About a Function Leading Economic System to Bifurcation Point

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Abstract: - Globalization, which replaced economic integration, has set new goals for economists. A unified economic zone removes distribution boundaries of a particular economy, changes scale of economic activity, and impact from external factors becomes an important element of economic system. Those facts not only explain a certain stage in creating economic relationships, but also entirely change major paradigms of economic concepts. As any physical or natural system, economy passes through order and chaos stages during its development.

Key-Words: - non-linearity, bifurcation, extremum of function, catastrophe theory

1 Introduction

An open economic system can never achieve balanced condition in long term. This is one of the paradigms which distinguishes a classic economic theory from a modern one. If a classic economic theory assumes existence of internal market mechanism, leading the market to balanced condition, on the other hand a modern theory of global economic zone implies several balanced conditions. Selection of a particular balanced condition is made by the state. Existence of several balanced conditions is based on a principle of non-linear behavior of a particular economy in a time of globalization. Non-linearity is a non-proportional reaction of internal components of the system to external irritants. A particular economic system can develop in two possible trajectories. This is evolutionary one, i. e. when an economic system operates in a mode of consistent system quality or develops in a linear mode. This trajectory is highly predictive and enables extrapolative forecasting. The second trajectory is bifurcation. In this case economic system becomes sensitive to external disturbances.

2 Problem Formulation

2.1 Methodological issues of nonlinear dynamics

Modern economic relationships cannot be explained from the perspective of classic economic theories. Bifurcation processes, attained size, existence of several sustainable trajectories cannot be explained from the perspective of classic economic theories. Market balancing mechanisms are not enough for market development in a time of globalization. Existence of several equal trajectories, appearing after passing through bifurcation points, is common for modern trends of national economies development. There can be a sequence of bifurcations going one after another. This can lead a system to unpredictive mode. Unpredictability occurs because number of possible trajectories for development of a particular economic system is growing in arithmetic progression with every new loop of bifurcations. At first there are two trajectories, on the next stage each one of those trajectories is divided by two, and the number grows to 4, then 8, etc. All of this makes non-linearity issues very important to study, and multidisciplinary approach makes them a research topic, helping understand similarity of economic processes and physical phenomena. (2) Without a doubt it makes an impact on methods to research modern economic processes.

When creating models of social and economic processes in the face of globalization, it is necessary to remember about non-linear behavior of systems, as well as determined (managed) character of the chaos which is inevitable during transition of the economic system from one state to another. Also we should remember that the smallest change in initial conditions can exponentially influence on behavior of economic system at the end of development trajectory. (1) Similar to physical systems, economic systems are dissipative too. Whereas in physics this means derivation of energy from the system, in economics this means extraction of economic resources from the system. Extraction of economic resources is reduction of proceeds from taxes, custom charges, etc. As external factors the

author of article states not only change of global prices on energy products, but also innovation performance. In order to formalize the task for usage of mathematical tool, regarding accuracy of measure units, we use metric of innovation investments. This value is measured in monetary units. Also it is necessary to focus on the fact that in the closed economic systems some variables depend on simultaneously changing metrics of other variables, i.e. on the current state of the system. Presence of variables with lag means that behavior of the system is defined not only by its current state, but also by trajectory through which this system has come to this state. This is not just a new paradigm, but also a new glance on economic task from the mathematic modelling perspective. Rather than using functional dependencies of some variables from the others, these models use functionality of variables' trajectories. Here trajectories are elements of mathematic models.

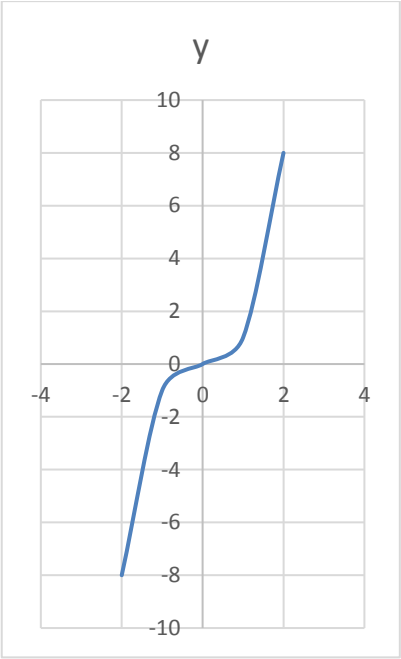


Fig.2

3 Problem Solution

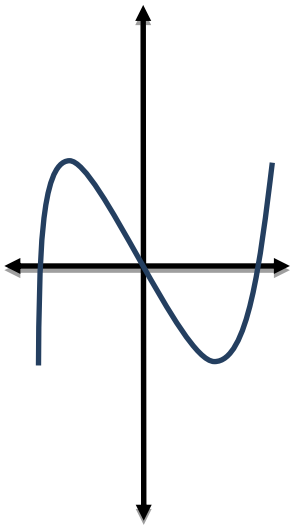
3.1 Cubic function and its properties

In author's opinion, the simplest function describing behavior of economic system based on such external factor as volume of investment into innovation, is a cubic function looking like this

$$f(x) = x^3 + \lambda x$$

Behavior of this function depends on parameter λ. If $\lambda < 0$, system behavior will correspond to Fig.1, if λ=0, system behaviour will correspond to Fig.2, system behaviour with $\lambda > 0$ is shown on Fig.3.

Fig.1



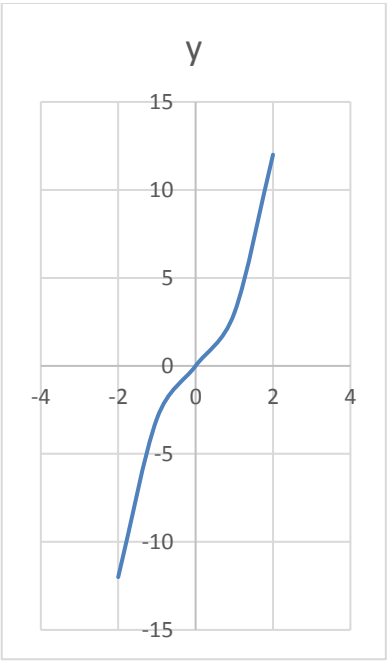


Fig.3

When $\lambda < 0$, every value (y) will correspond to 3 values (x), when $\lambda = 0$ function converts to an ordinary cubic parabola, when $\lambda > 0$ extremum points disappear. We can confirm this by equaling the derivatives of this function to null and resolving the equation based on them. In this case when λ is negative, $x = \sqrt{\frac{\lambda}{3}}$, $x_2 = -\sqrt{\frac{\lambda}{3}}$. $x_3 = 0$

If $\lambda > 0$, then X doesn't have valid roots, so in this case it doesn't have extremum points. This can be seen on the graph.
If $\lambda = 0$, this function converts to an ordinary cubic parabola, i.e. it is not typical. But if we consider a family (cubic ones in this case) of functions depending on a parameter (in our case this is λ), this parameter with particular values can make a cubic attribute prevailing. It means that behavior of all functions with parameter values close to λ is very similar.

Let's equal this function to null, making the following equation:

$$x^3 - \lambda x = 0$$

A fundamental attribute of this equation is that when λ parameter exceeds a particular threshold, number of solutions of this equation spasmodically grows from one to three. This indicates presence of bifurcations. In catastrophe theory this bifurcation is called a "fork".

3.2 The economic meaning of bifurcation

If we accept that this function shows dependency of earnings from sensitivity of particular economics to innovation investments, the parameter indicates investments. If it is positive, this means that investments come from production profits. Negative value of parameter shows external investment. I.e., production cannot receive investments from its own profits. As a result, with certain parameter values the system is subjected to catastrophe. Economic sense of the parameter can change depending on a certain economics and economic situation, but system behavior will be approximately the same. Another example of "fork" bifurcation: as we know from theory, upper limit of economic growth is full employment of all labor force, which leads to high composite demand. After that further growth of composite demand does not increase aggregate supply and correspondingly does not lead to growth of national product metrics. This is bifurcation point. As a result, demand on labor force grows on the job market, while labor force supply remains the same, leading to situation where growth of wages outruns growth of workforce productivity. In this case there are two scenarios. In the first scenario the government interferes into the process and creates conditions to attract highly qualified immigrant workforce to the country. This helps equalize demand on labor force with the supply, ensuring further growth of national economy. Population growth represented by immigrant workforce increases aggregate demand, and with other conditions being equal it increases aggregate supply and leads to economic growth.

Another scenario, without government interference, outrunning wages growth in comparison to national product growth, will lead to inflation. In the end this will cause drop of money purchasing power and decrease of aggregate demand. The next stage of economic crisis will be decrease of production volumes and growth of unemployment.

In the third scenario, after bifurcation point the government invests into economy, enabling workforce productivity growth, leading to growth of productivity with the same workforce, as well as growth of aggregate demand and aggregate supply. As a result, economy continues to grow.

As we can see, after bifurcation point there are three scenarios, two of which lead to economic growth, and the last one causes stagnation. Stagnation can occur without external interference, and economic growth requires governmental impact, i.e. external factor. In this regard we can also look at inflation impact on budget deficit. This problem arises when there are several equal inflation points, bringing money market and financial policy to equal state. Depending on at which point of equality on development trajectory the inflation metric stands, it can correspond to a certain position on money market. Also a certain monetary policy can lead to different changes in fiscal area depending on at which point of equality this monetary component is located. Also it is possible that the same values of state debt correspond to three metrics of monetary growth rate. When state debt metrics change, inflation values can change spasmodically. This is bifurcation. (3).

Quite often analysis of economic system behavior is performed in stochastic mode. This can be explained by random factors defining behavior of the system. In this case the approximating bifurcation point can be determined based on growing dispersion of parameters in a particular economic system. Apart from growing dispersion, also noises can appear or amplify. Non-linear economic systems are not prone to determinacy, but also can be influenced by random factors. As distinct from linear ones, in non-linear economic systems presence of random factors, even with null averages, can substantially impact on sustainability and further behavior of the system.

4 Conclusion

Mathematical methods which are used to model social and economic processes, can be conditionally divided to three groups. Those are balance methods, optimization methods and econometric models. Linear approach to solving tasks is common for all these models. If linearity principle is violated, non-linear function (usually, quadratic function) is linearized using methods of variable replacement or by taking of logarithm. This is convenient from mathematics point of view. Economic sense of non-linearity is much deeper, than in math. If an abstract non-linear task can be linearly approximated and receive an answer with satisfying approximation within particular boundaries, in terms of constant economic progress discrepancies can be comparable to results. This explains why there were no forecasts for the last economic crises beginning from 2000s financial crisis which began from mortgage crisis in USA.

Extensive economic development makes linearity and stationarity such formulas which are confirmed in retrospective. Consistency of statistic metrics of economic system within entire development trajectory allow a researcher to get rather accurate results in linear forecasts of non-linear processes using econometric models. Stationary solutions of linear equations describe processes which do not depend on initial conditions. Apart from solution simplicity, stationary solutions of linear equations have another advantage – simple identification of the major managing factor which influences on economic development of a particular country. For instance, for Azerbaijan economy this is petroleum component.

The specified methods for forecasting social and economic systems cannot be used in the age of globalization. A unified economic zone requires to change paradigms which served as a basis for researches and forecasting particular economic systems. This is explained by several reasons. Firstly, this is necessity to account for impact of external factors on behavior of economic system. External factors are global markets conjuncture, global prices on petroleum and other energy products, etc. Influence of external factors can be more meaningful than internal factors. Secondly, a lot of freedom levels in the system parameters. In other words, a lot of independent variables, describing behavior of the system. Thirdly, this is scaling effect which should always be accounted for. This leads to inconsistency in functions which describe a particular economic task.

In stochastic tasks it is necessary to know which distribution any given task is related to. We perceive all rows in economic models as properly distributed random values. All models which eventually resolve to econometrics should also be built on properly distributed random values. Those are models built based on Cobb-Douglas production function. We need to remember that proper distribution does not apply to spasmodic processes or so-called aggregation processes. These are processes development of which requires accumulation of energy or resources in our case. Information is very important for such complicated economic systems, as financial markets. Every consecutive trade on financial markets can depend on positive or negative information received some time earlier. And it is hard to predict what information is positive for a particular financial market, and what information is negative. This makes current events on a particular financial market dependent on events of the day before.

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