The Dual Aspects of Accounting Transactions and Asset Value Change in the Accounting Equation

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Abstract: - The purpose of the study is to analyze the accounting equation and the relation between assets and claims on the assets (liabilities plus stockholders’ equity), based on the dual aspects of accounting transactions. The methodology is rationalistic and analytical. The analysis consists of the application of the identity and characteristic functions, as well as a coordinate transformation. Results show that the accounting equation does not consist in a single addition, what would lead to inequality, but a series of addition functions when taking into account the dual aspects of accounting transactions. Due to the different number of summation dimensions on each side of the final equation, a coordinate transformation is applied resulting in a change in the value of assets; this value is not the same as that of the initial accounting equation.

Key-Words: Dual aspects, accounting transactions, accounting equation, balance sheet.

1 Introduction
This paper addresses the issue of the equality of assets to claim on the assets, in the balance sheet of financial statements. The idea that assets are equal to liabilities plus stockholders’ equity (claim on the assets) is crucial in financial statements. The bases for this idea are the dual aspect of the accounting transactions, the double-entry bookkeeping, and the accounting equation.

An accounting transactions must be recorded in two accounts with different signs in a double classification system [1], i.e. the recording of the same monetary units in two accounts with opposite signs. That is the recognition of its duality; every single transaction has two different properties, such as being a debit and a credit simultaneously. In accounting practice, the double-entry bookkeeping system is the way of registering the accounting transactions in financial statements, based on their dual aspects. Finally, the accounting equation is a checkpoint of the balance sheet correspondence with the accounting assumption and practice.

Some approaches to financial statements try to provide a different perspective, such as quantum accounting (see [2], [3]), and triple-entry bookkeeping ([4], [5]); however, they do not put into question either the dual aspect of accounting transactions or the accounting equation. Another way of depicting the financial reality of business is by fair value accounting; this method introduces a more comprehensive analysis of the accounting information [6]. It is, somehow, a critic of the dual aspects of accounting transactions, but without making it explicit. Due to the different assets and claims on the assets structures, another research proposed an inequality as the proper relationship between assets and claims on the assets [7].

The previous findings stir some issues that deserve attention.

2 Problem Formulation
The assets and claims on the assets sides of the accounting equation have different item structures; they do not comprise the same accounts. However, they have the same monetary units, by the dual aspects of the economic transactions.

It, therefore, seems comprehensible to review what the relationship between assets and claims on the assets is, taking into account the identity of the monetary units, which are located on both sides of the accounting equation.

2.1 Methodology
This research uses a rationalistic and analytical method. It is not based on empirical data but in the analysis of theoretical assumptions. It focuses on the accounting equation analysis using its basic mathematical operation. An identity function accounts for the dual aspect of accounting
transactions. A characteristic function relates both sides of the equation identifying those monetary units that are equal. These functions, as well as the rest of the analysis, take claims on the assets as the range and assets as the domain. A reformulation of the accounting equation is made; finally, a coordinate transformation shows the change in value in assets between the two systems that come up in the reformulated accounting equation. These coordinate systems are the sides of the equation.

The type of accounting valuation method assumed in the analysis is book value method.

3 Problem Solution

2.1 The dual aspects of accounting transactions

This analysis will only take the lowest level accounts, i.e. those that have monetary units $u_i$ with no aggregation into higher order accounts. Accordingly, the claims on the assets side of the equation includes accounts $C_i$ (accounts with monetary units $u_i$) and the assets side of the equation contains accounts $A_i$ (assets accounts with monetary units $u_i$).

It must be pointed out that claims on the assets do not comprise the typical aggregated liabilities and stockholder’s equity items or any other aggregation account, but only the lowest level items located beneath them. Doing it in this way does not change the result and provides a clearer explanation.

The accounts $A_i$ and $C_i$ are already sorted, because their sequence in balance sheet follows national or international standards. Thus, a correspondence exists between the natural numbers set $\mathbb{N}$ and the accounts $A_i$ and $C_i$; we can assign serial numbers $A_1, A_2, \ldots A_n$ and $C_1, C_2, \ldots C_n$ to these accounts. To the purposes of this research, the monetary units located in the accounts do not need to follow any order.

To the analysis, the value of the monetary unit is irrelevant; it can be the legal tender or any other, as long as it remains the same for all of the assets and claims on the assets accounts.

Every monetary unit $u_{ia}$ in an account $A_i$, is simultaneously located in an account $C_i$ as $u_{ic}$, but still it is the same monetary unit, so $u_{ia} = u_{ic}$. Accordingly, there must be a function relating the monetary units $u_{ia}$ of the accounts $A_i$ with the monetary units $u_{ic}$ of the accounts $C_i$, based on this characteristic. Due to the dual aspect of accounting transactions assumption and the unicity of the monetary units, an identity function $f$ must exist for every monetary unit $u_{ia}$ of all $A_i$ and $u_{ic}$ of all $C_i$. In fact, the accounting equation requires assets being equal to claims on the assets, and its mathematical expression is $A = C$, with $C$ being liabilities ($L$) plus stockholders’ equity ($E$), $C = L + E$.

The identity function can take two directions. The first one takes claims on the assets as the domain and assets as the range, $F: C \to A$. The second one takes assets as the domain and claims on the assets as the range, $F: A \to C$. None of this direction is the most relevant, and the duality assumption does not privilege any direction. To the purpose of this research, the analysis will assume the function $F: C \to A$.

2.2 The dual aspects of accounting transactions and the inequality between assets and claims on the assets

The function relating claims on the assets and assets takes each element $u_{ci}$ of every $C_i$ as the first component, and each element $u_{ai}$ of every $A_i$ as the second component of the pair $(u_{ci}, u_{ai})$. To every $u_{ci}$ corresponds an identity image $u_{ai}$ and $(u_{ci}, u_{ai})$; accordingly, it exists a unique monetary unit $u_{ia}$, named $u_{ia}$ in $C_i$ and $u_{ai}$ in $A_i$, located on a claim on the asset and asset accounts simultaneously. That is in accordance with the dual aspects of accounting transactions, so the identity function denotes that a monetary unit is equal to itself despite its location at different and opposite places.

Additionally, the monetary units $u_{ia}$ of a single $C_i$ are distributed in several $A_i$, i.e. the range of the function $f$ for a $C_i$ is not in a single $A_i$. That is so because financial statements do not classify the monetary units by identity groups, but by accounts. The classification is different on both sides of the equation because assets and claims on the assets have different accounts. Therefore, to contain all the images of a $C_i, f$ must be a family of functions for that $C_i$, each function having some (but not all) of the monetary units $u_{ci}$ of $C_i$ (domain) and some (but not all) of the monetary units $u_{ai}$ in an $A_i$ (range). To collect all the identity images of all $u_{ci}$ for every $C_i$ requires many functions.

To get some monetary units in each function, the analysis uses a characteristic function $1_{C_i}$. For every $C_i$ and $A_i$ the characteristic function $1_{C_i}$ assigns ‘1’ to the $u_{ai}$ of $A_i$, which are identity images of an $u_{ci}$ located in $C_i$, and ‘0’ to that that is not. Then, they are multiplied by the value of $u_{ai}$ of $A_i$, which is the value of the monetary unit in the financial statements. The full expression including the characteristic function and its multiplication by $u_{ai}$ is
\[ u_{ai}(C_i(u_{ci})) = u_{ai}(1 \mid u_{ci} = u_{ai}; 0 \mid u_{ci} \neq u_{ai}) \quad (1) \]

Since the images of all of the monetary units of each \( C_i \) are in several \( A_i \), each \( C_i \) range, comprising \( u_{a1}, u_{a2}, u_{a3}, \ldots, u_{an} \), is located in several \( A_i \). It is uncommon to find one \( A_i \) with all the images of a domain \( C_i \). The domains and the ranges show no one-to-one correspondence.

Let us put an example; given a \( C_i \) with \( n \) \( u_{ci} \) monetary units, it might exist an \( A_j \) with \( m_j \) \( u_{ai} \) and another \( A_j \) with \( m_j \) \( u_{ai} \) monetary units. Part of the \( A_j \) monetary units are an image of some monetary units of \( C_i \) and part of the monetary units of \( A_j \) are an image of some monetary units of \( C_i \). Based on this different ranges, the \( C_i \) \( n \) \( u_{ci} \) monetary units are divided into \( n_j \) and \( n_j \), each one corresponding to each range in \( A_j \) and \( A_j \), with \( n = n_j + n_j \). However, \( A_j \) has more monetary units than those that are an image of some of the monetary units of \( C_i \) and, then, the monetary units of \( A_j \) are \( m_1 = m_1 + m_1 \), with \( m_1 \) being images of some monetary units of \( C_i \) and \( m_1 \) being the images of some monetary units of \( C_i \). Similarly, \( A_j \) has more monetary units than those that are an image of some of the monetary units of \( C_i \) and, then, the monetary units of \( A_j \) are \( m_2 = m_2 + m_2 \), with \( m_2 \) being images of some monetary units of \( C_i \) and \( m_2 \) being no images of \( C_i \). In this way, the images of the monetary units of \( C_i \) spread over several partial ranges (\( m_1 \) of \( A_j \) and \( m_2 \) of \( A_j \)).

Now, instead of adding all of the asset values and all of the claims on the asset values, what would lead to the standard accounting equation, one can add domains on one side of the equation and ranges on the other side; it means to add by the identity of monetary units.

The linear accumulation of a single domain \( C_i \) with \( n \) monetary units is:

\[ SC_i = \sum_{i=1}^{n} u_{ci} \quad (2) \]

To every \( C_i \) there are several \( A_i \) with partial ranges of the function \( f \) so we can choose the first \( A_i \), in the order they are arranged in the balance sheet; its linear accumulation is:

\[ SA_i = \sum_{i=1}^{n} u_{ai} \quad (3) \]

It must be noted that all the monetary units of \( A_i \) that are not images of \( C_i \) were removed in this \( A_i \) by the characteristic function in (1), what results in \( SA_i < SC_i \) for each \( A_i, C_i \). That is the reason why \( SC_i \) is not equal to \( SA_i \), the addition of the same monetary units is not possible when considering a \( C_i \) and a sole \( A_i \) for that \( C_i \), because other ranges \( A_i \) for the domain \( C_i \) would require other functions \( f \) to have them added.

Extending the previous computation for a \( C_i \) to all the \( n \) \( C_i \) domains with \( m \) elements in each domain, their sum \( SC_T \) is:

\[ SC_T = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{cj} \quad (4) \]

Applying again one single function \( f \) that takes every domain \( C_i \) and one single range \( A_i \), not all of them, for every domain \( C_i \), the sum \( SA_p \) is:

\[ SA_p = \sum_{h=1}^{k} \sum_{i=1}^{n} \sum_{j=1}^{m} u_{ah}f_j \quad (5) \]

with \( k = \) sequence number for \( C_i \); \( n = \) sequence number for \( A_i \); \( m = \) number of images in a \( A_i \) of some of the monetary units in a \( C_i \); \( \exists m = \) a unique range in a unique \( A_i \) for a \( C_i \) exists and is found, and \( \exists n = \) the function \( f \) takes a single range \( A_i \) for every domain \( C_i \). As it happened with a single \( C_i \) in (2) and (3), it results in the following inequality:

\[ A_s \neq C_s \quad (6) \]

\[ \sum_{h=1}^{k} \sum_{i=1}^{n} \sum_{j=1}^{m} u_{ah}f_j \neq \sum_{i=1}^{n} \sum_{j=1}^{m} u_{cj} \quad (7) \]

This formula is an inequality because the images of every \( C_i \) are distributed in partial ranges \( A_i \) and a run of the function \( f \) only takes one range in an \( A_i \) for each \( C_i \). In doing so, the accounting equation in its standard formulation do not reflect the real relationship between assets and claims on the assets, which is, actually, an inequality. The standard equation is:

\[ A_s \neq C_s \quad (8) \]

\[ C_s = L + E \quad (9) \]

\[ A_s \neq L + E \quad (10) \]

Typically, \( A_s < C_s \), because ranges are restricted to those obtained in a unique \( f \) for each \( C_i \). To be both sides equal, the range that was obtained should be multiply by a coefficient, to artificially increase its value.
2.2 The Assets Value Transformation in Coordinate Systems

First of all, it must be noted that it is not possible to obtain the total value of assets by adding all $A_i$ in the usual form:

$$A_s = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{ahij}$$  \hspace{1cm} (11)

with $n =$ sequence number for $A_i; m =$ number of images of any $C_i$ in a $A_i$. That is so because the addition based on the dual aspects of accounting transactions must follow the financial statements classification, and its corresponding partition in function domains and ranges, and not to add all the ranges of different functions in one run. The addition of the ranges (it is a procedure with several runs) must stop in every run once the function finds a partial range.

To get together all the $m$ images of each one of the $n$ ranges for a particular $C_i$, the procedure must be run $n$ times, resulting in many partial functions. In this way, the procedure picks up all the images for a $C_i$. The recursive procedure to get all the $A_i$ images for all the $C_i$ is:

$$A_s = \sum_{h=1}^{k} \sum_{i=1}^{3n} \sum_{j=1}^{3m} u_{ahij}$$  \hspace{1cm} (12)

with $k =$ sequence number for $C_i; n =$ sequence number for $A_i; m =$ number of images in an $A_i$ of the monetary units in a $C_i; 3m =$ a range in a $A_i$ for a $C_i$ exists and is found, and $3n =$ the function $f$ takes, if it exists, a range in every $A_i$ for every domain $C_i$. It must be noted the difference of this equation with (5), where the equation states that a unique range exists in an $A_i$, while in (12) it states that a partial range exists in many $A_i$. The latter allows for running the procedure multiple times for every $C_i$ until all its ranges are found.

Finally, the equation:

$$A_s = C_s$$  \hspace{1cm} (13)

results in:

$$\sum_{h=1}^{k} \sum_{i=1}^{3n} \sum_{j=1}^{3m} u_{ahij} = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{cij}$$  \hspace{1cm} (14)

One can take both sides of the equation as coordinate systems with three (asset side) and two (claims on the asset side) dimensions, and transform the three summation terms on the left side into two summation terms to make both coordinate systems equal. To this, the computation must introduce scaling parameters $s_i$ and $c_i$; slope and constant respectively. The dimensional systems have a common axis for both terms, the axis $C_i$, which can be taken as the $y$-axis in the three-dimensional system. Then, it is needed to project the values in the axis $x (a_i)$ and $z (a_i)$ of the three-dimensional system onto the axis $x (a_i)$ and $y (a_i)$ in the new two-dimensional system. The equations must be linear, like the accounting equation simple calculation, and they are in the form:

$$a_i=s_i a_i+c_i$$  \hspace{1cm} (15)

Let us take the axis $x (a_i)$, in the three dimensional system, as the partition of all the $A_i$ by image groups of every $C_i$: the axis $z (a_i)$ is the monetary units $u_{ahij}$, in that system too. Once the $y$ axis ($C_i$ accounts) is removed, and substituting in (15) the computations for each monetary unit are:

$$A_s=s_x A_s+c_x$$

$$u_{xy}=s_x u_{ahij}+c_x$$

Each function $f$: $C_i \rightarrow A_i$ creates a set of parameters ($s_i$, $c_i$, $s_z$, $c_z$) for some monetary units of the domain $C_i$, and the range found in an $A_i$. All monetary units in each function have the same computation parameters, because they belong to the same partial function $f$: $C_i \rightarrow A_i$. The scaling parameters for the new axis $A_s$, in the two dimension system, are $s_x$ and $c_x$; they are position transformations in an ordered sequence of ranges, starting with the range 1 in an $A_i$ for the first domain $C_i$ and ending with the last range in an $A_i$ for the last domain $C_n$. It is important to note that this transformation does not change the value of the monetary units but the position of the domains $A_i$; however, it will have an impact when aggregating accounts.

Also, the new axis $u_{xy}$ is the value of the monetary units of each domain $A_i$ in the new system, and it is a transformation of the original value $u_{ahij}$ in the three-dimensional system. The parameters $s_x$ and $c_x$ are not the same as those for $A_i$; their value come from the fact that the monetary units in an $A_i$ could be images of different $C_i$, and to collect them involves several functions for each $C_i$. Accordingly,
the values of $s_z$ and $c_z$ must depend on each $C_i$ function, and all of them could be different.

Accordingly, the accounting equation is as follows:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} u_{cij} = \sum_{i=1}^{n_1} \sum_{j=1}^{m_1} u_{alj} + \cdots + \sum_{i=1}^{n_n} \sum_{j=1}^{m_n} u_{alj}$$  \hspace{1cm} (17)

with $n = \text{sequence number for } C_i$; $m = \text{monetary units } u_c$ in each $C_i$; $n_j = \text{sequence number for } A_i$ ranges in the new system; $m_1 = \text{monetary units } u_a$ in each $A_i$ range in the new system.

Substituting the monetary units in the new system for their coordinate transformation

$$\sum_{i=1}^{n} \sum_{j=1}^{m} u_{cij} = \sum_{i=1}^{n_1} \sum_{j=1}^{m_1} (s_z u_{ahij} + c_z)_{ij} + \cdots + \sum_{i=1}^{n_n} \sum_{j=1}^{m_n} (s_z u_{ahij} + c_z)_{ij}$$

$$\hspace{1cm} (18)$$

with $n = \text{sequence number for } C_i$; $m = \text{monetary units } u_c$ in each $C_i$; $n_j = \text{sequence number for } A_i$ in the new two coordinate system; $m_1 = \text{monetary units } u_a$ in each $A_i$ in the new two coordinate system; $u_{ahij}$ the monetary unit value in the old three coordinate system, and $s_z$ and $c_z$ the slope and constant transformation parameter values.

The $s_z$ and $c_z$ parameter values need a definition for each group of monetary unit images $A_i$ in the three coordinate system; they depends on the range of every $C_i$ for which they are images. This computation results in a change of the asset values; as claims on the assets do not change their value, to keep the equality with the new terms, the asset value must change. Some of the assets can increase their value and others can decrease it; however, a transformation from a three coordinate system to a two coordinate system cannot keep the same scale.

4 Conclusion

By introducing the simply mathematical computations of the accounting equation combined with a coordinate transformation, the results confirm that the accounting equation, based on the dual aspects of accounting transactions, is more than just an addition, but a sequence of functions. That is of great relevance.

These mathematical equations, even they are just a successive layers of computations, are intended to be the basis for more complex mathematical analysis.

References: