$$\dot{x} = \nu \tag{7}$$

$$\dot{\nu} = \frac{T_L}{\alpha R} + \frac{T_R}{\alpha R} + \frac{F_{dL}}{\alpha} + \frac{F_{dR}}{\alpha} + \frac{(J_{mo}+J_{po})-ml^2}{\alpha(J_{mo}+J_{po})} f_p - \frac{m^2 g\, l^2}{\alpha(J_{mo}+J_{po})}\theta \tag{8}$$

$$\dot{\theta} = \omega \tag{9}$$

$$\dot{\omega} = \frac{\left(\frac{mlT_L}{R} + \frac{mlT_R}{R} + mlF_{dL} + mlF_{dR}\right) + (mg\, l\theta + f_p l)}{\beta} + \frac{\left(M+m+4M_w+\frac{2J_w}{R^2}\right) + ml\, f_p}{\beta} \tag{10}$$

$$\dot{\delta} = \Omega \tag{11}$$

$$\dot{\Omega} = \frac{D}{2}\left[\frac{\frac{T_L}{R} - \frac{T_R}{R} + F_{dL} - F_{dR}}{J_\delta + \frac{D^2}{2}\left(\frac{J_W}{R^2} + M_W\right)}\right] \tag{12}$$

Where

$$\alpha = M + m + 4M_w + \frac{2J_w}{R^2} + \left[\frac{m^2\, l^2}{(J_{mo}+J_{po})}\right]$$

$$\beta = (J_{mo}+J_{po})\left(M + m + 4M_w + \frac{2J_w}{R^2}\right) + m^2\, l^2$$

The general state-space representation of a continuous LTI system can be written in the following form:

$$\begin{bmatrix} \dot{x} \\ \dot{\nu} \\ \dot{\theta} \\ \dot{\omega} \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \nu \\ \theta \\ \omega \\ \delta \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ B_{21} & B_{22} & B_{23} & B_{24} & B_{25} \\ 0 & 0 & 0 & 0 & 0 \\ B_{41} & B_{42} & B_{43} & B_{44} & B_{45} \\ 0 & 0 & 0 & 0 & 0 \\ B_{61} & B_{62} & B_{63} & B_{64} & 0 \end{bmatrix} \begin{bmatrix} T_L \\ T_R \\ F_{dL} \\ F_{dR} \\ f_p \end{bmatrix} \tag{13}$$

Where

$$A_{23} = -\frac{m^2 g\, l^2}{\alpha(J_{mo}+J_{po})}, \quad A_{43} = \frac{\left(M+m+4M_w+\frac{2J_w}{R^2}\right)mgl}{\beta}$$

$$B_{21} = B_{22} = \frac{1}{R\alpha}, \quad B_{23} = B_{24} = \frac{1}{\alpha}$$

$$B_{25} = \frac{(J_{mo}+J_{po})-m\, l^2}{\alpha(J_{mo}+J_{po})}, \quad B_{41} = B_{42} = \frac{ml}{R\beta}$$

$$B_{43} = B_{44} = \frac{ml}{\beta}, \quad B_{45} = \frac{\left(M+m+4M_w+\frac{2J_w}{R^2}\right)l + ml}{\beta}$$

$$B_{61} = \frac{D}{2R}\left[\frac{1}{J_\delta + \frac{D^2}{2}\left(\frac{J_\omega}{R^2} + M_\omega\right)}\right], \quad B_{62} = -B_{61}$$

$$B_{63} = \frac{D}{2}\left[\frac{1}{J_\delta + \frac{D^2}{2}\left(\frac{J_\omega}{R^2} + M_\omega\right)}\right], \quad B_{64} = -B_{63}$$

3 System Analysis

The TWIPMR system can be defined as a MIMO system where ($T_L, T_R, F_{dL}, F_{dR}, f_p$) the inputs are and ($x, \dot{x}, \theta, \dot{\theta}, \delta, \dot{\delta}$) are the outputs states. The system transfer functions are summarized in table 2. The transfer functions have at least one or more unstable poles. The open loop step and impulse responses are shown in Fig 3. It can be clearly seen that all responses are diverging and the system is unstable. Also a rapid divergence in the output is observed when a little variations in the input signal is occurred. [6],[7]. Consequently, in order to avoid this degradation in stability and tracking performance, the decoupling optimal controller is designed as will be explained in the next section.

Table 2: Transfer Function Matrix

	TL	TR	F_{dR}	F_{dL}	f_p
x	$\frac{1.3\,s^2 - 15.}{s^4 - 11.2s^2}$	$\frac{1.3\,s^2 - 15.}{s^4 - 11.2s^2}$	$\frac{0.2\,s^2 - 2}{s^4 - 11.2s^2}$	$\frac{0.2\,s^2 - 2}{s^4 - 11.2s}$	$\frac{0.2\,s^2 - 2}{s^4 - 11.2s}$
$\dot{x}$	$\frac{1.3\,s^2 - 15.}{s^3 - 11.2s}$	$\frac{1.3\,s^2 - 15.}{s^3 - 11.2s}$	$\frac{0.2\,s^2 - 2}{s^3 - 11.2s}$	$\frac{0.2\,s^2 - 2}{s^3 - 11.2s}$	$\frac{0.2\,s^2 - 2}{s^3 - 11.2s}$
θ	$\frac{1.58}{s^2 - 11.2}$	$\frac{1.58}{s^2 - 11.2}$	$\frac{0.27}{s^2 - 11.2}$	$\frac{0.27}{s^2 - 11.2}$	$\frac{1.03}{s^2 - 11.2}$
$\dot{\theta}$	$\frac{1.58\,s}{s^2 - 11.2}$	$\frac{1.58\,s}{s^2 - 11.2}$	$\frac{0.27\,s}{s^2 - 11.2}$	$\frac{0.27\,s}{s^2 - 11.2}$	$\frac{1.03s}{s^2 - 11.2}$
δ	$\frac{18.85}{s^2}$	$-\frac{18.85}{s^2}$	$\frac{3.2}{s^2}$	$\frac{-3.2}{s^2}$	0
$\dot{\delta}$	$\frac{18.85}{s}$	$-\frac{18.85}{s}$	$\frac{3.2}{s}$	$\frac{-3.2}{s}$	0