The Dual Aspect of Accounting Transaction and the Assets-Claims on Assets Equivalence

FERNANDO JUÁREZ
Escuela de Administración
Universidad del Rosario
Autopista Norte, No. 200
COLOMBIA
fernando_juarez2@yahoo.com

Abstract: - The purpose of the study is to analyze the justification of the assets-claims on assets equivalence, based on the dual aspect of accounting transactions. Assets and claims on assets were considered as sets; the analysis used the axiomatic method, a definition of accounting axioms, and a test of the equality of the assets-claims on assets cardinality by a bijective functionThe results show that the assets cardinality is not equal to the cardinality of claims on assets when taking into account the dual aspect of accounting transactions.

Key-Words: - Dual aspect, accounting transactions, axiomatic method, cardinality, balance sheet.

1 Introduction

The paper addresses the issue of assets-claims on assets equivalence, using the axiomatic method and the concept of cardinality.

The accounting equation expresses the assets-claims on assets equality, and it is a consequence of the dual aspect of the accounting transactions. The accounting equation is a mathematical expression, and it needs analyzing by mathematical methods, but the dual aspect of accounting transactions is, somehow, a type of assumption in accounting and requires analysis by axiomatic method. This method is appropriate for any science to analyze structures [1] and assumptions, and so, it is to analyze the accounting assumptions.

The dual aspect of accounting transactions means that every accounting transaction has to be recorded in two accounts with different signs in a double classification system [2].

The notion that assets must equal liabilities plus stockholders' equity, as the accounting equation states, based on the dual aspect of accounting transactions, is an accounting principle. However, other approaches exist that put into question the principles of accounting (see [3], [4], [5], [6]). Besides, the fair value approach ([7], see [8] for a critic) is, somehow, a critic of the dual aspect that supports the accounting equation.

Meanwhile, the applications of the axiomatic method to accounting are significant. Accounting was linked to logic and set theories [9] pointing out the importance of the duality approach ([9], pp. 101–105). Other authors built consistent and complete axiomatic systems ([10], [11], [12]), preserving the dual aspect of accounting transactions [see 13]. Moreover, the analysis of financial statements can include other logics, such as belief, circumscription, paraconsistent logics and dialogic, with a different view of the accounting equation ([14], [15], [16]).

2 Problem Formulation

Axiomatic method is useful to analyze the assumptions of accounting; also, the equivalence of assets to claims on assets and the dual aspect of the accounting equations, analyzed by the axiomatic method, still deserve more attention.

Accordingly, the purpose of this paper is to examine the assets-claims on assets equivalence, using the axiomatic method and the analysis of the cardinality of sets to test their equivalence.

2.1 Methodology

This research uses the axiomatic method, which comprises set theory and predicate logic to develop rationales and conclusions. Predicate logic was the language used to formulate the axiomatic theory of Zermelo and Fraenkel (ZF) (see [17], [18], [19]) that supports this analysis.

3 Problem Solution

3.1 Primitives and Axioms of the Zermelo–Fraenkel theory

ZF theory has two primitives, membership ∈ and set $\{x_i\}$. This theory comprises nine well-defined axioms to operate with sets. ZF theory deals only with sets and not with elements not linked to any set (urelements); thus, the members of a set are always, in turn, sets. This theory remains the most prevalent set theory, and it addresses infinite and finite sets.

The analysis will use the following ZF axioms: a) the axiom of union that allows grouping sets into another set; b) the axiom of specification that allows creating sets based on the properties of their elements; and c) the axiom of replacement which states that . The ZF theory also assumes the definition of subset as a set that is a member of another set.

3.2 Accounting axioms

The application of the axiomatic method to accounting requires a list of specific accounting axioms and a primitive of the system. The primitive is the monetary unit u_i; it is the value unit used to value every asset or claim on the asset.

The accounting axioms follow.

Accounting axiom 1. The elements of any set of assets and claims on assets are sets that contain sets of monetary units. Therefore

$$\forall A \,\forall C \forall u_i [(\forall A_i \forall C_i \,(u_i \in A \mid u_i \in C) \rightarrow (u_i \in A_i \mid u_i \in C_i)] \quad (1)$$

with A = assets, C = claims on assets, A_i = elements (subsets) of assets, C_i = elements (subsets) of claims on assets, and u_i = monetary units. A special type of set is the single monetary unit $\{u_i\}$. This axiom expresses that a set X (A_i or C_i) has sets of monetary unit, i.e. $X = (\{u_i\}, \{u_i\}, \{u_i\})$.

The monetary unit can be in the legal tender or any other unit; it does not make any difference to the analysis, so it does not need additional definition; the only requirement is that the type of monetary unit must be the same for all sets.

Accounting axiom 2. Every monetary unit $\{u_i\}$ is different to another monetary $\{u_j\}$ unit.

$$\forall u_i \,\forall u_j [u_i \neq u_j] \quad (2)$$

This axiom is necessary, because if the monetary units were equal, a set containing ten monetary units would be equal to a set containing just one. Therefore, to any pair of monetary units $\{u_i\}$ and $\{u_j\}$

$$\forall u_i \,\forall u_j \,\forall x_i [(u_i \in x_i \wedge u_j \in x_i) \rightarrow u_i \neq u_j] \quad (3)$$

$$\forall u_i \,\forall u_j \,\forall x_i \,\forall y_i [(u_i \in x_i \wedge u_j \in y_i) \rightarrow u_i \neq u_j] \quad (4)$$

Accounting axiom 3. This axiom represents the dual aspect of the accounting transactions. Every monetary unit is allocated to a single asset set and claim on assets set, simultaneously. That is

$$\forall u_i \,\exists! C_i \,\exists! A_i \,\exists A \,\exists C \,[u_i \in A \wedge u_i \in C \rightarrow (u_i \in A_i \wedge u_i \in C_i)] \quad (5)$$

Therefore, a monetary unit $\{u_i\}$ can belong to two different sets A_i and C_i simultaneously.

3.3 The structure of assets and claims on assets

Assets and claims on assets comprise sets that contain monetary units (sets) (accounting axiom 1) and not sets that contain other sets containing monetary units (sets). It means that no aggregation accounts exist in this structure. It only takes on the lowest level accounts on the balance sheet.

The axiom of specification combined with the axiom of union creates this structure consisting only of the lowest levels accounts. While the details of this combination are not relevant to this research, the specification axiom needs some explanation.

The specification axiom allows allocating monetary units to any set A_i of assets or C_i of claims on assets; according to accounting axiom 3, every monetary unit is in both of them.

In the case of assets the specification axiom is

$$\forall A_i \exists A \exists u_a \,[u_a \in A \leftrightarrow (u_a \in A_i \wedge \phi_a)] \quad (6)$$

with ϕ_A: u_a monetary unit considered an A_i asset under an accepted definition. It is a property that all elements of a set A_i must have.

In the same manner, to claims on assets

$$\forall C_i \,\exists C \,\exists u_c \,[u_c \in C \leftrightarrow (u_c \in C_i \wedge \phi_C)] \quad (7)$$

with ϕ_C: u_c monetary unit considered a C_i claim on assets under an accepted definition. Again, this is a property that all elements of a set C_i must have.

There are definitions of what an asset or claim on assets are; however, these definitions are not relevant to the analysis, as long as they are consistent throughout the Balance Sheet.

By the accounting axioms 2 and 3, the sets C_i are disjoint sets, and no monetary unit $\{u_{ci}\}$ is a member of C_i and C_j simultaneously. The same rationale is valid for the sets A_i, so they are disjoint too. However, C_i and A_i contain the same monetary units, and they are not disjoint.

The accounting axioms do not require that all the monetary units of a single C_i be in a specific A_i (accounting axiom 1); thus, the monetary units of a single C_i could be in several A_i or those of a single A_i could be in several C_i.

3.4 Analysis of the cardinality of assets and claims on assets

Assets (A) and claims on assets (C) might be equivalent. Thus, the value of A would be equal to the value of C although they have different elements or subsets. The value of a set is called its cardinality, and it refers to the number of elements that a set has, no matter what these elements are.

The cardinality of each A_i and C_i is the number of the monetary units they have. The cardinality of a set with n monetary units is depicted as $|n|$ because every monetary unit is unique (accounting axiom 2). Thus, the cardinality of a set or an item in the balance sheet is the amount of monetary units allocated to it.

A cardinality property is that the cardinality of the union of disjoint sets is the sum of the cardinality of each set. As shown, the sets C_i are disjoint, and the cardinality of the set C is the sum of the cardinalities of its n subsets C_i

$$|C| = \sum_{i=1}^{n} |C_i| \tag{8}$$

It is the same rationale for the cardinality of A, which contains all the subsets A_i. It is the sum of the cardinalities of its n disjoint subsets A_i

$$|A| = \sum_{i=1}^{n} |A_i| \tag{9}$$

In the case that C = Liabilities and Equity, and A = current and non-current assets, the cardinality of both of them, A and C, would be 2, which is not the value sought in this research. To use the cardinality of the set union correctly, it has to operate with all the lowest level accounts A_i and C_i, without any aggregation. The cardinality of interest is the total monetary value of the sets A and C, which contain subsets A_i and C_i that, in turn, contain subsets of monetary units $\{u_i\}$; then, the cardinality relationship to test is

$$|A| = |C| \tag{10}$$

Thus, for the cardinality of the total assets to be equal to that of the total claims on assets, it must be

$$\sum_{i=1}^{n} |A_i| = \sum_{i=1}^{m} |C_i| \tag{11}$$

with $n \neq m$ in the usual arrangement of the balance sheet.

The cardinality equality of two sets requires a bijection so that they have the same number of elements; this is called equinumerosity. Thus, there must be a bijection between A and C to determine that they have the same cardinality. Consequently, there must be a function f from C to A that assigns a member A_i to a member of C_i, in such a way that A_i is an image of C_i by the function f. The assets-claims on assets equivalence assumes that all the monetary units of claims on assets are in assets too (accounting axiom 3). Thus, and by the dual aspect of accounting transactions (accounting axiom 3), a crucial function f is the one that links every monetary unit in a C_i to the same monetary unit in an A_i. In that case, $|A_i| = |C_i|$ for all A_i C_i.

Despite another function could determine a relationship between A_i and C_i, the function f, based on the accounting axiom 3, expresses the more direct A_i C_i relation between them.

The function f of C to A must be bijective. To accomplish this, every set C_i must have an image A_i, and the images A_i and A_j of any two C_i and C_j, must be different in A, i.e. $f(C_i) \neq f(C_j)$ (injective function). Furthermore, all the sets A_i must have a pre-image or reverse image C_i in C (surjective function), and $f^{-1}(C_i) \neq f^{-1}(C_j)$. If the function f does not meet these conditions, it is not bijective, and the cardinalities of C and A are not the same.

According to the replacement axiom, the image of a set is contained within another set; this axiom is as follows:

$$\forall A \forall w_{1 \ldots\ldots} \forall w_n \, [\forall x (x \in A \rightarrow \exists!\, y \;\; f) \rightarrow \exists B \forall x \, (x \in A \rightarrow \exists y (y \in B \wedge f))] \tag{12}$$

with f: a function between sets. That means that every element x of the set A is related to an element y of the set B by f. The application to the assets-claims on assets equality is

$$\forall C \forall w_{i\ldots\ldots} \forall w_n [\forall C_i(C_i \in C \rightarrow \exists! A_i\ f) \rightarrow \exists A \forall A_i (C_i \in C \rightarrow \exists A_i(A_i \in A \wedge f))] \quad (13)$$

This axiom guarantees that every C_i in C must have an image A_i in A. The function f must be a bijection.

Accounting axiom 3 determines the allocation of every monetary unit to C_i and A_i; however, as this axiom does not require that the monetary units of a single C_i be all allocated to a single A_i, it is possible that a C_i has some of its monetary units distributed into two sets A_i and A_j. If this happens, C_i would not have an A_i image, because some of its monetary units would not be the same as the monetary units of, let us say, A_i, because they would be in A_j. Consequently, the function is not a bijection, and that is in contradiction with the requirement of a bijective function for C and A, which is to have the same cardinality. The lack of a bijective function leads to the conclusion that

$$|A| \neq |C| \quad (14)$$

The cardinalities of assets and claims on assets are not equal. The dual aspect of accounting transactions (accounting axiom 3) the structure of assets and claims on assets, and the function that assigns the same elements to the sets C_i and A_i, lead to a cardinality change.

Despite another function could determine a relationship between A_i and C_i, the function f, based on the accounting axiom 3, expresses the more direct A_i C_i relation between them.

4 Conclusion

The intention of this research was to test the equivalence of the assets and claims on assets sets, based on the dual aspects of the accounting transactions. The analysis made use of the axiomatic method and set cardinality, what led to the conclusion that assets and claims on assets are not equivalents.

Still, more research on the topic is needed, and its implications need exploring.

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