Stability Analysis of Photovoltaic Systems Using Krasovskii's Method Based on The Second Method Of Liapunov

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Abstract: - A single diode model incorporates diffusion and recombination characteristics of charge carriers, but neglects the recombination in the space charge zone. If such an effect has to be taken into consideration, double diode model has to be used. The double diode model can exhibit good accuracy resulting in accurate prediction of photovoltaic system's performance. A novel approach to analyse the stability of double diode model based photovoltaic systems which relies on second method of Liapunov that uses Krasovskii's method is proposed in this paper. Nominal operating condition is considered and stability is analysed in accordance with the devised Liapunov function.

Key-Words: - PV system, Double diode model, Liapunov, Krasovski, Stability

1 Introduction

Solar energy is a clean energy source since it is completely natural one .Considerable progress is made in the last few decades in the area of renewable energy systems such as wind, tidal and solar energy systems. Out of all the resources available, the solar energy provides prospective solution for the energy crisis and is found to be trustworthy. Solar energy has become an essential part of our life and man is learning to harness so as to replace the traditional sources of energy. Solar power is expected to become the world's largest source of electricity by 2050, with solar photovoltaics (PV) accounting for 16 percent to the global overall consumption. Solar PV is becoming the mainstream electricity source in the last two decades. Since 2000, worldwide growth of photovoltaics has averaged 40% per year and total installed capacity has reached 139 GW at the end of 2013. The International Energy Agency predicted that global solar PV capacity could reach 3,000 GW in 2010 and four years later, in 2014, the agency reassessed and estimated that the capacity would reach 27% of global electricity generation by 2050.

Photovoltaic systems are gaining importance in today's world and it becomes major concern to analyse the stability of the system as the installations are increasing day by day. When the atmospheric condition changes, the behaviour of the PV system changes. The dynamic analysis of the PV systems become essential as the equilibrium points of the system changes with change in the light emitted by the sun. The behaviour of PV systems can be described by set of non linear equations, which includes bypass and blocking diodes models and is characterized by a sparse Jacobian matrix [3]. Small signal stability is defined as the ability of the system to maintain synchronism when it is subjected to small disturbances [5]. Small signal stability analysis is to examine the stability of the system under various PV penetration levels [7]. Researchers [10] have investigated the transient behavior of the transmission system in response to various disturbances related to PV generation. The small signal analysis of microgrids with wind generators has been well documented in literature [2], [8]. The small signal model of an isolated PV array delivering a load is developed in [8] while, [1] have analyzed the small signal stability of large scale PV generation at the transmission level. In [9], a Lyapunov function is obtained considering the dynamics of both a conventional generator and a dynamic load.

In this paper the stability of double diode model based system is analysed using Krasovskii's method based on second method of Liapunov. The mathematical model for the Photovoltaic system and Krasovoskii's method that suits for non linear systems are presented in this paper. Two equilibrium points are obtained using Newton-Raphson method and the stability is assessed.

2 Mathematical modeling of Photovoltaic system

Mathematical modeling is the process of constructing mathematical objects. A mathematical object could be a system of equations, a set of numbers, a stochastic process, a geometric or algebraic structure or an algorithm. Photovoltaic (PV) system that incorporates double diode is taken into account and attempted to model mathematically. Typically, Photovoltaic systems are characterized such that constant power is injected into the grid. Here the dynamics of the PV systems with constant power load is considered and internal resistive components are neglected.

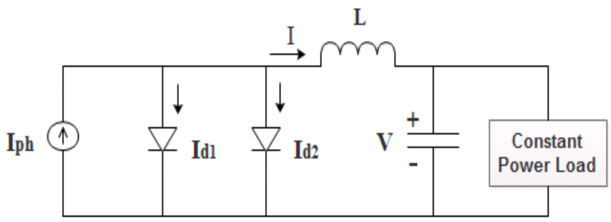

Fig.1 Equivalent circuit of double diode model based PV system

The assumption of constant power load is practical because the Photovoltaic cells are connected to the grid and they supply constant power in planning stage. The equivalent circuit of double diode model based PV system is shown in Fig.1.

Apply Kirchoff's current law at node 1

$$I_{ph} - I_{d1} - I_{d2} - I = 0 \qquad (1)$$

$$I_{ph} - 2I_s\left\{e^{\alpha\left(V+L\frac{dI}{dt}\right)} - 1\right\} - I = 0 \qquad (2)$$

At node 2

$$I - c\frac{dV}{dt} - \frac{P}{V} = 0 \qquad (3)$$

In equation (2), the exponential terms comes due to the diode current. Finally, the dynamics of the systems are given as follows.

From (3) we get

$$\frac{dV}{dt} = \left[\frac{1}{c}\right]I - \left[\frac{P}{cV^2}\right]V \qquad (4)$$

From (6.2) we get

$$\frac{dI}{dt} = \left(\frac{1}{L}\right)\left(\frac{1}{\alpha}ln\left[\frac{I_{ph}-I}{2I_s}\right] + 1\right) - \left(\frac{1}{L}\right)V \qquad (5)$$

Here, $\propto = \frac{q}{N_s KT}$, q is the charge of electron,
$K = 1.3807 * \frac{10^{-3}J}{K}$ is the Boltzmann's constant,
N_s = 72 is the number of cells connected in series in the PV system,
$T = 298\ K$ is the temperature,
$Iph = 4.7A$ for irradiance at 1000W/m^2 is the light generated current,
$P = 150\ W$ is the constant power drawn by load,
$I_s = 9e^{-11}A$ is the saturation current of the diode,
$L = 1\mu H$is the series inductance of the PV system,
$C = 10mF$is shunt capacitance of the PV system,
I and $V$are the output current and output voltage of PV system respectively.

Equations (4) and (5) represent a nonlinear, time-varying system that defines the dynamics of photovoltaic system and can be written in the following form

$$\dot{x} = f(x,t) \qquad (6)$$

Therefore, the equilibrium point of (6) must satisfy $f(x,t) = 0$.

The i-v characteristic curve may shift with changing atmospheric condition, thus there is the probability of two solutions which can be obtained by solving equations (7) and (8). Both the equations are non linear equations.

$$\left(\frac{1}{L}\right)\left(\frac{1}{\alpha}ln\left[\frac{I_{ph}-I}{2I_s}\right] + 1\right) - \left(\frac{1}{L}\right)V = 0 \qquad (7)$$

(1)

$$I - \frac{P}{V} = 0 \qquad (8)$$

From (7), we get two equations such as

$$f(V) = 4.7V - 150 + (18 * 10^{-11})V - (e^{0.5408V})((18 * 10^{-11})V) \qquad (9)$$

$$f(I) = 4.7 - I + (18 * 10^{-11}) - \left(e^{\frac{81.12}{t}}\right)(18 \times 10^{-11}) \qquad (10)$$

Equations (9) and (10) when solved numerically using Newton-Raphson method yields following two solutions

$$(\bar{V}, \bar{I}) = [(41.667,3.5999), (31.9540,4.6942)]$$

3 Krasovskii's Method for formulation of Lyapunov functions

If asymptotic stability of equilibrium states of non linear systems is to be examined, stability analysis for the linearised models of non linear systems is inadequate. Therefore the non linear systems are to be investigated without linearization. One of the methods available for the purpose is Krasovskii's method that is based on second method of Liapunov [12]. By this method sufficient conditions for asymptotic stability can be tested.

A system defined by $\dot{x} = f(x)$, where x is a n-dimensional vector is considered.Let us assume that f(0) = 0 and f(x) is differentiable with respect to x_i where, i = 1,2,3,…..,n. Jacobian matrix F(x) for the system can be written as

$$F(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad (11)$$

Defining Hermitian matrix as $\hat{F}(x) = F^T(x)F(x)$, where $F^T(x)$ be the conjugate transpose of F(x). The equilibrium state $\boldsymbol{x}$ = 0 is asymptotically stable if the Hermitian matrix is negative definite. A Liapunov function for the system is $S(x) = f^T(x)\ f\ (x)$. If in addition $f^T(x)\ f(x) \rightarrow \infty$ as $\| x \| \rightarrow \infty$, then the equilibrium state is asymptotically stable.

Proof:

If $\hat{F}(x)$ is negative definite for all $x \neq 0$, the determinant of $\hat{F}$ is nonzero everywhere except at x = 0. There is no other equilibrium state than x = 0 in the entire state space. Since $f(0) = 0$, $f(x) \neq 0$ for $x \neq 0$, and $S(x) = f^T(x)\ f\ (x)$ is positive definite.

Now, $\dot{S}$ can be obtained as

$$\begin{aligned} \dot{S}(x) &= \dot{f}^T(x)f(x) + f^T(x)\dot{f}(x) \\ &= [F(x)f(x)]^T f(x) + f^T(x)F(x)f(x) \\ &= f^T(x)[F^T(x) + F(x)]f(x) \\ &= f^T(x)\hat{F}(x)f(x) \end{aligned}$$

If $\hat{F}(x)$is negative definite, it is seen that $\dot{S}(x)$is negative definite. Therefore, $S(x)$is a Liapunov function.and the origin is asymptotically stable.

4 Stability analysis using Liapunov function

Liapunov functions are applicable for the analysis of the equilibrium states of the system.

The Jacobian matrix of the PV system is given by,

$$F = \begin{bmatrix} -\frac{1}{\alpha L}\left[\frac{2I_S}{I_{ph}-I+2I_S}\right] & -\frac{1}{L} \\ \frac{1}{C} & \frac{P}{cV^2} \end{bmatrix} \quad (12)$$

$$F^T = \begin{bmatrix} -\frac{1}{\alpha L}\left[\frac{2I_S}{I_{ph}-I+2I_S}\right] & \frac{1}{C} \\ -\frac{1}{L} & \frac{P}{cV^2} \end{bmatrix} \quad (13)$$

The matrix $\hat{F}$ is

$$\hat{F} = F + F^T = \begin{bmatrix} -\frac{2}{\alpha L}\left[\frac{2I_S}{I_{ph}-I+2I_S}\right] & -\left[\frac{1}{L} - \frac{1}{C}\right] \\ -\left[\frac{1}{L} - \frac{1}{C}\right] & \frac{2P}{cV^2} \end{bmatrix} \quad (14)$$

The matrix$\hat{F}$ is negative definite for $V > 0, I < I_{ph}$. Therefore, the Lyapunov function for PV system is

$$S = f^T f \quad (15)$$

$$S = [f_1\ f_2]\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad (16)$$

$$S = {f_1}^2 + {f_2}^2 \quad (17)$$

Where,

$$f_1 = \left(\frac{1}{L}\right)\left(\frac{1}{\alpha} ln\left[\frac{I_{ph}-I}{2I_S}\right] + 1\right) - \left(\frac{1}{L}\right)V \quad (18)$$

and

$$f_2 = \left[\frac{1}{c}\right]I - \left[\frac{P}{cv^2}\right]V \quad (19)$$

The derivative of S from Eq. (17) is written as

$$\dot{S} = 2\left[f_1\dot{f}_1 + f_2\dot{f}_2\right] \quad (20)$$

$\dot{f}_1$ and $\dot{f}_2$are given by the expressions

$$\dot{f}_1 = \frac{1}{\propto L}\left[\frac{2I_S}{I_{ph}-I+2I_S}\right] \quad (21)$$

$$\dot{f}_2 = \frac{1}{c} \quad (22)$$

The appropriate values of V and I obtained using Newton-Raphson method are utilized to find the values of S and $\dot{S}$ using equations (17) and (20). The values and condition of the system at equilibrium points are listed in Table.1.

Table 1. Evaluation of stability.

Voltage(V)	Current (I)	$\dot{S}$	Condition
41.6671	3.5999	56.06	Unstable
31.9540	4.6942	-117.92	Stable

Out of the two equilibrium points, one yields stable condition and the other does not. The stability analysis using Liapunov function claims that the PV system with light generated current can deliver the maximum current of 4.6942 A for the constant power load of 150W. Since, the two diode model considers recombination in the space charge zone which the single diode model neglects, the maximum current that the system can deliver for it to be stable is very close to the desired value of current under nominal operating conditions.

5 Conclusion

The double diode model based PV system is considered and stability is analysed using the Liapunov function formulated by employing Krasovoskii's method. As the system includes recombination characteristics of the charge carriers in the space charge zone, an accurate result is obtained. A set of non linear equations represent the system, hence the stability is analysed directly. The same concept can be applied to the systems that employ various loads.

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