costs in a multilayer network model based on the results of a plan formulated at the abstract level that sets the daily volumes.

Figure 3 shows the overall structure of the proposed system. The lower part, which determines the minimum cost flow in the multilayer network model, is solved using a traditional method. In this case, however, it is necessary to prepare the cost coefficients for each arc in advance by forming a database, and it becomes difficult to tune costs effectively. In view of this, our proposed method utilizes an interactive multi-objective planning method as a user interface, and as aspiration levels are obtained interactively, Pareto solutions to the problem are presented in the form of abstract plans. If an abstract plan that satisfies the designers can be created, a cost generator is then employed to produce
costs for the arc variables of the multilayer network model, as described above, based on the abstract plan. Then by determining the minimum cost flow under these dynamically generated costs, it is possible to obtain a detailed plan that reflects the various requirements of the system without having to tune costs in advance by trial and error.

As outlined above, separating the problem into separate layers—an abstract level (abstract planning level), which deals with daily volumes, and a detailed level (time-series flow planning level), which deals with hourly flow—enables the application of solution methods that take advantage of the characteristics of each particular layer. At the same time, it allows a substantial reduction in the computational effort required to solve the problem.

### 3.2 Multi-objective planning method at the abstract level

The daily pipeline flow volume \( X \) and demand volume \( D \) are defined as follows.

\[
X = \sum_{t=1}^{T} x(t) \quad (3)
\]

\[
D = x(0) + \sum_{t=1}^{T} b(t) \quad (4)
\]

Based on these, if we apply the conservation of flow rule on a daily basis, Eq. (2) can be simplified further to obtain the following:

\[
A_2 X = D \quad (5)
\]

Now, if we make the intake volume from the water source an objective at the abstract level, then the objective function in terms of the daily intake volume \( X_i \) from a given intake pipeline \( i \) can be simply expressed by the following equation:

\[
f_i(X) = X_i \quad (6)
\]

Here, representing the factor \( w_i \), for normalizing the difference between the objective function and aspiration level as the equation below, in terms of the ideal point \( f^*_i \) and the worst point \( f_i \), becomes second nature.

\[
w_i = \frac{1}{f^*_i - f_i} \quad (7)
\]

In some cases, \( f^*_i \) and \( f_i \) can be determined by an optimization calculation, but because this is inefficient in practice, the ideal point and worst point can be considered heuristically, resulting in the following:

\[
f_i^* = 0 \quad (8)
\]

\[
f_i = U_i - L_i \quad (9)
\]

Note that \( U_i \) and \( L_i \) are the upper and lower limit values for daily operation of pipeline \( i \), respectively. That is, \( L_i = l_i T \), \( U_i = u_i T \).

Thus, the auxiliary min-max problem in the multi-objective planning method becomes the following:

\[
z + \alpha \sum_{i \in N_{obj}} (X_i - \bar{X}_i)/(U_i - L_i) \rightarrow \min.
\]

\[
s.t. \quad X_i - (U_i - L_i) z \leq \bar{X}_i \quad (\forall i \in N_{obj}) \quad (10)
\]

\[
A_2 X = D
\]

\[
L \leq X \leq U
\]

where \( N_{obj} \) is the index set for pipelines directly connected to the intake points and \( \bar{X} \) is the aspiration level vector for \( f_i(x) \).

Of the objective functions \( f^{(k)}(x) \) with respect to the \( k^{th} \) solution \( X^{(k)} \), \( f^{(k)}_q \) represents a function that the designer wishes to improve. Therefore, at this point, the designer is asked to input a new aspiration level \( \bar{f}_q \), and a parametric linear planning problem for \( \Delta f^{(k)}_q = \bar{f}_q - f^{(k)}_q \) is defined as follows:

\[
z \rightarrow \min.
\]

\[
s.t. \quad X_q \leq X^{(k)}_q + \theta \Delta f^{(k)}_q
\]

\[
X_i - (U_i - L_i) z \leq X^{(k)}_i \quad (i \in N_{obj} \setminus \{q\}) \quad (11)
\]

\[
A_2 X = D
\]

\[
L \leq X \leq U
\]

In problem (11), up to \( \theta = 1 \) the solution \( X^{(k+1)} \) is presented by following a Pareto surface, and the
optimal basis inverse matrix when \( \theta = 0 \) can be obtained by applying the theory of sensitivity analysis to the final tableau in problem (10). From the obtained optimal basis inverse matrix, we can directly determine the upper limit \( \theta_{\text{max}} \) of the optimal value \( \theta \); if \( \theta_{\text{max}} < 1 \), then the Pareto curve bends until reaching the point of satisfying the new aspiration level, so at the point \( \theta = \theta_{\text{max}} \), the dual simplex method is applied to form a new optimal basis inverse matrix. This same calculation process is repeated until \( \theta_{\text{max}} \geq 1 \). If \( \theta_{\text{max}} \geq 1 \), \( \theta = 1 \) represents the point of aspiration level attainment.

Applying the above process, it is possible to plan the flow of the whole network interactively, based on daily intake volume.

### 3.3 Multi-stage Primal Method at the Detailed Level

If a daily flow of \( X_i \) is planned at the abstract level for pipeline \( i \), which is subject to smoothing, we can designate \( T_i^e \) as a period of time within a day during which the pipeline is usable. Here, “pipeline up time” is the sum of all the periods of time that water flows freely within the range defined by the upper and lower limits of water system operation. Conversely, “pipeline down time” is the sum of all the periods of time that water flow is interrupted for reasons such as inspections and pump stoppages. From these definitions, we obtain the following equation:

\[
X_i = \sum_{t=1}^{T} x_i(t) \quad (12)
\]

If flow smoothing is conducted ideally, the pipeline flow is maintained constant whenever the pipeline is in up time, and this constant value \( x_e \) can be defined as follows:

\[
x_e = \frac{X_i}{T_i^e} \quad (13)
\]

Thus, the desired flow volume per hour can be set as follows:

\[
goal_i(t) = \begin{cases} 
  x_e & (t: \text{pipeline up time}) \\
  0 & (t: \text{pipeline down time}) 
\end{cases} \quad (14)
\]

In this way, it is even possible to achieve flow smoothing while maintaining the abstract level plan. Hence, by creating a cost function, as shown in Fig. 4, so that a flow that satisfies Eq. (14) is a minimum cost flow, it is possible to flexibly determine an operation plan that satisfies the system operation requirements using a traditional method. By formulating problem in accordance with this cost, as described previously, the ideal minimum cost flow is defined as the flow value that restores the water levels of reservoirs to their original value after time \( T \), where the flow volume is constant in each of the pipelines subject to smoothing. If we restrict the variables used here to integer values, it is possible to determine a solution quickly, using a basic factorization method and a multi-stage primal method that utilizes integrality of the solution.

![Fig.4 An example of auto-generated cost function](image_url)

![Fig.5 A water supply network used for evaluation](image_url)

### 4 Numerical experiment

#### 4.1 Assumptions

To verify the proposed method, we formulated a plan using data from an existing water supply system, as shown in Fig. 5 (79 arcs, 48 nodes, 11 reservoirs, and 4 intake points per layer, over a planning period of 24 hours). We assumed the most typical weekday pattern for water demand. We compared the following three methods of computation.

**Multi-Stage Integer Programming (MSIP)**

This established method combines a multi-stage primal method with a smoothing process performed as a post-processing step.
Multi-Objective Programming (MOP) only
This is a multi-objective planning method applied under a multi-stage network model that defines objectives for water level restoration for 6 reservoirs having sufficient effective storage volumes and for flow smoothing for 12 pipelines.

Proposed
This proposed method separates the system modeling into two layers, an abstract level and a detailed level, and applies a multi-stage primal method of solution after automatically generating costs in the multilayer network model.

4.2 Computation time
Table 2 shows the results of a comparison of computing times. The CPUs used were an x58 processor rated at SPECint_base95=4 and an x86 processor rated at SPECfp_base95=2. The computation time for the proposed method was calculated as the sum of the time needed to formulate the initial abstract plan (time until solution for \( k=1 \) was obtained) and the time needed to formulate the detailed plan (same as for MSIP); it does not include the time for the repeated trade-off analysis done by the system designers when formulating the abstract plan to obtain a solution for \( k \geq 2 \). As made clear by Table 2, solving a large-scale problem such as that of a multilayer network model using the versatile revised simplex method necessitates a huge amount of computational effort. On top of this, when a large-scale problem is formulated using a multi-objective planning method, the objective functions become complex and large, which would appear to further increase the required computational effort. On the other hand, because with our proposed method the computation needed for the multi-objective planning method is greatly reduced and it is possible to utilize the multi-stage primal method, which requires little computation, the end result is a very substantial savings in computing time.

4.3 Abstract Level Planning Results
As described above, the established MSIP method requires only a small amount of computational effort, but it is necessary to prepare appropriate cost values in advance. To improve on this point, our proposed method applies an interactive multi-objective planning method at the abstract level. Table 3 shows an example of an abstract level plan. The objective functions represent the water intake volumes from four intake points (A through D). The top stage is the specified aspiration level, and the bottom stage is the plan values calculated under the top stage values. In the table, \( k \) represents the iteration of the solution produced. Now, let us assume a formulated plan capable of applying water intake restrictions at intake point D as a countermeasure in the event of a drought. Aspiration levels are input to specify how much water intake is desired at each of the intake points. Here, it is not necessary to pay attention to Eq. (2). Because there is a desire to restrict intake from intake point D, the aspiration level here is set to 0. The first obtained solution (\( k=1 \)) is the best approximation based on calculation of the extended Chebyshev distance for the given aspiration level vector. Or in other words, since the initial aspiration levels were all too difficult with respect to the prevailing demand, the optimum system solution is obtained by minimizing the maximum value of “objective non-achievement sensitivity” as normalized for each objective.

Table 3 An example of the tradeoff analysis on abstract level (Quantity of water intaken : ton)

<table>
<thead>
<tr>
<th>Solution Number</th>
<th>Intake A</th>
<th>Intake B</th>
<th>Intake C</th>
<th>Intake D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k=1 ) Aspiration</td>
<td>24,000</td>
<td>200,000</td>
<td>15,000</td>
<td>0</td>
</tr>
<tr>
<td>Plan</td>
<td>27,785</td>
<td>230,202</td>
<td>22,335</td>
<td>9,320</td>
</tr>
<tr>
<td>( k=2 ) Aspiration</td>
<td>27,785</td>
<td>237,701</td>
<td>24,156</td>
<td>0</td>
</tr>
<tr>
<td>Plan</td>
<td>20,000</td>
<td>243,965</td>
<td>25,677</td>
<td>0</td>
</tr>
<tr>
<td>( k=3 ) Aspiration</td>
<td>20,000</td>
<td>243,965</td>
<td>25,677</td>
<td>0</td>
</tr>
</tbody>
</table>

The designers find the initial solution unsatisfactory because the intake volume from point D could not be set to 0, so they once again set the aspiration level for the intake volume from point D to 0. The differences relative to the initial solution are that we now enter trade-off mode (as of \( k=1 \)), and that it is not necessary to set aspiration levels for all objectives. The points where no aspiration level is set are sacrificed in return for achieving the specified aspiration levels. As a result, in the second solution...
(k=2), the flow values do satisfy the requirement that the intake volume from D be 0, but at the price that the intake volumes from intake point B and C increase.

4.4 Detailed Level Planning Results
The time-series data of the detailed level plan results are shown in Fig. 6. As an example, the results of the proposed method are shown. Figure 6 shows the planning results over 24 hours for a particular pipeline subjected to flow smoothing. The horizontal axis indicates time, and the vertical axis indicates the hourly flow rate (ton/hr). We can see that smoothing was achieved within a variation range of ±300 tons relative to the average flow rate.

Fig.6 An example of time-series flow plan

5 Conclusion
We explained the validity of separating the plan into an abstract level and detailed level, and we proposed a method for automatically generating the costs in a multilayer network model based on the results of a plan formulation at the abstract level, which sets daily volumes. In addition, separating the problem into an abstract level that deals with daily quantities and a detailed level that deals with hourly flow rates, we could make use of solution techniques that take advantage of the characteristics of each layer. To flexibly adjust for the aspiration levels of the water system designers, we utilized an interactive multi-objective planning method as a user interface. That is, as the aspiration levels are interactively obtained, a Pareto solution can be determined to serve as the abstract level plan. If an abstract level plan that satisfies the water system designers can be created, a cost generator can then generate the costs of the arc variables of the multilayer network model, based on the abstract plan. Then by determining the minimum cost flow under these dynamically generated costs, it is possible to derive a detailed plan that reflects the various requirements without the need to tune costs in advance by trial and error.

Using data from an existing water supply network, we performed a verification of the plan formulation method. The results confirmed that all reservoirs absorb demand fluctuations by varying their water level, that after 24 hours the water level is restored to its original value, and that the above design goals above are even satisfied in terms of flow smoothing. It was also possible to greatly reduce computational effort and shorten computing time.

References: