

# Trajectory Control of a Variable Loaded Servo System by using Fuzzy Iterative Learning PID Control

OMER AYDOGDU, MEHMET LATIF LEVENT  
Department of Electrical and Electronics Engineering,  
Selcuk University,  
Alaeddin Keykubat Campus, 42075, Selcuklu, Konya,  
TURKEY  
oaydogdu@selcuk.edu.tr, mlatif@selcuk.edu.tr,  
<http://www.selcuk.edu.tr/muhendislik/en>

*Abstract:* - In this study, trajectory control of the variable loaded servo system is performed by using a Fuzzy Logic based Iterative Learning Control (ILC) method. As the iterative learning control structure, a Iterative Learning PID (IL-PID) Controller is used in the study. Also, a fuzzy adjustment mechanism has been added to the control system for specify the initial parameter of the IL-PID controller. So, with combining the fuzzy based parameter adjustment mechanism and the IL-PID controller, Fuzzy Iterative Learning PID (Fuzzy IL-PID) controller is designed to improving the system performance. In the designed system, thanks to the fuzzy adjustment mechanism, the IL-PID controller parameters such as  $K_p$ ,  $K_i$ , and  $K_d$  values are automatically adjusted to the appropriate values initially. To illustrate the effectiveness of the proposed fuzzy IL-PID controller, trajectory control of the variable loaded servo system was performed by using both Fuzzy PID and Fuzzy IL-PID control methods under the same conditions separately, and the obtained results were compared. It is seen from the experimental results, the proposed Fuzzy IL-PID control method is to better compensate the system effect as time varying loads and has reduced the steady-state error more than other method in iterations progresses.

*Key-Words:* - Fuzzy PID Control, Fuzzy IL-PID Control, Trajectory Control, Variable Loaded Servo System

## 1 Introduction

In practice, conventional PID controllers are widely used in many control systems due to the simple, robust and effective structure. Although, PID controller gives very good results in the control of linear, time-invariant systems, it gives not enough good results in the control of non-linear, time varying systems. Also, the effectiveness of the PID controllers depends on the precisely modeling of the system and the settings of the controller parameters. Conventional methods such as Ziegler-Nichols used to set the parameters of PID controllers, only helps to determine the controller parameters at the beginning in design stage. Besides, these methods do not provide the ability to on-line setting to the parameters of the controller in time [1]. Therefore, the PID controllers are insufficient when applied to time-varying system and it cannot operate at the desired performance.

On the other hand, fuzzy logic controller working with intuitive knowledge has been used frequently in recent years as an alternative to conventional PID controllers. Fuzzy logic system is a basic method based on human thinking and knowledge that can be applied successfully in many

areas. Fuzzy logic is propounded with an article published in 1973 by L. Zadeh [2]. The most important advantage of fuzzy logic method is their intuition and experience of the operator can transmit to the controller by means of fuzzy set theory. As a feature, the advantages of fuzzy control in the control system can be listed as follows [3];

- It is robust against noise,
- It gives better results for model uncertainty and changing system parameters,
- It has very good transient state response, etc.

In spite of these advantages, the fuzzy control methods are ineffective in the control of nonlinear systems, and not enough, especially to minimize the steady-state error of the system, as with conventional PID controller. Therefore, Iterative Learning Control (ILC) method has been proposed to eliminate deficiencies in both the PID and fuzzy control systems [4].

ILC method provides very convenient and effective results for periodically repeated in the control applications such as trajectory control of a robot arm. In studies now, ILC systems are divided into two

categories as traditional ILC and adaptive ILC methods. Traditional ILC has a learning mechanism that produces the control inputs based on the error and the previous input values. Adaptive ILC method uses the advantages of both the ILC and adaptive control to produce the control inputs [5]. In the last decade, many Adaptive ILC applications have been developed for unstable and nonlinear systems [6].

Proposed Fuzzy IL-PID control method in the study is an adaptive ILC method and it has been developed to obtain optimal PID parameters of the ILC mechanism. In the study, the main purpose of the combination of the fuzzy logic control and the ILC method to benefit from the advantages of both fuzzy logic and ILC methods and, to improve the performance of the ILC system [6]. In this method, obtained initial PID control parameters as  $K_p, K_i, K_d$  by using fuzzy logic method has been used in the ILC system.

## 2 Modeling of the Variable Loaded Servo (VLS) System

In industrial applications, permanent magnet DC (PMDC) motors are widely used as an actuator for electromechanical energy conversion [7]. It directly provides rotary motion or moment and, coupled with wheels or drums and cables, can provide transitional motion or force. In this study, the VLS system actuated with PMDC motor which has the electric circuit of the armature and the variable loaded body diagram of the rotor are shown in Fig. 1.

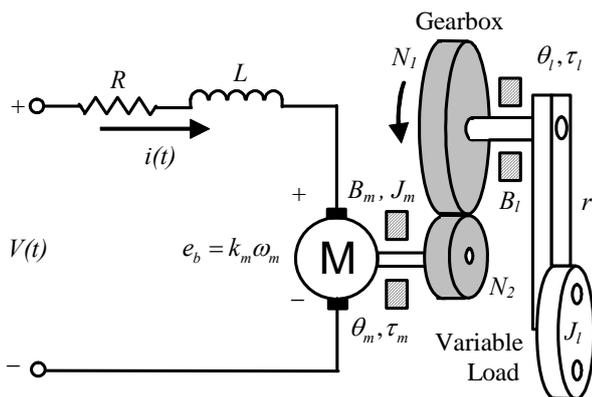


Fig. 1. Equivalent circuit of the VLS system

Variable loaded servo plant and PMDC motor parameters are given in Table 1. We can write the following differential equations based on Newton's law combined with Kirchhoff's law from the Fig. 1 [8];

$$V(t) = Ri(t) + L \frac{di(t)}{dt} + e_b(t), \quad (1)$$

$$\tau_m(t) = J_m \frac{d^2\theta_m(t)}{dt^2} + B_m \frac{d\theta_m(t)}{dt} + \tau_l(t), \quad (2)$$

where;  $i(t)$  is armature current,  $e_b(t)$  is back emf voltage,  $\tau_m(t)$  is motor torque,  $\tau_l(t)$  is load torque,  $\theta_m(t)$  is angle of the armature. The motor torque,  $\tau_m(t)$  is related to the armature current,  $i(t)$  by a constant factor  $k_t$  and the back emf,  $e_b(t)$  is related to the rotational velocity of armature,  $\omega_m(t)$  or  $\frac{d\theta_m(t)}{dt}$  by a constant factor  $k_m$ , as given the following equations;

$$\tau_m(t) = k_t i(t), \quad (3)$$

$$e_b(t) = k_m \omega_m(t) = k_m \frac{d\theta_m(t)}{dt}. \quad (4)$$

Table 1. Variable loaded servo system and load parameters

Symbol	Definition	Value
$V$	Voltage of the DC motor	6 Volt
$R$	Motor armature resistance	2.6 ohm
$L$	Motor armature inductance	0.18 mH
$k_t$	Motor torque constant	0.00767 N.m/A
$k_b$	Motor back-EMF constant	0.00767 Volt/(rad/sec)
$N_g$	Total gear ratio (N2/N1)	70
$\eta_g$	Gearbox efficiency	0.90
$J_m$	Motor inertia	$4.6 \times 10^{-7}$ kg.m <sup>2</sup>
$B_m$	Motor viscous damping coefficient	$\cong 0$ (negligible)
$J_{l\_in}$	Initial load and gearbox moment of inertia	$4.83 \times 10^{-7}$ kg.m <sup>2</sup>
$J_{l\_sub}$	Subsequent load and gearbox moment of inertia	$4.83 \times 10^{-6}$ kg.m <sup>2</sup>
$B_{l\_in}$	Initial load viscous damping coefficient	$4.41 \times 10^{-6}$ N.m/(rad/sec)
$B_{l\_sub}$	Subsequent load viscous damping coefficient	$3.41 \times 10^{-5}$ N.m/(rad/sec)

In the VLS system, rotational angle of the load,  $\theta_l(t)$ , transmitted by the gear box from armature angle,  $\theta_m(t)$ , and load torque,  $\tau_l(t)$ , may be expressed as;

$$\theta_l(t) = \frac{1}{N_g} \theta_m(t), \quad (5)$$

$$\tau_l(t) = \frac{1}{N_g \eta_g} \left( J_l \frac{d^2\theta_l(t)}{dt^2} + B_l \frac{d\theta_l(t)}{dt} \right), \quad (6)$$

where;  $N_g$  is total gear ratio and  $\eta_g$  is gearbox efficiency. Using equations (1-6), simulink model of the VLS system with variable load parameters ( $J_l, B_l$ ) has been obtained as shown in Fig. 2.

### 3 Trajectory Control of the Variable Loaded Servo System

In this study, trajectory control of the variable loaded servo system as modeling above has been performed in Matlab-Simulink. First, the servo system is controlled by the Fuzzy PID controller. Next, the same system has been controlled by Fuzzy IL-PID controller, and the simulation results have been compared.

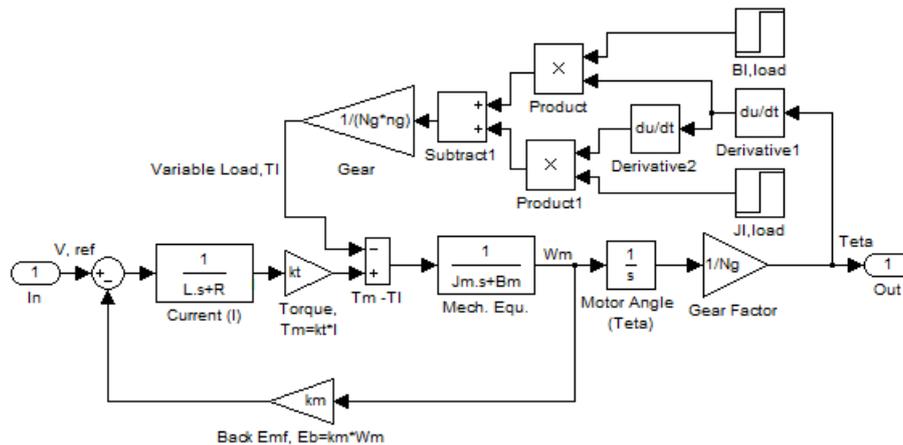


Fig. 2. Simulink model of the VLS system with variable load ( $J_l, B_l$ )

#### 2.1 Fuzzy PID Control

In industrial applications, PID controllers expressed as equation (7) are effective controllers widely used in especially linear time invariant system [9].

$$u(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{de(t)}{dt} \quad (7)$$

where;  $u(t)$  is the control signal,  $e(t)$  is the error value of between desired reference input and actual system output signal. Also, as seen from the equation (7), the design parameters of the PID controller are proportional gain ( $K_p$ ), integral gain ( $K_i$ ) and derivative gain ( $K_d$ ) values.

In the literature, experimental approaches such as Ziegler-Nichols method is often used in the PID controller design procedure. Many times however, it needs to be made further fine tune the parameters to achieve the optimal design. In this respect, determination of design parameters takes a long time.

In addition, various methods such as fuzzy system can be used to determine the PID control parameters. In the fuzzy PID method, the PID controller parameters are set using the fuzzy rule base system. Fuzzy PID method is an adaptive method, thence, provides better operating of the

PID controller in unexpected parameter changes and also increases system performance [10].

Trajectory control of the VLS system by using Fuzzy PID Controller is shown in Fig. 3. As shown in Figure, a Mamdani type fuzzy block with two inputs (system error,  $e(t)$  and derivative of the error,  $\dot{e}(t)$ ) and three outputs ( $K_p, K_i$  and  $K_d$ ) has been used in the Fuzzy PID Controller. Also, PID controller parameters have been set automatically on-line by the fuzzy system.

Generally, Mamdani type fuzzy logic system consists of four basic components names as fuzzification, rule base, inference engine and defuzzification [11]. In the fuzzifier process, the crisp input values ( $e(t), \dot{e}(t)$ ) are converted into fuzzy values [12]. In the study, for fuzzification process, five triangular membership function labeled as Negative Big ( $NB$ ), Negative ( $N$ ), Zero ( $Z$ ), Positive ( $P$ ), Positive Big ( $PB$ ) are defined for each input values. Before the fuzzification process, two input values has been normalized to the range of  $[-2, 2]$  values. Also, the fuzzy values obtained in fuzzy inference mechanism must be converted to crisp output ( $K_p, K_i, K_d$ ) values by a defuzzifier process. For this purpose, the center-of-gravity method has been used and four triangular membership functions labeled as Zero ( $Z$ ), Low ( $L$ ), Medium ( $M$ ), High ( $H$ ) are defined for each output in the defuzzification process. Also, three output values has been denormalized to the range of  $[0, 2]$  values.

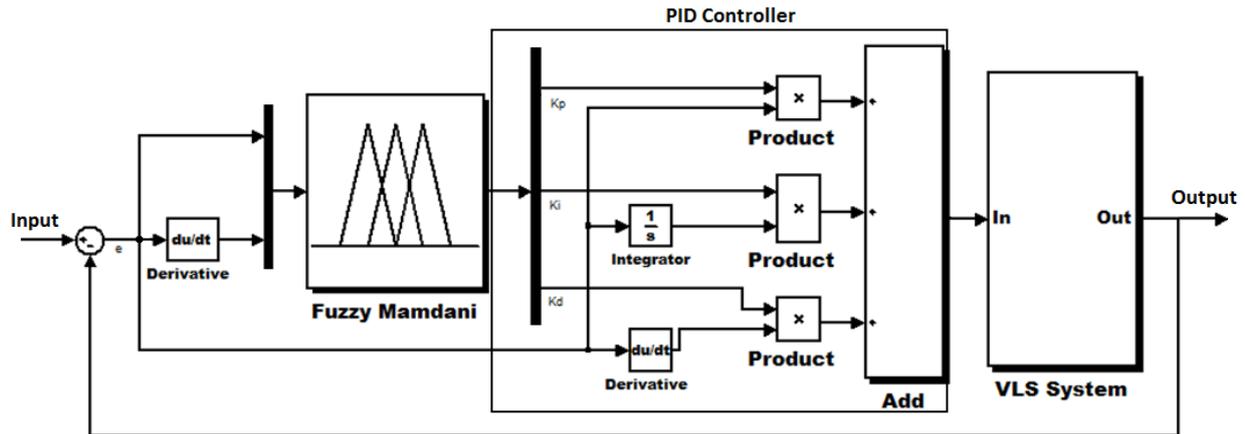


Fig. 3. Block diagram of VLS control system by using Fuzzy PID controller

In the study, the PID controller parameters ( $K_p$ ,  $K_i$ ,  $K_d$ ) have been set to the appropriate value with the fuzzy block. So, fuzzy system rule base is formed with operator experience, according to the desired system response. Fuzzy rule base sets defined for  $K_p$ ,  $K_i$ ,  $K_d$  have been given in Table 1, Table 2, and Table 3 respectively.

Table 2. Rule base for  $K_p$  output

$de \setminus e$	NB	N	Z	P	PB
NB	H	M	Z	L	L
N	H	M	Z	L	H
Z	H	H	H	H	H
P	H	H	L	M	H
PB	H	M	L	M	Z

Table 3. Rule base for  $K_i$  output

$de \setminus e$	NB	N	Z	P	PB
NB	L	H	H	M	L
N	L	M	H	M	L
Z	L	L	M	L	Z
P	L	M	M	M	L
PB	L	H	H	M	B

Table 4. Rule base for  $K_d$  output

$de \setminus e$	NB	N	Z	P	PB
NB	Z	H	H	H	L
N	Z	H	H	M	M
Z	L	L	H	L	H
P	L	H	H	M	M
PB	L	H	H	H	L

Min-Max compositional rule is used in the developed fuzzy logic system. In the decision

making operation, minimum inference method has been applied in the rule base given in above tables.

### 2.2 Fuzzy IL-PID Control

Iterative Learning Control (ILC) improves the transient response and reduces the error trajectory of the time varying system as time progresses. By means of iterative learning mechanism, control system become more stable and more robust. Also, ILC is an effective control method with excellent tracking performance. Another advantage of the ILC system, the distorting effects that will affect the control system is able to compensate. Basic block diagram of the ILC system is shown in Fig. 4 [13,14].

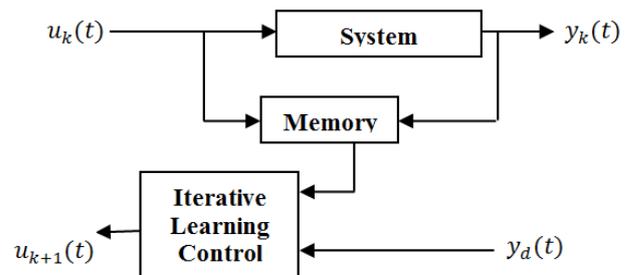


Fig. 4. Basic block diagram of the ILC system

In Fig. 4;  $u_k(t)$  is the current control signal,  $u_{k+1}(t)$  is the next control signal,  $y_k(t)$  is the output signal and  $y_d(t)$  is desired output signal. The basic expression of the control signal for the ILC system defined by Arimoto et al. [15] is given in equation (8).

$$u_{k+1}(t) = u_k(t) + \Gamma \dot{e}_k(t), \tag{8}$$

where;  $\Gamma$  is the learning gain value. Equation (8) is adapted to the PID controller parameters, Iterative

Learning PID (IL-PID) controller can adjust their parameters recursively created. Accordingly, recursive equations can be written as follows for IL-PID controller [14,15].

$$K_{p_{k+1}} = K_{p_k} + (\Phi e(t) + \Gamma \dot{e}(t) + \Psi \int e(t) dt) \quad (9)$$

$$K_{i_{k+1}} = K_{i_k} + (\Phi e(t) + \Gamma \dot{e}(t) + \Psi \int e(t) dt) \quad (10)$$

$$K_{d_{k+1}} = K_{d_k} + (\Phi e(t) + \Gamma \dot{e}(t) + \Psi \int e(t) dt) \quad (11)$$

where;  $K_{p_k}$ ,  $K_{i_k}$  and  $K_{d_k}$  are the current parameters of the PID controller,  $K_{p_{k+1}}$ ,  $K_{i_{k+1}}$  and  $K_{d_{k+1}}$  are the next parameters of the PID controller, and  $\Phi$ ,  $\Gamma$ ,  $\Psi$  are the learning gain values for proportional, derivative, and integral control respectively. When equations (9-11) are examined, it is seen that PID

controller parameters are continuously adjusted based on system error. Under normal conditions, steady-state error can take positive and negative values. Therefore, it is very difficult to adjust the PID control parameters by IL method. In literature, the absolute value of the steady-state error is used to solve this problem. Thus, the PID parameters are more easily moved to the most appropriate value [16]. Also, the initial values of the PID control ontroller parameters determined by fuzzy logic block and thence system is achieved in less time self-adjustment. Block diagram of the Fuzzy IL-PID control system proposed in the study is given in Fig. 5 below. As seen from Fig. 5, the initial values for the IL block are produced in the Fuzzy Mamdani block and PID controller parameters ( $K_p$ ,  $K_i$ ,  $K_d$ ) are updated in the IL block. Trajectory control of Variable Loaded Servo (VLS) system is performed by PID controller which has continuously updated parameters in the IL block.

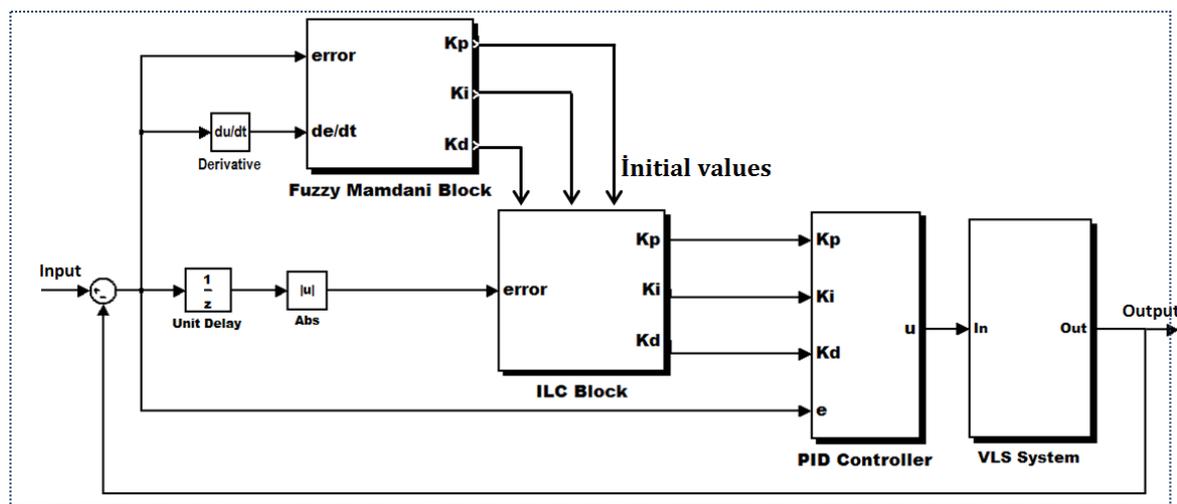


Fig. 5. Trajectory control of VLS system with Fuzzy IL-PID controller

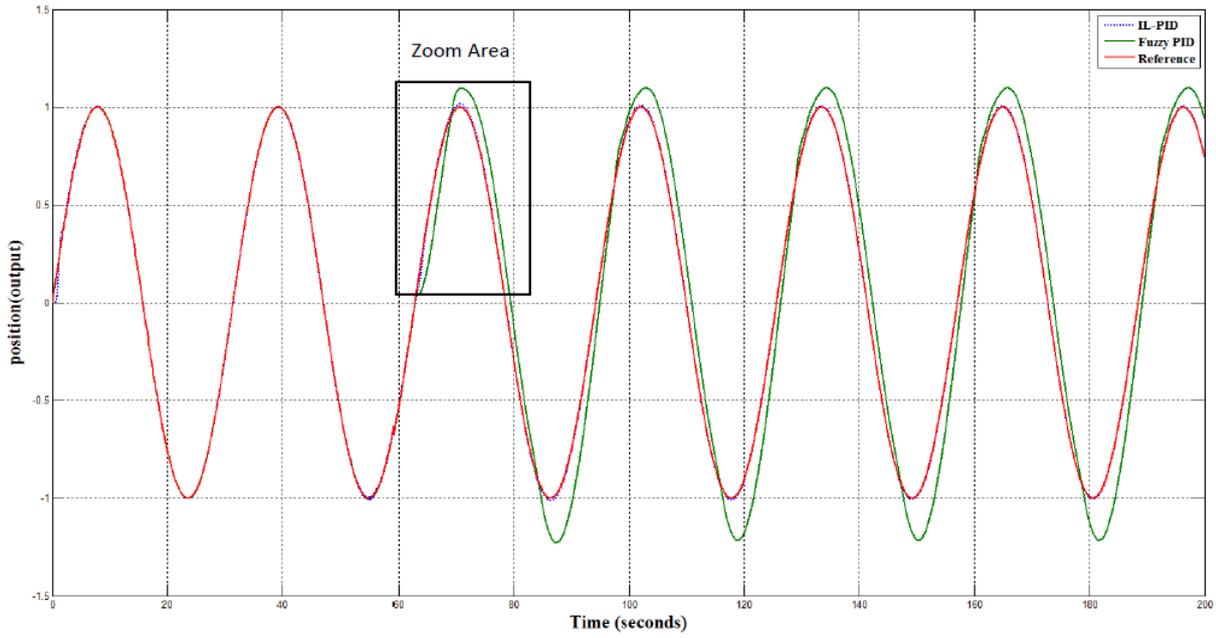
### 4 Result and Discussion

In this study, firstly, sinusoidal trajectory control of the VLS system was performed with the fuzzy PID controller. Then, the same trajectory control system was carried out with Fuzzy IL-PID controller under the same conditions. Also, output responses and trajectory errors of the system and parameter variations for each controller have been observed. The system output responses to both controllers are included in Fig. 6 for sinusoidal reference input. As shown in figure, output response of the Fuzzy PID control system has a little trajectory error for initial load, but after changing the load of the servo system at time 63

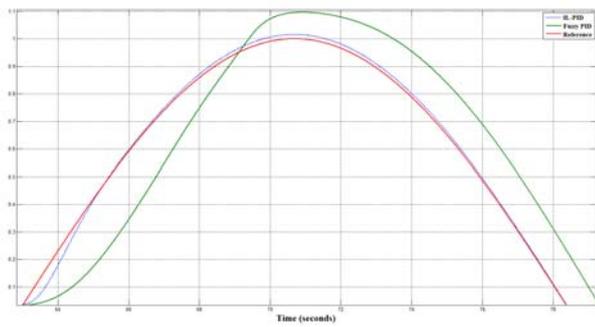
seconds, the trajectory error has increased and so on.

On the other hand, it is seen that the trajectory error of the Fuzzy IL-PID control system decreases gradually as time progresses and system output reaches to the reference trajectory though iterative learning mechanism after changing the load of the system at time 63 seconds. Here, it has been understood that Fuzzy IL-PID control method is to quickly compensate the effect of load changes.

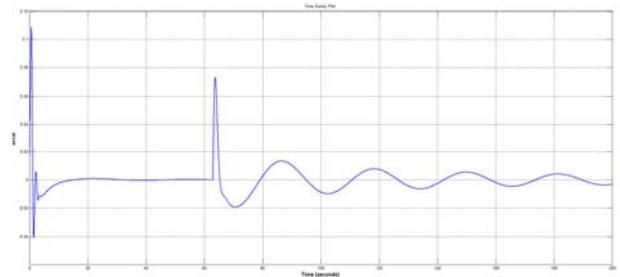
Zoom area in Fig. 6 is given in Fig. 7 to better observe the system responses. Trajectory error of the Fuzzy PID and Fuzzy IL-PID control system is shown in Fig. 8 and Fig. 9, and also, parameter changes of the PID control for Fuzzy PID and Fuzzy IL-PID control system is shown in Fig. 10 and Fig. 11, respectively.



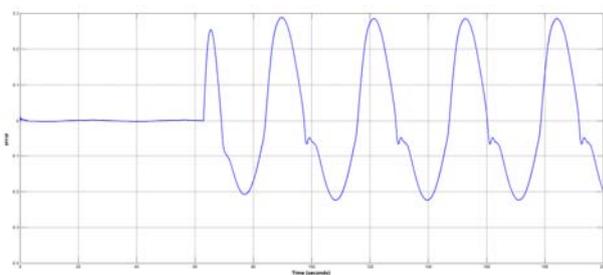
**Fig. 6.** Trajectory outputs of the VLS system



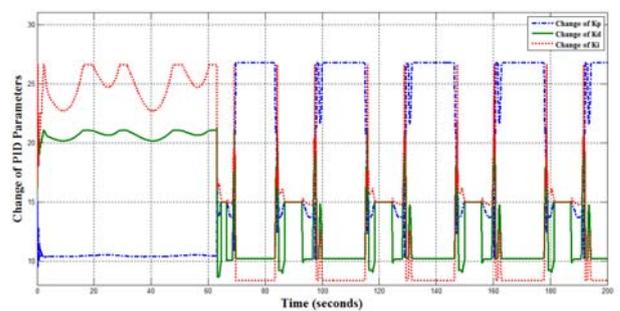
**Fig. 7.** Zoom area in Fig. 6



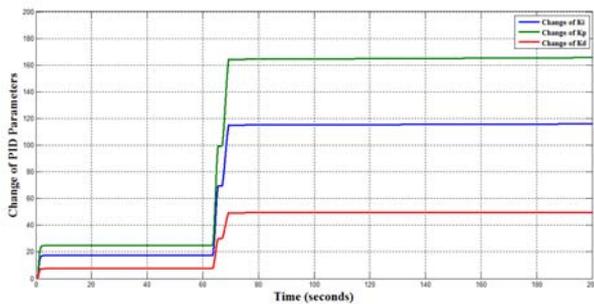
**Fig. 9.** Trajectory error of the Fuzzy IL-PID control system



**Fig. 8.** Trajectory error of the Fuzzy PID control system



**Fig. 10.** Parameter changes of the PID control for Fuzzy PID control system



**Fig. 11.** Parameter changes of the PID control for Fuzzy IL-PID control system

## 5 Conclusions

In the industrial applications, minimization of trajectory error of the DC motor driven servo control system is essential for quality production process. Especially, if the system changes as a variable load over time, the trajectory control of the servo system is more difficult. In this study, Fuzzy IL-PID control method which can be recursively updated controller parameters as time progresses is presented to solve this problem. To illustrate the comparative effectiveness of the proposed method, trajectory control of the time varying servo system has been executed for sinusoidal input by using both Fuzzy IL-PID and Fuzzy PID controllers. Experimental results show that the proposed Fuzzy IL-PID control method makes better compensated the effects of time-varying load and also more reduces the steady state error than other method as time progresses. Finally, Fuzzy IL-PID control method provides better control by continuously updating their control parameters iteratively.

## 6 References

- [1] S. M. Baek, T. Y. Kuc, An adaptif PID learning Control of DC motors, *IEEE International Conference on Computational Cybernetics and Simulation*, Orlando, FL, 1997, vol. 3, pp. 2877-2882.
- [2] L. A. Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, *IEEE trans. on systems, Man and Cybernetics*, vol. 3, no. 1, pp.28-44, 1973.
- [3] O. Elshazly, M. El-Bardini, Development of self-tuning fuzzy iterative learning control for controlling a mechatronic system, *International Journal of Information and Electronics Engineering*, vol. 2, no. 4, pp. 565-569, July, 2012.
- [4] Y. M. Pok, K. H. Liew, and J. X. Xu, Fuzzy PD iterative learning control algorithm for improving tracking accuracy, *IEEE International Conference on Systems, Man and Cybernetics*, San Diego, CA, 1998, vol. 2, pp. 1603-1608.
- [5] J. Wei, Adaptive iterative learning control for a class of nonlinear time-varying systems with unknown delays and input dead-zone, *IEEE/CAA Journal of Automatica Sinica*, vol. 1, no. 3, pp. 302-314, July, 2014.
- [6] C. R. Hu, S. S. Lin, H. Z. Sheng, A new discrete-time adaptive ILC for nonlinear systems with time-varying parametric uncertainties, *Acta Automatica Sinica*, vol. 34, no. 7, pp. 805-808. 2008 (in Chinese).
- [7] O. Alkan, O. Aydogdu, Fuzzy model reference learning control of a time-varying rotary servo systems, *Proceedings of Second International Conference on Informatics*, Çanakkale, (ICI'2011), 2011, pp. 1-7.
- [8] C. Kasnakoglu, Modeling and control of flow problems by adaptation-based linear parameter varying models, *Turkish Journal of Electrical Engineering & Computer Sciences*, vol. 18, 2010.
- [9] M.A. Fadil, PID Controller for Micro-Unmanned Air Vehicle (Micro-UAV), BS thesis, Mekanikal-Aeronautik, Universiti Teknologi, Kuala Lumpur, Malaysia, 2012.
- [10] R. E. Precup, S. Preitl, E. M. Petriu, J. K. Tar, and J. Fodor, Iterative learning-based fuzzy control system, in *IEEE International Workshop on Robotic and Sensors Environments*, Ottawa, Canada, 2008, pp. 25-28.
- [11] M. Dotoli, B. Maione and B. Turchiano, Fuzzy-Supervised PID Control: Experimental Results, in *1st European Symposium on Intelligent Technologies*, Tenerife, Spain, EUNITE 2001, pp. 31-35.
- [12] G. Feng, A Survey on Analysis and Design of Model-Based Fuzzy Control Systems, *IEEE Trans. on Fuzzy Sys.*, vol. 14, no. 5, pp. 676-697, October, 2006.
- [13] C. C. Lee, Fuzzy Logic in Control Systems: Fuzzy Logic Controller-Part I, *IEEE Transactions on Systems, Man and Cybernetics*, vol. 20, no. 2, pp. 404-418, March/April, 1990.
- [14] H. S. Ahn, Y. Q. Chen and K. L. Moore, Iterative Learning Control: Brief Survey and Categorization, *IEEE Transactions On Systems, Man, And Cybernetics, Part C: Applications And Reviews*, vol. 37, no. 6, pp 1099-1121, November, 2007.
- [15] S. Arimoto, S. Kawamura, F. Miyazaki, Bettering operation of dynamic systems by learning: A new control theory for

- servomechanisin and mechatronics systems, *The 23rd IEEE Conference on Decision and Control*, Las Vegas, NV, 1984, pp. 1064-1063.
- [16] M. A. Fadil, N. A. Jalil and I. Z. Mat Darus, Intelligent PID controller using iterative learning algorithm for active vibration controller of flexible beam, *IEEE Symposium on Computers & Informatics*, Langkawi, ISCI, 2013, pp. 80-85.