Variations of the Rectangular Fuzzy Assessment Model and Applications to Human Activities

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Abstract: **-** The Rectangular Fuzzy Assessment Model (RFAM) is based on the popular in fuzzy mathematics COG defuzzification technique. In the paper at hands the RFAM and its variations GRFAF, TFAM and TpFAM, developed for treating better the ambiguous assessment situations being at the boundaries between two successive assessment grades, are presented and their outcomes are compared to each other through the corresponding outcomes of a traditional assessment method of the bi-valued logic, the Grade Point Average (GPA) index. Examples are also presented illustrating our results.

Keywords: **-** Grade Point Average (GPA) index, Centre of Gravity (COG) defuzzification technique, Rectangular Fuzzy Assessment Model (RFAM), Generalized RFAM (GRFAM), Triangular FAM (TFAM), Trapezoidal FAM (TpFAM).

1. Introduction

In 1999 Voskoglou [1] developed a fuzzy model for the description of the process of learning a subject matter in classroom with the help of the ***possibilities*** of student profiles and later he assessed the student learning skills by calculating the corresponding system's ***total possibilistic uncertainty*** [2]. Meanwhile, Subbotin et al. [3], based on Voskoglou's model [1], adapted properly the frequently used in fuzzy mathematics ***Center of Gravity (COG) defuzzification technique*** and used it as an alternative assessment method of student learning skills, later named as the ***Rectangular Fuzzy Assessment Model (RFAM)***. Since then, Voskoglou and Subbotin, working either jointly or independently, applied the RFAM and three variations of it, the ***GRFAM,*** the ***TFAM*** and the ***TpFAM***, for assessing several human or machine (Decision – Making and Case-Based Reasoning with the help of computers, etc.) activities, e.g. see [4 - 15], , etc.

In the present work the outcomes of the RFAM and of its variations are compared to each other through the corresponding outcomes of a traditional assessment method of the bi-valued logic, the ***Grade Point Average (GPA)*** index. The rest of the paper is formulated as follows: In Section II we present the GPA method. In Section II we sketch the RFAM, while in Section IV we briefly describe the three variations of the TRFAM, developed for treating better the ambiguous assessment situations being at the boundaries between two successive assessment grades. In Section V the outcomes of the RFAM and its variations are compared to each other through the outcomes of the GPA index and examples are presented illustrating our results. The last Section VI is devoted to our conclusion and to some hints for future research on the subject.

2. The GPA Index

The calculation of the mean value of the individual scores of a group of objects (e.g. students, players, machines, etc.) characterizing their performance with respect to an action is the traditional method for assessing the group's ***mean performance.***

On the other hand, the GPA index is a very popular in the USA and other Western countries assessment method calculating the group's ***quality performance***. The GPA index, a weighted average in which greater coefficients (weights) are assigned to the higher scores, is calculated by the formula

$$\text{GPA} = \frac{0n_F + 1n_D + 2n_C + 3n_B + 4n_A}{n} \quad \textbf{(1)},$$

where n is the total number of the group's members and n_A, n_B, n_C, n_D and n_F denote the numbers of the group's members that demonstrated excellent (A), very good (B), good (C), fair (D) and unsatisfactory (F) performance respectively [16].

Formula (1) can be also written in the form

$$\text{GPA} = y_2 + 2y_3 + 3y_4 + 4y_5 \quad \textbf{(2)},$$

where $y_1 = \frac{n_F}{n}$, $y_2 = \frac{n_D}{n}$, $y_3 = \frac{n_C}{n}$, $y_4 = \frac{n_B}{n}$ and $y_5 = \frac{n_A}{n}$ denote the ***frequencies*** of the group's members which demonstrated unsatisfactory, fair, good, very good and excellent performance respectively.

In case of the ***worst*** performance ($n_F = n$) formula (1) gives that GPA = 0, while in case of the ***ideal*** performance ($n_A = n$) it gives GPA = 4. Therefore we have in general that

$$0 \leq \text{GPA} \leq 4 .$$

Consequently, values of GPA greater than 2 could be considered as indicating a more than satisfactory performance.

3. The Rectangular Fuzzy Assessment Model (RFAM)

A commonly used in fuzzy mathematics technique is the defuzzification of a given ***fuzzy set (FS)*** by calculating the coordinates of the COG of the level's section contained between the graph of the FS's membership function and the X - axis [17].

In [15] we have described in detail the use of the COG technique as an assessment method and we have applied it for evaluating the student understanding of the polar coordinates in the plane. The graph of the membership function in this case is that presented in Figure 1. It is recalled here that the design of this graph was achieved by replacing the elements of the universal set

$$U = \{A, B, C, D, F\}$$

of the linguistic characterizations introduced in Section II by the real intervals [4, 5], [3, 4), [2, 3), [1, 2) and [0, 1) respectively, corresponding to a scale of numerical scores assigned to each linguistic characterization..

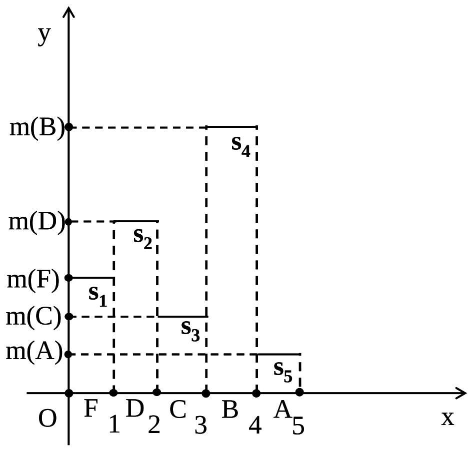

Figure 1: The graph of the COG technique

In Figure 1 the area of the level's section contained between the graph and the X - axis is equal to the sum of the areas of the rectangles S_i, i=1, 2, 3, 4, 5. Due to the shape of this graph we have named the above method as the ***Rectangular Fuzzy Assessment Model (RFAM).***

Note that the membership function $y = m(x)$ can be defined, according to the user's personal goals, in any compatible to the common sense way. However, in order to obtain assessment results compatible to the corresponding results of the GPA index, we have defined $y = m(x)$ in terms of the ***frequencies*** introduced in Section II.

Then, using the well known from Mechanics formulas for calculating the coordinates of the COG it is straightforward to check [15] that the coordinates of the COG in this case are given by the formulas

$$x_c = \frac{1}{2}(y_1+3y_2+5y_3+7y_4+9y_5),$$

$$y_c = \frac{1}{2}(y_1^2+y_2^2+y_3^2+y_4^2+y_5^2) \quad \textbf{(3)},$$

with x_1 = F, x_2 = D, x_3 = C, x_4 = B, x_5 =A and

$$y_i = m(x_i) = \frac{n_{x_i}}{n},$$

i = 1, 2, 3, 4, 5, where obviously

$$\sum_{i=1}^{5} m(x_i) = 1.$$

Further, using elementary algebraic inequalities and performing elementary geometric observations [15] one obtains the following assessment criterion:

- *Among two or more groups the group with the greater x_c performs better.*

- *If two or more groups have the same $x_c \geq 2.5$, then the group with the greater y_c performs better.*
- *If two or more groups have the same $x_c < 2.5$, then the group with the smaller y_c performs better.*

As it becomes evident from the above criterion, a group's performance depends mainly on the value of the x-coordinate of the COG of the corresponding level's area, which is calculated by the first of formulas (3). In this formula, greater coefficients (weights) are assigned to the higher grades. Therefore, the COG method focuses, similarly to the GPA index, on the group's ***quality performance.***

In case of the ***ideal*** performance (y_5 =1 and y_i = 0 for i $\neq$ 5) the first of formulas (3) gives that $x_c = \frac{9}{2}$. Therefore, values of x_c greater than $\frac{9}{4}$ = 2.25 could be considered as demonstrating a more than satisfactory performance.

4. The Variations GRFAM, TFAM and TpFAM of the RFAM

A group's performance is frequently represented by numerical scores in a climax from 0-100. These scores can be assigned to the linguistic labels of *U* as follows: A (85-100), B(75-84), C (60-74), D(50-59) and F (0-49) [1].

Nevertheless, ambiguous cases appear frequently in practice, being at the boundaries between two successive assessment grades; e.g. something like 84-85%, being at the boundaries between A and B. In an effort to treat better such kind of cases, Subbotin [8] "moved" the rectangles of Figure 1 to the left, so that to share common parts (see Figure 2). In this way, the ambiguous cases, being at the common rectangle parts, belong to both of the successive grades, which means that these parts must be considered ***twice*** in the corresponding calculations.

The graph of the resulting fuzzy set is now the bold line of Figure 2. However, the method mentioned in Section II for calculating the coordinates of the COG of the area contained between the graph and the X-axis is not the proper one here, because in this way the common rectangle parts are calculated only once. The right method for calculating the coordinates of the COG in this case was fully developed by Subbotin & Voskoglou [9] and the resulting framework was called the ***Generalized Rectangular Fuzzy Assessment Model (GRFAM).*** The development of GRFAM involves the following steps:

1. Let y_1, y_2 , y_3, y_4 , y_3 be the ***frequencies*** of a group's members who obtained the grades F, D, C, B, A respectively. Then $\sum_{i=1}^{5} y_i$ = 1 (100%).

2. We take the heights of the rectangles in Figure 2 to have lengths equal to the values of corresponding frequencies. Also, without loss of generality we allow the sides of the adjacent rectangles lying on the OX axis to share common parts with length equal to the 30% of their lengths, i.e. 0.3 units.[2]

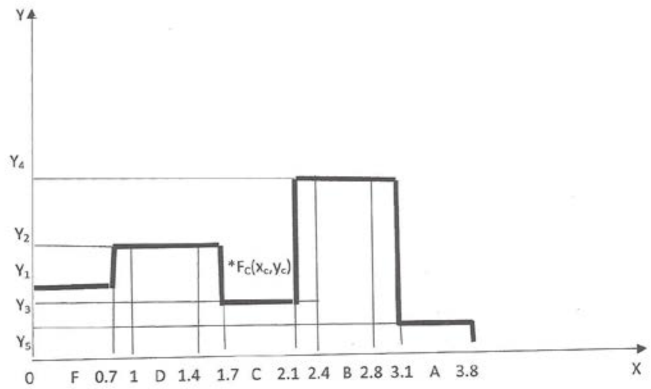

Figure 2: Graphical representation of the GRFAM

3. We calculate the coordinates (x_{c_i}, y_{c_i}) of the COG, say F_i , of each rectangle, i = 1, 2, 3, 4, 5 as follows: Since the COG of a rectangle is the point of the intersection of its diagonals, we have that $y_{c_i} = \frac{1}{2} y_i$. Also, since the x-coordinate of each COG F_i is equal to the x- coordinate of the middle of the side of the corresponding rectangle lying on the OX axis, from Figure 2 it is easy to observe that

[1] This way of assignment, although it satisfies the common sense, it is not unique; in a more strict assessment, for example, one could take A(90-100), B(80-89), C(70-79), D(60-69) and F (0-59), etc.

[2] Since the ambiguous assessment cases are situated at the boundaries between the adjacent grades, it is logical to accept a percentage for the common lengths of less than 50%.

$$x_{c_i} = 0.7\mathrm{i} - 0.2.$$

4. We calculate the coordinates (X_c, Y_c) of the COG F of the whole area considered in Figure 2 as the resultant of the system of the GOCs F_i of the five rectangles from the following well known [20] formulas

$$X_c = \frac{1}{S}\sum_{i=1}^{5} S_i x_{c_i}\ ,\ Y_c = \frac{1}{S}\sum_{i=1}^{5} S_i y_{c_i} \quad \textbf{(4)}.$$

In the above formulas *Si*, i= 1, 2, 3, 4, 5 denote the areas of the corresponding rectangles, which are equal to y_i . Therefore

$$S = \sum_{i=1}^{5} S_i = \sum_{i=1}^{5} y_i = 1$$

and formulas (4) give that

$$X_c = \sum_{i=1}^{5} y_i(0.7i - 0.2)\ ,\ Y_c = \sum_{i=1}^{5} y_i\left(\frac{1}{2}y_i\right)$$

or

$$X_c = (0.7\sum_{i=1}^{5} iy_i) - 0.2\ ,\ Y_c = \frac{1}{2}\sum_{i=1}^{5} y_i^2 \quad \textbf{(5)}.$$

5. We determine the area in which the COG F lies as follows: For i, j = 1, 2, 3, 4, 5, we have that $0 \le (y_i - y_j)^2 = y_i^2 + y_j^2 - 2y_iy_j$, therefore $y_i^2 + y_j^2 \ge 2y_iy_j$, with the equality holding if, and only if, $y_i = y_j$. Therefore

$$1 = \left(\sum_{i=1}^{5} y_i\right)^2 = \sum_{i=1}^{5} y_i^2 + 2\sum_{\substack{i,j=1,\\ i\neq j}}^{5} y_i y_j \le \sum_{i=1}^{5} y_i^2 + 2\sum_{\substack{i,j=1,\\ i\neq j}}^{5} (y_i^2 + y_j^2)$$

$$= 5\sum_{i=1}^{5} y_i^2$$

or

$$\sum_{i=1}^{5} y_i^2 \ \ge\ \frac{1}{5} \quad \textbf{(6)},$$

with the equality holding if, and only if,

$$y_1 = y_2 = y_3 = y_4 = y_5 = \frac{1}{5}.$$

In case of the equality the first of formulas (5) gives that $X_c = 0.7(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + \frac{5}{5}) - 2 = 1.9$.

Further, combining the inequality (6) with the second of formulas (5), one finds that

$$Y_c \ge \frac{1}{10}.$$

Therefore the ***unique minimum*** *for* Y_c corresponds to the COG F_m (1.9, 0.1).

The ***ideal case*** is when $y_1 = y_2 = y_3 = y_4 = 0$ and $y_5 = 1$. Then formulas (5) give that X_c = 3.3 and $Y_c = \frac{1}{2}$. Therefore the COG in this case is the point F_i (3.3, 0.5).

On the other hand, the ***worst case*** is when $y_1 = 1$ and $y_2 = y_3 = y_4 = y_5 = 0$. Then from formulas (5) one finds that the COG is the point F_w (0.5, 0.5).

Therefore, the area in which the COG F lies is the area of the triangle $F_w F_m F_i$ (Figure 3).

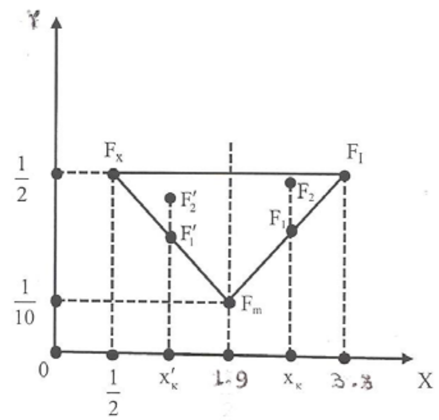

Figure 3: The triangle where the COG lies

6. From elementary geometric observations on Figure 3 one obtains the following assessment criterion:

- *Between two groups, the group with the greater X_c performs better.*
- *If two groups have the same $X_c \ge 1.9$, then the group with the greater Y_c performs better.*
- *If two groups have the same $X_c < 1.9$, then the group with the lower Y_c performs better*

From the first of formulas (5) it becomes evident that the GRFAM measures the ***quality*** group's ***performance***.

Also, since the ideal performance corresponds to the value X_c = 3.3, values of X_c greater than $\frac{3.3}{2}$ = 1.65 could be considered as indicating a more than satisfactory performance.

At this point one could raise the following question: Does the ***shape*** of the membership function's graph of the assessment model affect the assessment's conclusions? For example, what will happen if the rectangles of the GRFAM will be replaced by isosceles triangles? The effort to answer this question led to the development of the ***Triangular Fuzzy Assessment Model (TFAM)***, created by Subbotin & Bilotskii [4] and fully developed by Subbotin & Voskoglou [6].

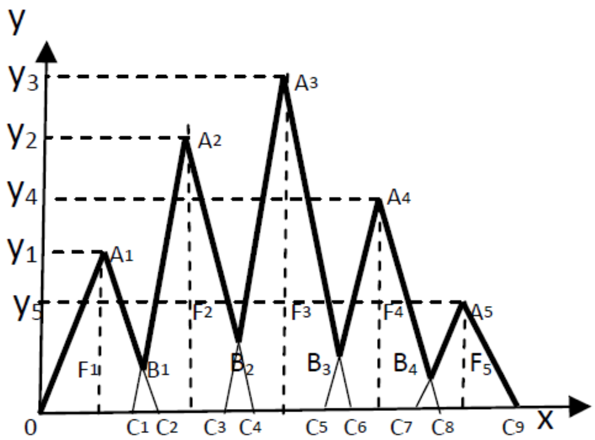

Figure 4: Graphical Representation of the TFAM

The graphical representation of the TFAM is shown in Figure 4 and the steps followed for its development are the same with the corresponding steps of GRFAM presented above. The only difference is that one works with isosceles triangles instead of rectangles. The final formulas calculating the coordinates of the COG of TFAM are:

$$X_c = (0.7\sum_{i=1}^{5} iy_i) - 0.2\,, \qquad Y_c = \frac{1}{5}\sum_{i=1}^{5} y_i^2 \quad \textbf{(7)}$$

and the corresponding assessment criterion is the same with the criterion obtained for GRFAM.

An alternative to the TFAM approach is to consider isosceles trapezoids instead of triangles [6, 7]. In this case we called the resulting framework ***Trapezoidal Fuzzy Assessment Model (TpFAM).*** The corresponding scheme is that shown in Figure 5.

In this case the *y* - coordinate of the COG F_i, i=1, 2, 3, 4, 5, of each trapezoid is calculated in terms of the fact that the COG of a trapezoid lies on the line segment joining the midpoints of its parallel sides *a* and *b* at a distance *d* from the longer side *b* given by

$$d = \frac{h(2a+b)}{3(a+b)},$$

where *h* is its height [18]. Also, since the *x*-coordinate of the COG of each trapezoid is equal to the x-coordinate of the midpoint of its base, it is easy to observe from Figure 5 that

$$x = 0.7i - 0.2.$$

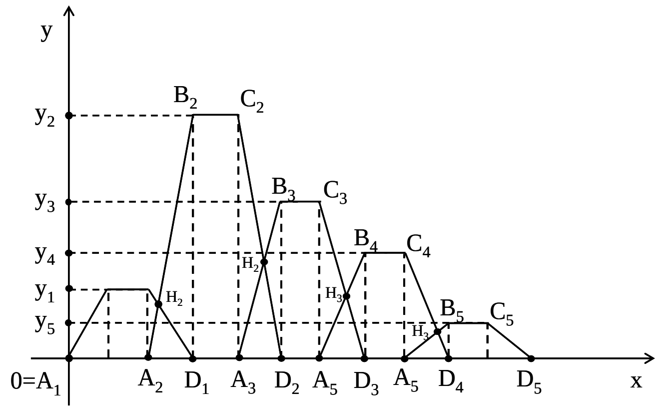

Figure 5: The TpFAM's scheme

One finally obtains from formulas (4) that

$$X_c = (0.7\sum_{i=1}^{5} iy_i) - 0.2\,, \quad Y_c = \frac{3}{7}\sum_{i=1}^{5} y_i^2 \quad \textbf{(8)}$$

and the assessment criterion remains the same again.

5. Comparison of the Assessment Models

One can write formulas (5), (6) and (7) in the unified form:

$$X_c = (0.7\sum_{i=1}^{5} iy_i) - 0.2\,, \quad Y_c = a\sum_{i=1}^{5} y_i^2 \quad \textbf{(9)}\,,$$

where $a = \frac{1}{2}$ for the GRFAM, $a = \frac{1}{5}$ for the TFAM and

$a = \frac{3}{7}$ for the TpFAM. Combining formulas (9) with the common assessment criterion stated in Section 4 one obtains the following result:

Theorem 1: The three variations of the COG technique, i.e. the GRFAM, the TFAM and the TpFAM are equivalent to each other assessment models.

Further, the first of formulas (9) can be written as

$$X_c = 0.7(y_1 + 2y_2 + 3y_3 + 4y_4 + 5y_5) – 0.2$$

$$= 0.7\,[(y_2 + 2y_3 + 3y_4 + 4y_5) + \sum_{i=1}^{5} y_i\,] – 0.2.$$

Therefore, by formula (2) one finally gets that

$$X_c = 0.7(\text{GPA} + 1) – 0.2 = 0.7\text{GPA} + 0.5 \quad \textbf{(10).}$$

In the same way, the first of formulas (3) for RFAM can be written as

$$x_c = \frac{1}{2}(y_1 + 3y_2 + 5y_3 + 7y_4 + 9y_5) = \frac{1}{2}(2\text{GPA} + 1),$$

or

$$x_c = \text{GPA} + 0.5 \quad \textbf{(11)}.$$

We are ready now to prove:

Theorem 2: If the values of the GPA index are different for two groups, then the GPA index, the RFAM and its variations (GRFAM, TFAM and TpFAM) provide the same assessment outcomes on comparing the performance of these groups.

Proof: Let G and G′ be the values of the GPA index for the two groups and let x_c, $x_c{}'$ be the corresponding values of the x-coordinate of the COG for the RFAM. Assume without loss of generality that G>G′, i.e. that the first group performs better according to the GPA index. Then, equation (11) gives that $x_c > x_c{}'$, which, according to the first case of the assessment criterion of Section III, shows that the first group performs also better according to the RFAM.

In the same way, from equation (10) and the first case of the assessment criterion of Section IV, one finds that the first group performs better too according to the equivalent assessment models GRFAM, TFAM and TpFAM.-

The following result shows that Theorem 2 remains true even in case of the same GPA index.

Theorem 3: If the values of the GPA index are the same for two groups, then the RFAM and its variations GRFAM, TFAM and TpFAM provide the same assessment outcomes on comparing the performance of these groups.

Proof: Since the two groups have the same value of the GPA index, equations (10) and (11) show that the values of X_c and x_c are also the same. Therefore, one of the last two cases of the assessment criteria of Sections III and IV could happen. The possible values of x in these criteria lie in the intervals $[0, \frac{9}{2}]$ and [0, 3.3] respectively, while the critical points correspond to the values $x_c = 2.5$ and $X_c = 1.9$ respectively. Obviously, if both values of x are in [0, 1.9), or in $[2.5, \frac{9}{2}]$, then the two criteria provide the same assessment outcomes on comparing the performance of the two groups. Assume therefore that $1.9 < X_c$ and $x_c < 2.5$. Then, due to equation (10),

$$1.9 < X_c \Leftrightarrow 1.9 < 0.7\text{GPA} + 0.5 \Leftrightarrow 1.4 < 0.7\text{GPA}$$

$$\Leftrightarrow \text{GPA} > 2.$$

Also, due to equation (11),

$$x_c < 2.5 \Leftrightarrow \text{GPA} + 0.5 < 2.5 \Leftrightarrow \text{GPA} < 2.$$

Therefore, the inequalities $1.9 < X_c$ and $x_c < 2.5$ cannot hold simultaneously and the result follows.-

Combining Theorems 2 and 3 one obtains the following corollary:

Corollary 4: The RFAM and its variations GRFAM, TFAM and TpFAM provide always the same assessment results on comparing the performance of two groups.

The following example ([9], Section 4, paragraph vii) shows that in case of the same GPA values the application of the GPA index ***could not*** lead to logically based conclusions. Therefore, in such situations, our criteria of Sections 3 and 4 become useful due to their logical nature.

Example 5: The student grades of two Classes with 60 students in each Class are presented in Table 1

Table 1: Student Grades

Grades	Class I	Class II
C	10	0
B	0	20
A	50	40

The GPA index for the two classes is equal to

$$\frac{2*10+4*50}{60}=\frac{3*20+4*40}{60}\approx 3.67,$$

which means that the two Classes demonstrate the same performance in terms of the GPA index. Therefore equation (10) gives that

$$X_c = 0.7*3.67 + 0.5 \approx 3.07,$$

while equation (11) gives that $x_c \approx 4.17$ for both Classes. But

$$\sum_{i=1}^{5} {y_i}^2 = (\frac{1}{6})^2+(\frac{5}{6})^2=\frac{26}{36}$$

for the first and

$$\sum_{i=1}^{5} {y_i}^2 = (\frac{2}{6})^2+(\frac{4}{6})^2=\frac{20}{36}$$

for the second Class. Therefore, according to the assessment criteria of Sections 3 and 4 the first Class demonstrates a better performance in terms of the RFAM and its variations.
Now which one of the above two conclusions is closer to the reality? For answering this question, let us consider the ***quality of knowledge***, i.e. the ratio of the students received B or better to the total number of students, which is equal to $\frac{5}{6}$ for the first and 1 for the second Class. Therefore, from the common point of view, the situation in Class II is better.
Nevertheless, many educators could prefer the situation in Class I having a greater number of excellent students. Conclusively, in no case it is logical to accept that the two Classes demonstrated the same performance, as the calculation of the GPA index suggests.

The next example shows that, although by Corollary 4 the RFAM, GRFAM, TFAM and TpFAM provide always the same assessment results on comparing the performance of two groups, they ***are not equivalent*** assessment models.

Example 6: Table 2 depicts the results of the final exams of the first term mathematical courses of two different Departments, say D_1 and D_2, of the School of Technological Applications (future engineers) of the Graduate T. E. I. of Western Greece. Note that the contents of the two courses and the instructor were the same for the two Departments.

Table 2: Results of the two Departments

Grade	D_1	D_2
A	1	1
B	3	6
C	11	13
D	9	10
F	6	5
No. of students	30	35

The GPA index is equal to

$$\frac{1*9+2*11+3*3+4*1}{30}\approx 1.47$$

for D_1 and

$$\frac{1*10+2*13+3*6+4*1}{35}\approx 1.66$$

for D_2. Therefore, the two Departments demonstrated a less than satisfactory performance (since GPA < 2), with the performance of D_2 being better.
Further, equation (10) gives that $X_c \approx 1.53$ for D_1 and $X_c \approx 1.66$ for D_2. Therefore, according to the first case of the assessment criterion of Section IV, D_2 demonstrated, with respect to GRFAM, TFAM and TpFAM, a better performance than D_1. Moreover, since

$$1.53 < \frac{3.3}{2} = 1.65 < 1.66,$$

D_1 demonstrated ***a less than satisfactory*** performance, while D_2 demonstrated ***a more than satisfactory*** performance.
In the same way equation (11) gives that $x_c \approx 1.97$ for D_1 and $x_c \approx 2.16$ for D_2. Therefore, according to the first case of the assessment criterion of Section III, D_2 demonstrated, with respect to RFAM, a better performance than D_1. But in this case, since for both Departments X_c <

$\frac{4.5}{2}$ = 2.25, ***both*** Departments demonstrated ***a less than satisfactory*** performance.

Remark: Note that, if GPA > 2, then

X_c = 0.7GPA + 0.5 > 0.7 * 2 + 0.5 = 1.9 > 1.65

and

x_c = GPA + 0.5 > 0.2 + 0.5 = 2.5> 2.25.

Therefore, the corresponding group's performance is more than satisfactory with respect to GRFAM, TFAM, TpFAM and RFAM.

However, if GPA < 2 , then X_c < 1.9 and x_c < 2.5, which do not guarantee that X_c < 1.65 and x_c < 2.25. Therefore the assessment characterizations of RFAM and the equivalent to each other GRFAM, TFAM, TpFAM ***can be different only when GPA < 2***.

6. Conclusion

From the discussion performed in this paper it becomes evident that the RFAM and the equivalent to each other GRFAM, TFAM and TpFAM, although they provide always the same assessment outcomes on comparing the performance of two groups, they are not equivalent assessment models. Further, the assessment outcomes of the above models are also the same with those of the GPA index, unless if the value of the GPA index is the same for both groups. In the last case the GPA index could not lead to logically based conclusions. Therefore, in this case either the use of RFAM or of its variations must be preferred.

Other fuzzy assessment methods have been also used in earlier author's works like the measurement of a system's uncertainty [19] and the application of the fuzzy numbers [20]. These methods, in contrast to the previous ones which focus on the corresponding group's quality performance, they measure its mean performance. The plans for our future research include the effort to compare all these methods to each other in order to obtain the analogous conclusions.

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