**Could the coefficients re-estimation solve the industry or time specific issues?**

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**Abstract:** The aim of this paper is to examine discrimination performance of three bankruptcy prediction models in environments and periods different from the ones utilized by deriving the models. We compared selected models’ accuracy in the original setting and present conditions. Secondary aim was to examine a way of possible increasing of the discrimination performance of models by the recalculation of the classification functions. Discrimination performance of the models and financial ratios was tested on companies operating in manufacturing business. Results conclusively demonstrate that the discrimination accuracy of bankruptcy models deteriorates significantly in different environments. The classification function of each model was recalculated using the data from Czech manufacturing companies. For the adjustment of models’ coefficients the same methods, as used originally by theirs authors, were applied, i.e. the probit method, the linear discrimination analysis and the logit method. The results shown, that the re-estimation of model coefficient could lead to its higher classification accuracy in alternative conditions. We can dedicate that recalculating of the classification rules is one of the ways to increase discrimination performance of the bankruptcy prediction models in different environment.

**Key-Words:** Bankruptcy models, Discrimination capability, Financial ratios, Model robustness, Manufacturing, Linear discrimination analysis, Probit and Logit method, AUC value

1 **Introduction: The topic importance**

Bankruptcy of a company has a profound negative effect on all involved stakeholders. It follows that causes of bankruptcy and its timely prediction has been of utmost interest both in theoretical and applied research. The cumulative losses associated with the bankruptcy are defined as costs of bankruptcy.

Since bankruptcies are often result of excessive level of indebtedness, identifying an optimal capital structure has been the most important topic of previous research along with identifying the magnitude of bankruptcy costs. In this context Altman writes: „If bankruptcy costs are relatively significant then it may be argued that at some point the expected value of these costs outweighs the tax benefit derived from increasing leverage.“ – see [3]. Similarly, Kraus and Litzenberger proved that the finding the optimal capital structure must arise as a compromise between savings realized from tax shield and magnitude of potential costs of financial distress [23]. Both of these quantities are a rising function of indebtedness.

Previous research recognized two separate types of costs of bankruptcy – direct and indirect. Among direct costs we identify costs of insolvency proceedings as well as managers’ and employees’ salaries for the period of administrative claims associated with financial distress – see [43]. Indirect costs have the highest impact on the company value, since they represent loss of potential revenues. Opler and Titman describe the indirect costs as a twofold loss of credibility – in the face of both the customers and the suppliers [34]. Customers are less likely to trust the company to provide quality service and guarantee and suppliers may enforce higher costs in fear for repayment of their unsettled claims. Another indirect bankruptcy costs cited in [31] are tightening of investment spending and eventual sale of assets. Chen and Merville identified risk of lowering of prices by competitors in order to attain market share of the company in danger of bankruptcy [9]. Additionally, according [43] managers’ demands for higher salaries as a compensation for increased risk of loss of employment. Altman reached the following conclusion, based on his empirical analysis: „In
many cases bankruptcy costs exceed 20% of the value of the firm measured just prior to bankruptcy and even in some cases measured several years prior. On average, bankruptcy costs ranged from 11% to 17% of firm value up to three years prior to bankruptcy". – see [3].

1.1. Testing various methods of improving the discrimination accuracy of models
Absence of a sufficient number of observations concerning bankrupt companies tends to favour the models created in different environments or even in another period against the creation of one’s own models. Altman created the first bankruptcy model. In response to these works, more bankruptcy models were created - see [1], [2], [4], [13], [33], [37], [39], [40], [41], [45], and many others. The Altman model is among the most cited and hence the most known model. The original version of the Altman model was intended only for companies listed on the capital market. Later the modification of the model was published for companies not listed in the capital market: the so-called revised Z-score [4]: which became very popular even in our conditions. The modification of the model that dates from 1983 enabled its wider use, which was probably contributed to by the simplicity of the formula. The popularity of the model is summarized by [28], according to whom the Altman model (see [2]) was still robust, even though it had been developed more than 30 years ago. This view was also confirmed by other studies – see [5], [15], [26], [38]. Conversely, some authors have come to the opposite conclusion – see [44], [16]. The results of these researches show that discriminative accuracy of models significantly decreases if the model applies in another industry, period or economic environment than their original environment – see [16], [32], [35], [44]. Some authors assume that accuracy of bankruptcy prediction models depends on the situation of country’s economy: According to their results bankruptcy prediction models are more accurate when GDP of the country grows at low rate, i.e. growth of economy is not very high [24]. Kaplinski claims that bankruptcy prediction models should be adjusted to the economic conditions of the given country or even industry [21]. A possible explanation for this could be that the significance of bankruptcy predictors is not stable over time or that these predictors are specific for a given time, place and industry. Such arguments are motivating efforts aimed at creating new bankruptcy prediction models.

In our paper we test the current accuracy of the Zmijewski, Springate and Tserng model in the conditions of alternative conditions under which the models were created, namely in the conditions of Czech manufacturing companies. Moreover, the aim is to find whether the accuracy could be enhance by re-estimating the classification function of the model.

2 Sample and methods used
The sample includes the financial statements of 1,508 companies in the manufacturing industry (NACE rev. 2 main section C), operating in Czech republic, of which 628 companies are financially healthy (active), and 880 companies, which went bankrupt in the following year (bankruptcy). In the sample, all companies were included whose data were contained in the database and which went bankrupt in the period 2007 – 2012. The number of observation in each sample is shown in table below.

Table 1 Number of observations

<table>
<thead>
<tr>
<th></th>
<th>Learn</th>
<th>Test</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bankrupt</td>
<td>432</td>
<td>196</td>
<td>628</td>
</tr>
<tr>
<td>Active</td>
<td>635</td>
<td>245</td>
<td>880</td>
</tr>
<tr>
<td>Sum</td>
<td>1067</td>
<td>441</td>
<td>1508</td>
</tr>
</tbody>
</table>

Source: Our own analysis of data from the Amadeus database
As the aim of the paper is not only to test the model, but also to derive and test an adjusted classification rule the sample needed to be split into the learning and test sample. The sample was randomly split into the learning subsample (70% of the data) and test subsample (30%).

In course of this research, we tested three different models. Moreover, these models apply different classification methods. Namely, we test the Zmijewski model [45] which applies the probit method, the Springate model which applies linear discrimination analysis [20] and the Tserng model (see [46]) which applies the logit method. Authors [46] publish four alternatives of their model, in this paper we apply the model number 3, as this version of the model is suitable for publically unquoted companies.

2.2 Zmijewski model
The model could be described by following formula:
\[ p = \Phi(X), \]
(1)
where
\[ X = -4.3 -4.5\cdot EAT/TA+5.7\cdot TL/TA+0.004\cdot CA/CL \]
(2)
and
\( p \) – predicted probability of bankruptcy, \( \Phi \) – cumulative distribution function of standard normal distribution, \( EAT \) – earning after taxes, \( TA \) – total assets, \( TL \) – total liabilities, \( CA \) – current assets, \( CL \) – current liabilities

2.3 Springate model
The model could be described by following formula:
\[ S = 1.3\cdot NWC/TA+3.07\cdot EBIT/TA+0.66\cdot EBT/CL+0.4\cdot S/TA \]
(3)
Where
\( NWC \) – net working capital, \( EBIT \) – Earnings before interest and taxes, \( EBT \) – earning before taxes, \( S \) – sales.

Bankrupt if \( S < 0.862 \)

2.1 Tserng model
In this paper we test model number 3 (see [46]), this model takes a following form:
\[ P = 1/(1+\exp(-T)) \]
(4)
where
\[ T = -0.109\cdot CA/CL+1.978\cdot TL/TA-0.268\cdot S/TA-4.793\cdot EBIT/TA-3.456 \]
(5)
The probit and the logit model are applications of the inverse density function of the normal or logistic distribution. The probit model can be written in the form, see [17]:
\[ P_i = \int_{-\infty}^{\alpha + \beta x} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} t^2 \right) dt \]
(6)
\( \alpha, \beta \) are estimated parameters, \( x \) is the vector of independent predictors (here financial indicators), \( P_i \) is the probability of default (bankruptcy).

The logit model can be written in the form, see [17]:
\[ P_i = \frac{\exp(\alpha + \beta x)}{1+\exp(\alpha + \beta x)} \]
(7)
The Linear discriminant analysis (LDA) is a special kind of discriminant analysis, which adds the assumption of identical covariance matrices \( \Sigma_k \). Under these assumption the discriminant rule, based on the Mahalanobis distance, can be written as follows [18]:

For active:
\[ x^T \Sigma_{1}^{-1}(\mu_{1} - \mu_{2}) > 1/2(\mu_{1} + \mu_{2})^T \Sigma_{1}^{-1}(\mu_{1} - \mu_{2}) \]
(8)
For bankrupt:
\[ x^T \Sigma_{1}^{-1}(\mu_{1} - \mu_{2}) < 1/2(\mu_{1} + \mu_{2})^T \Sigma_{1}^{-1}(\mu_{1} - \mu_{2}) \]
(9)
Where
\( x \) is the vector of independent predictors, where \( x = (x_1, x_2, \ldots, x_n) \), \( \mu_k \) is the vector of mean values of the quantity \( x \) \(-\)th group, \( \Sigma_k \) is the covariance matrix of the \( k \)-th group, \( \pi_1 \) or. \( \pi_2 \) is apriori the probability of units belonging to the group corresponding to the range group 1 or 2.

3 Results
The question of this research is how much the original classification function fits the alternative conditions. The models were applied in alternative economic condition (country). The classification rule of each model was recalculated by using the data from the learning subsample. For this adjustment an original method was used, i.e. for Zmijewski model the probit method was used, for Springate model the linear discrimination analysis.
was used and finally the logit method was used in case of Tserng model.

3.1 Re-estimated function for Zmijewski model
The details of recalculating the Zmijewski model are following:

Table 2 Details of re-estimated Zmijewski model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>2.01343</td>
<td>0.143203</td>
<td>197.6843</td>
<td>0.000000</td>
</tr>
<tr>
<td>EAT/TA ***</td>
<td>0.03858</td>
<td>0.014499</td>
<td>7.0811</td>
<td>0.007790</td>
</tr>
<tr>
<td>TL/TA ***</td>
<td>-2.00860</td>
<td>0.168889</td>
<td>141.4442</td>
<td>0.000000</td>
</tr>
<tr>
<td>CA/CL</td>
<td>0.03714</td>
<td>0.025738</td>
<td>2.0820</td>
<td>0.149041</td>
</tr>
</tbody>
</table>

Note: ***statistically significant at 1% level. Source: Our own analysis of data from the Amadeus database

Zmijewski model applies three variables, the return on assets (EAT/TA), the total indebtedness (TL/TA) and current ratio (CA/CL) and the constant. The return on assets and total indebtedness are statistically significant at 1% level, however the current ratio is not significant at any standard level.

The adjusted function of the Zmijewski model, which should better fit the data, is following:

\[ Z_{\text{re-estimated}} = 0.0386 \times \text{EAT/TA} - 2.009 \times \text{TL/TA} + 0.037 \times \text{CA/CL} + 2.013 \]  

(10)

3.2 Re-estimated function for the Springate model
The classification function was recalculated for the Springate model by using the linear discrimination analysis, as it is the same method as originally applied. Details are listed in following table.

Table 3 Details of re-estimated Springate model

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>WC/TA ***</td>
<td>0.9931</td>
<td>24.55</td>
<td>0.000001</td>
<td>0.885</td>
</tr>
<tr>
<td>EBIT/TA **</td>
<td>0.9724</td>
<td>5.816</td>
<td>0.016083</td>
<td>0.497</td>
</tr>
<tr>
<td>EBT/CL**</td>
<td>0.9714</td>
<td>4.859</td>
<td>0.027755</td>
<td>0.366</td>
</tr>
<tr>
<td>S/TA ***</td>
<td>0.9807</td>
<td>13.28</td>
<td>0.000283</td>
<td>0.882</td>
</tr>
</tbody>
</table>

Note: ***statistically significant at 1% level, **statistically significant at 5% level. Source: Our own analysis of data from the Amadeus database

All the variables of the model are statistically significant. The relative size of net working capital (WC/TA) and the total assets turnover (S/TA) are statistically significant at 1% level, and return on assets (EBIT/TA) and the ratio of earning before tax and current liabilities (EBT/CL) are significant at 5% level.

The re-estimated function for the Springate model is following:

\[ S_{\text{re-estimated}} = -0.0762 \times \text{WC/TA} + 0.029 \times \text{EBIT/TA} - 0.0293 \times \text{EBT/CL} - 0.0179 \times \text{S/TA} \]  

(11)

Bankrupt if \[ S_{\text{re-estimated}} > -0.8808 \]

3.3 Re-estimated function for the Tserng model
The model was re-estimated by using the logit method.

Table 4 Details of re-estimated Zmijewski model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>4.2945</td>
<td>0.3901</td>
<td>121.158</td>
<td>0.000000</td>
</tr>
<tr>
<td>CA/CL</td>
<td>0.0448</td>
<td>0.0883</td>
<td>0.2570</td>
<td>0.612214</td>
</tr>
<tr>
<td>TL/TA ***</td>
<td>-4.6061</td>
<td>0.4134</td>
<td>124.088</td>
<td>0.000000</td>
</tr>
<tr>
<td>S/TA ***</td>
<td>0.0952</td>
<td>0.0302</td>
<td>0.0001</td>
<td>0.993447</td>
</tr>
</tbody>
</table>

Note: ***statistically significant at 1% level. Source: Our own analysis of data from the Amadeus database

The model incorporates five variables, the current ratio (CA/CL), the total indebtedness (TL/TA), the total assets turnover (S/TA) and return on assets (EBIT/TA). Only the return on assets and the total indebtedness are statistically significant variables of the model, they are significant at 1% level. The current ratio and the total assets turnover are not significant at any standard level. The re-estimated function of this model could be written in following way:

\[ T_{\text{re-estimated}} = 0.045 \times \text{CA/CL} - 4.606 \times \text{TL/TA} - 0.0004 \times \text{S/TA} + 0.095 \times \text{EBIT/TA} + 4.295 \]  

3.4 Comparing the models accuracy
For testing the accuracy of the models, the ROC curves and the Area Under Curve (AUC) were applied. Both, the original version of the models and the re-estimated versions of the models were tested on the test sample (30% of the data), to ensure that both versions of models are tested out-of-sample. The models were tested for different time prior bankruptcy, from a year prior bankruptcy (further
referred as time t+1) up to five years prior bankruptcy (time t+5).

Table 5 AUC values of the tested models

<table>
<thead>
<tr>
<th>Model</th>
<th>Time</th>
<th>Original</th>
<th>Re-estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zmijewski</td>
<td>t+1</td>
<td>0.436</td>
<td>0.535</td>
</tr>
<tr>
<td></td>
<td>t+2</td>
<td>0.485</td>
<td>0.508</td>
</tr>
<tr>
<td></td>
<td>t+3</td>
<td>0.542</td>
<td>0.469</td>
</tr>
<tr>
<td></td>
<td>t+4</td>
<td>0.568</td>
<td>0.476</td>
</tr>
<tr>
<td></td>
<td>t+5</td>
<td>0.556</td>
<td>0.454</td>
</tr>
<tr>
<td>Springate</td>
<td>t+1</td>
<td>0.825</td>
<td>0.830</td>
</tr>
<tr>
<td></td>
<td>t+2</td>
<td>0.678</td>
<td>0.679</td>
</tr>
<tr>
<td></td>
<td>t+3</td>
<td>0.630</td>
<td>0.625</td>
</tr>
<tr>
<td></td>
<td>t+4</td>
<td>0.651</td>
<td>0.642</td>
</tr>
<tr>
<td></td>
<td>t+5</td>
<td>0.584</td>
<td>0.584</td>
</tr>
<tr>
<td>Tserng</td>
<td>t+1</td>
<td>0.854</td>
<td>0.901</td>
</tr>
<tr>
<td></td>
<td>t+2</td>
<td>0.789</td>
<td>0.888</td>
</tr>
<tr>
<td></td>
<td>t+3</td>
<td>0.742</td>
<td>0.851</td>
</tr>
<tr>
<td></td>
<td>t+4</td>
<td>0.766</td>
<td>0.829</td>
</tr>
<tr>
<td></td>
<td>t+5</td>
<td>0.659</td>
<td>0.756</td>
</tr>
</tbody>
</table>

Source: Our own analysis of data from the Amadeus database

Speaking about the original version of the model we can say, that the AUC values (for t+1) for Springate and Tserng model are relatively high (0.825 vs. 0.854), however the AUC value for Zmijewski is very low, only 0.436.

The modified version showed slightly better results. In case of Tserng model, the AUC values of the re-estimated version of the model is higher in all the analysed periods prior bankruptcy. The original version reached highest AUC value of 0.854 (in t+1), however the re-estimated version reach (in the same period) 0.901.

The same applies for Springate model, but only for period t+1 and t+2, in other periods the re-estimated version of the model do not reached higher values of AUC. Very similar results could be found in case of the Zmijewski model, where the re-estimated version reached better values only for period t+1 and t+2.

4 Discussion

There is no consensus in the current literature about the issue of historical bankruptcy models accuracy in case that the models are applied under other than original conditions. These authors have indicated that the predication accuracy of bankruptcy models falls markedly when they are applied to a different industry, period or economic environment than their original environment [16], [32], [35], [44].

When speaking about the change in the model accuracy, from theoretical point of view, there are three possible explanations. First, there is a shift in cut-off score, second, the coefficients of the model are not suitable or third, the model incorporates variables that are not significant under the analysed conditions (periods or economic environment).

In this paper, the accuracy of the models were analysed in terms of ROC curves, respectively the AUC values. This allow us to abstract from the current set of cut-off score. Analysing the other two issues requires to re-estimate the models and tested the re-estimated version of the model on the same data set. For this purpose use the same methods as was originally used by the authors of the analysed models. It was found, that the re-estimation of model coefficient could lead to higher classification accuracy in alternative conditions, at least in two periods prior bankruptcy. This effect was more significant in cases of Tserng and Zmijewski model, rather than in case of Springate model. The possible explanation for that is that only in case of Springate model all the model’s variables are significant. Re-estimating the model coefficient might increase the weight of significant indicators and on the other hand to decrease the weight of the insignificant.

In case of Zmijewski model the current ratio, which measures the company’s solvency, does not represent a significant variable. The same applies for the Tserng model, which incorporates also the current ratios and is not significant too. Springate model incorporates other measure of company’s solvency – the relative size of net working capital (WC/TA), which represents a significant variable. The WC/TA ratio is frequently used in bankruptcy models, see [1], [39] or [44]. Further analysis of the models variables showed that there are two more significant variables, the return on assets (EBIT/TA of EAT/TA) and the total indebtedness (TL/TA). The importance of these ratios is highlighted by the fact, that they are significant in different models, which applies different methods. This is in line with other authors’ results. As EBIT/TA ratio is the strongest predictor of most of Altman’s models and the one that often appears in other studies, e.g. [27], [30], [36]. EBIT/TA is one of two accounting indicators that stood the test of Shumway’s criticism regarding the relevance of financial indicators [39].
5 Conclusion
The presented paper dealt with the accuracy of bankruptcy prediction models and their application under alternative conditions. It was found, that the classification functions of the models do not fit the alternative conditions. Based on that, there is a need of re-estimating the models for current conditions. This need is more obvious in cases, where the variables of the model do not represent significant predictors of bankruptcy. In the presented research, it was abstracted of the possible shift of grey zone borders, as the accuracy was evaluated in terms of ROC curves.

References:


