Robust Predictive Controller Based on an Uncertain Fractional Order Model

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Abstract: This paper focuses on Robust Fractional Predictive Control (RFPC) for fractional order dynamic systems with real parametric uncertainties to take into account the uncertain behavior of physical process. Based on worst case strategy, the control law is obtained by resolution of a non convex min-max optimization problem which takes into account the uncertainties on the fractional order model parameters. The performance of the proposed predictive controller are illustrated with practical results of a thermal system and compared to the Fractional Predictive Control (FPC) with fixed parameters.

Keywords: Robust predictive control; uncertain fractional systems; min-max optimization problem; thermal system.

1. Introduction

The Model Predictive Control (MPC) algorithms is become a mature control strategy for many years because it can handle a large class of dynamic systems such as non-minimum phase systems, open loop unstable systems, delayed and multivariable systems [1]. Consequently, the strategy of MPC is widely encountered in the industrial processes [2]. The MPC is a control technique that optimizes a performance criterion and uses a model to predict the process output future behavior. Hence, the presence of the model is necessary for the predictive control development, but in reality, models are determined with uncertain parameters, which can provide poor closed loop performances [1]. To improve closed loop performances, one can use the robust model predictive control (RMPC). The control law is determined by solving a min-max optimization problem where a quadratic criterion is minimized with respect to its worst-case in order to take into account the set of all possible plant uncertainties [3].
Recently, fractional calculus has been attached the attention of many researchers in engineering science from an application point of view [4]. Many physical systems have shown a dynamic behavior of fractional order. The first fractional dynamic system to be widely recognized is the thermal system. Some other fractional systems can be found in the electromagnetic waves systems, electrode-electrolyte polarization, viscoelastic, etc. [5]. Firstly, the idea of designing a non-integer controller was proposed by Oustaloup in 1988 [6-7]. In 1994, the fractional order PID controller proposed by Podlubny [1]. Since then, there are many different fractional-order controller strategies [8-10]. In recent years, the robustness of fractional order systems has been utilized in order to take into account the parametric variations of uncertain fractional order systems [11-13].
In [14], the authors proposed an application of the fractional predictive controller a fixed-parameter system model. The purpose of our work is to develop the robust predictive control of uncertain fractional order systems. The output deviation approach is used to design the j-step ahead output predictor, and the corresponding control law is obtained by the resolution of a min-max optimization problem which takes into account the uncertainties of the fractional order model parameters.
The outline of this paper is organized as follows. In section 2, a problem formulation and some definitions of fractional order systems are introduced. The needed steps to find the optimal control law of RFPC for

fractional order systems are introduced in section 3. The experimental results on a thermal system are exhibited in section 4 to illustrate the effectiveness of the proposed approach. Finally, conclusion is given in last section.

2. Preliminary and problem formulation

Fractional calculus, known also as non-integer calculus, is a generalization of integration and derivation to fractional order fundamental operators ${}_{t0}D_t^{\alpha}$ where $\alpha \in R$, t_0 and t are the limitation. In the development of fractional calculus, there are several definitions of fractional order differentiations and integrations [15]. The Grünwald-Letnikov's (G-L) definition is the most known definition for fractional control and its application [16-17], it has defined as:

$$ {}_{t0}D_t^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{i=0}^{(t-t_0)/h} (-1)^i \binom{\alpha}{i} f(t-ih) \quad (1)$$

with $\alpha \in \mathrm{R}^+$, h is the sampling period and $\binom{\alpha}{i}$ means:

$$\binom{\alpha}{i} = \frac{\alpha(\alpha-1)\dots(\alpha-i+1)}{i!}$$

Expression (1) may be used to numerically evaluate the integral or the derivative of fractional order using some suitably chosen value of sampling rate as follows [18].

$$ {}_{t_0}D_t^{\alpha} f(t) = \frac{1}{h^{\alpha}} \sum_{i=0}^{(t-t_0)/h} (-1)^i \binom{\alpha}{i} f(t-ih) \quad (2)$$

The series are contrasted with a number of terms which increases when h decreases. For real implementation, by using the short memory principle [1], expression (2) can be rewritten using only the recent past values of $f(t)$ as:

$$ {}_{t_0}D_t^{\alpha} f(t) = \frac{1}{h^{\alpha}} \sum_{i=0}^{N} (-1)^i \binom{\alpha}{i} f((k-i)h) \quad (3)$$

where N is an integer.

Generally, a fractional model can be described by a fractional differential equation characterized by the following form:

$$\sum_{l=0}^{L} a_l D_t^{\alpha_{al}} y(t) = \sum_{m=0}^{M} b_m D_t^{\alpha_{b_m}} u(t) \quad (4)$$

where $(a_l, b_m) \in R^2$, and $(\alpha_{al}, \alpha_{bm}) \in R_+^2$

In reality this equation is described with uncertain parameters, which means that the parameters a_l and b_m are bounded and uncertain:

$a_l \in \left[\underline{a}_l \ ,\ \bar{a}_l\right]$ and $\mathrm{b}_m \in \left[\underline{b}_m \ ,\ \bar{b}_m\right]$

$\underline{a}_l$, $\bar{a}_l$, $\underline{b}_m$ and $\bar{b}_m$ are respectively, the low and high values of a_l and b_m.

The use of the numerical approximation (3), allows rewriting equation (4) as follows [19].

$$y(k) = \frac{1}{\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}}} \sum_{m=0}^{M} \frac{b_m}{h^{\alpha_{b_m}}} \sum_{i=0}^{k} (-1)^i \binom{\alpha_{b_m}}{i} u(k-i) - \frac{1}{\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}}} \sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}} \sum_{i=1}^{k} (-1)^i \binom{\alpha_{a_l}}{i} y(k-i) \quad (5)$$

But the presence of uncertainties in the fractional order system model can lead the controller to be unstable or to have poor closed loop performances. In order to robustify the controller against the uncertainty model parameters and to handle a large class of systems, we will propose the FRPC that is based on the use of an uncertain fractional order model which is obtained by using the G-L definition given by equation (5).

3. Fractional robust predictive control

In this section, we provide the needed steps to find the optimal control law using the new proposed RFPC approach of uncertain descriptor fractional systems. Therefore, the G-L method depicted in section 2 will be used to obtain the fractional order model and we will be based on the output deviation method to compute the j-step ahead output predictor value as well as the cost function. For obvious reasons and without loss of generality, we will express $y(k)$ in terms of $u(k-1)$, and depending on the input deviation. By considering a noise sequence with zero mean and finite variance, the expression (5) becomes:

$$\Delta y(k) = \frac{1}{\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}}} \sum_{m=0}^{M} \frac{b_m}{h^{\alpha_{b_m}}} \sum_{i=0}^{k} (-1)^i \binom{\alpha_{b_m}}{i} \Delta u(k-1-i) - \frac{1}{\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}}} \sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}} \sum_{i=1}^{k} (-1)^i \binom{\alpha_{a_l}}{i} \Delta y(k-i) + e(k) \quad (6)$$

$\Delta = 1 - q^{-1}$, is an integral action introduced in order to obtain, in closed loop, a nil steady state error.

By using the relation (6), we obtain the predicted output of the system in $k+1$:

$$\hat{y}(k+1/k) = y_l(k+1) + \alpha_1 \Delta u(k) \quad (7)$$

where: $\alpha_1 = \frac{1}{\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}}} \sum_{m=0}^{M} \frac{b_m}{h^{\alpha_{b_m}}}$

and $y_l(k+1)$ is the free response of the system:

$$y_l(k+1) = y(k) + s_1 - s_2 \quad (8)$$

$$s_1 = \frac{1}{\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}}} \left(\sum_{m=0}^{M} \frac{b_m}{h^{\alpha_{b_m}}} \sum_{i=1}^{k+1} (-1)^i \binom{\alpha_{b_m}}{i} \Delta u(k-i) \right)$$

$$s_2 = \frac{1}{\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}}} \left(\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}} \sum_{i=1}^{k+1} (-1)^i \binom{\alpha_{a_l}}{i} \Delta y(k+1-i) \right)$$

The 2-step ahead predictor is given by:

$$\begin{aligned} \hat{y}(k+2/k) &= y(k+1) + \alpha_1 \Delta u(k+1) + \beta_1 \Delta u(k) \\ &+ \beta_2 \Delta y(k+1) + s_3 - s_4 \end{aligned} \quad (9)$$

where:

$$\beta_1 = \frac{-1}{\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}}} \sum_{m=0}^{M} \frac{b_m}{h^{\alpha_{b_m}}} \alpha_{b_m} ; \beta_2 = \frac{1}{\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}}} \sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}} \alpha_{a_l}$$

$$s_3 = \frac{1}{\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}}} \left(\sum_{m=0}^{M} \frac{b_m}{h^{\alpha_{b_m}}} \sum_{i=2}^{k+2} (-1)^i \binom{\alpha_{b_m}}{i} \Delta u(k+1-i) \right)$$

$$s_4 = \frac{1}{\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}}} \left(\sum_{l=0}^{L} \frac{a_l}{h^{\alpha_{a_l}}} \sum_{i=2}^{k+2} (-1)^i \binom{\alpha_{a_l}}{i} \Delta y(k+2-i) \right)$$

as: $\Delta y(k+1) = y(k+1) - y(k)$

then:

$$\begin{aligned} \hat{y}(k+2/k) &= (1+\beta_2) y(k+1) + \alpha_1 \Delta u(k+1) + \beta_1 \Delta u(k) \\ &- \beta_2 y(k) + s_3 - s_4 \end{aligned} \quad (10)$$

If we replace $\hat{y}(k+1/k)$ by its expression (7), we obtain:

$$\begin{aligned} \hat{y}(k+2/k) &= (1+\beta_2) y_l(k+1) + \alpha_2 \Delta u(k) \\ &+ \alpha_1 \Delta u(k+1) - \beta_2 y(k) + s_3 - s_4 \end{aligned} \quad (11)$$

where: $\alpha_2 = ((1+\beta_2)\alpha_1 + \beta_1)$

we set: $y_l(k+2) = (1+\beta_2) y_l(k+1) - \beta_2 y(k) + s_3 - s_4$

then:

$$\hat{y}(k+2/k) = y_l(k+2) + \alpha_1 \Delta u(k+1) + \alpha_2 \Delta u(k) \quad (12)$$

Consequently, the expression of the j-step ahead predictor $\hat{y}(k+j/k)$ is as follows:

$$\hat{y}(k+j/k) = \sum_{i=1}^{j} \alpha_{j-i+1} \Delta u(k+i-1) + y_l(k+j) \quad (13)$$

Predictive control involves the optimization of a cost function which indicates how well the process output follows the desired trajectory. This function may be expressed by the future errors between setpoint, and output signal and the future incremental control signal. The objective function is given by:

$$J(\Delta U, \Psi) = \sum_{j=1}^{N_2} (\hat{y}(k+j/k) - yc(k+j))^2 + \lambda \sum_{i=0}^{N_1-1} \Delta u(k+i)^2 \quad (14)$$

where N_1, N_2 and λ denote the control horizon, the prediction horizon and the weighting factor, respectively, and $\hat{y}(k+j/k)$ is given by the relation (13) and $yc(k+j)$ denotes the set point at time $k+j$.

The set Ψ represents the set of uncertain parameters.

$$\Psi = \left\{ a_l ,\ b_m / a_l \in \left[\underline{a}_l \ ,\ \bar{a}_l \right] \text{ and } \mathrm{b}_m \in \left[\underline{b}_m \ ,\ \bar{b}_m \right] \right\}, \quad l \in [0\ ,\ L] \text{ and } \mathrm{m} \in [0\ ,\ M]$$

The output sequence on the prediction horizon N_2 is written as follows:

$$Y = G \Delta U + Y_l \quad (15)$$

where:

$$Y = \left[\hat{y}(k+j/k), ..., \hat{y}(k+N_2/k \right]^T$$

$$\Delta U = \left[\Delta u(k), ..., \Delta u(k+N_1-1) \right]^T$$

$$Y_l = \left[y_l(k+1), ..., y_l(k+N_2) \right]^T$$

The G matrix is illustrated as follows:

$$G = \begin{bmatrix} \alpha_1 & 0 & 0 \cdots & 0 \\ \alpha_2 & \alpha_1 & 0 \cdots & 0 \\ \vdots & \vdots & \ddots & \\ \alpha_{N_2} & \alpha_{N_2-1} & \cdots \alpha_{N_2-N_1+1} & \end{bmatrix} ; \ \dim(G) = (N_2, N_1)$$

The cost function of equation (14) is expressed as:

$$J(\Omega, \Psi) = (G\Delta U + Y_l - Y_c)^T (G\Delta U + Y_l - Y_c) + \lambda \Delta U^T \Delta U \quad (16)$$

In the case of a fractional model with fixed-parameter, the optimal control of the Fractional Predictive Controller (FPC) is obtained by minimizing the cost function given by (16). So, this optimal control is given by the following expression:

$$\Delta U = [G^T G + \lambda I]^{-1} G^T [Y_c - Y_l] \quad (17)$$

For the fractional order model with real uncertain parameters the RFPC is founded using the worst case strategy. The control sequence represents the best solution to the worst case of all possible models opposite the uncertainties. Consequently, the optimal control law can be obtained by the resolution of the following min-max problem [20]:

$$\min_{\Delta U} \ \max_{\Psi} J(\Delta U,\ \Psi) \quad (18)$$

The min-max problem is resolved in two steps. The first step consists to calculate the maximum of the performance criterion $J(\Delta U,\Psi)$ compared to the uncertainties parameters of the set Ψ. Starting with an initial solution, we search the solution of following function in taking into account constraints on the parameters model.

$$J^*(\Delta U) = \max_{\Psi} J(\Delta U,\Psi) \qquad (19)$$

The second step concerns the minimization of the criterion $J(\Delta U,\Psi^*)$ in taking into account the solution found in (19) and the control sequence constraints:

$$J_2 = \min_{\Delta U} J^*(\Delta U) \qquad (20)$$

In this case, the j-step ahead predictor $\hat{y}(k+j/k)$ is evaluated using the parameters model Ψ^* obtained in the first step.
The optimization problem can be solved using the standard optimization technique. In this paper, we have exploited the 'fmincon' function defined in MATLAB.

4. Experiment results

In order to illustrate the robustness of the new approach developed in this paper, we have considered a thermal system depicted in figure 1. The thermal system is composed by an aluminum rod of 41 cm length and 2 cm section. The rod is submitted to a heating resistor and thermally isolated to insure a unidirectional transfer of the heat flux. The input signal of this system is a thermal flux which is generated by a heating resistor. It is fixed in one of the cylinder's extremities and controlled by a computer with USB data acquisition module. The power interface separating the controller from the heating resistance is a PWM converter with an input voltage varying from 0 to 5V. The output signal of the system is the cylinder temperature measured with a distance 'd' from the heated surface by an LM35DZ sensor. The sensor signal is amplified and conditioned in a stage realized to these purposes to obtain an output voltage varying from 0 to 5v. The thermal system is considered as a semi-infinite dimension because its length is more important compared to its section.
In order to demonstrate the non integer behavior of this thermal system, the rod is modeled with the following assumptions:

- The aluminum rod is perfectly isolated.
- The aluminum rod is considered as a semi-infinite dimension.
- At rest, the rod is at ambient temperature.
- Losses on the surface where the thermal flux is applied are neglected.

Indeed, the thermal flux throughout a metallic rod can be defined with fractional order model [21-22]. In literature, several approaches have been proposed to model the phenomenon of a thermal system. Cois [23] was shown that the model of this phenomenon is of fractional order medium which has a commensurable order of 0.5.

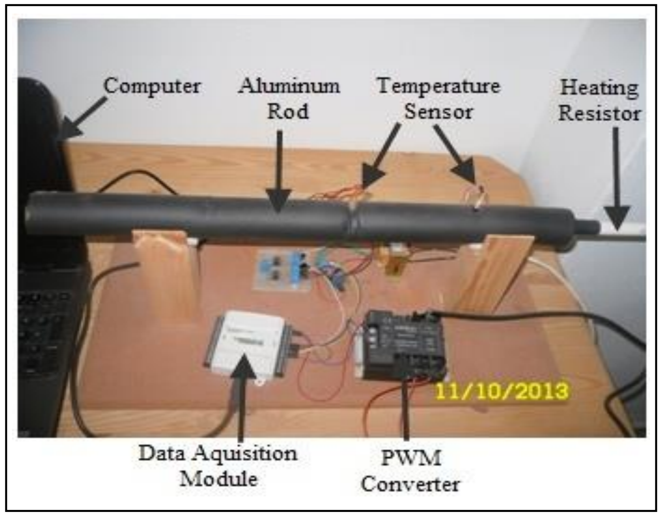

Fig. 1. Thermal system

4.1. Identification

The uncertain model of this system is established by an interpolation of several local models obtained for different temperature sensor positions. Hence, the goal here is to control the temperature at two points (P_1 and P_2) of the rod, measured at distances of $d_1 = 6\ cm$ and $d_2 = 15\ cm$ from one of the ends.
To determine the thermal system model, we have applied to the heating resistor a Pseudo Random Binary Sequence (PRBS) and we have saved the temperature values captured at the points P_1 and P_2. The evolution of the input and the two outputs are depicted in figure 2. For displaying reasons, we have multiplied the input sequence by 10. We note that for identification procedure the sample time is equal to 10 seconds.
Furthermore, we have used the Simplified Refined Instrumental Variable for Continuous-time Fractional models (SRIVCF) method to estimate a fractional order model [24]. Based on saved data represented on figure 2, we have determined the system transfer functions $H_1(s)$ and $H_2(s)$ corresponding to P_1 and P_2 respectively, which are given by:

$$H_1(s) = \frac{1.82}{37.14s^{1.5} + 90.5s + 11.19s^{0.5} + 1} e^{-25s} \quad (21)$$

$$H_2(s) = \frac{1.4}{216.69s^{1.5} + 188.42s + 3.08s^{0.5} + 1} e^{-55s} \quad (22)$$

In order to test the performance of both fractional models, we perform another input excitation sequence and we measure the corresponding temperature at P_1 and P_2. As represented in the validation data of figure 3, the identified models give satisfactory results.

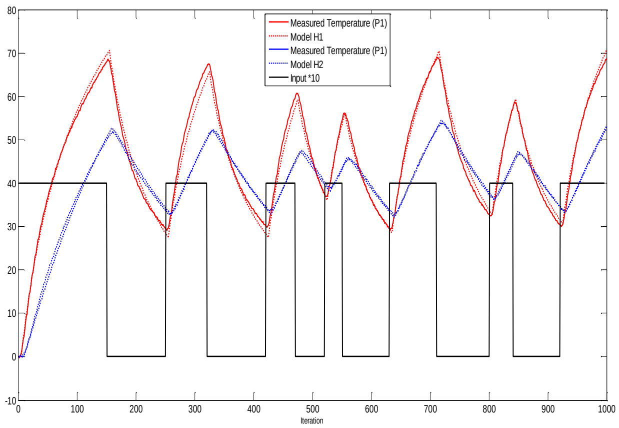

Fig. 2. Identification data.

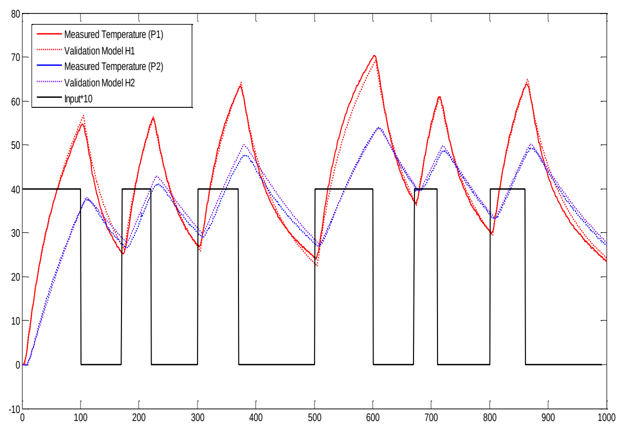

Fig. 3. Validation data.

From the obtained transfer functions we have determined the nominal model (average transfer function) of the system which is given by:

$$G_n(s) = \frac{1.61}{126.915s^{1.5} + 139.46s + 7.135s^{0.5} + 1} e^{-40s} \quad (23)$$

4.2. Controller design

The goal here is to control the temperature of the metallic rod in spite the system dynamic modification due to the sensor position changement from P_1 to P_2 at the sample time 300. For a better comparison, two types of controllers are tested: a Fractional Predictive Controller (FPC) and the proposed Robust Fractional Predictive Controller (RFPC). In all experiences, the sample time is equal to 20 seconds and the designed predictive controller parameters are fixed as follows:

$$N_1 = 1,\ \mathrm{N}_2 = 15 \text{ and } \lambda = 1$$

The FPC controller is designed based on the average transfer function given in equation (23).

This function is expressed by equation (5), where:

$$\begin{cases} h = 0.1;\ L = 3;\ M = 0;\ b_0 = 1.61; \alpha_{b_0} = 0; \\ a_0 = 1;\ a_1 = 7.135;\ a_2 = 139.46;\ a_3 = 126.915 \\ \alpha_{a_0} = 0;\ \alpha_{a_1} = 0.5;\ \alpha_{a_2} = 1;\ \alpha_{a_3} = 1.5 \end{cases}$$

In this case, the optimal control of the FPC is obtained by equation (18).

The synthesis of the RFPC controller requires taking into account the variation range of model parameters uncertainties. From the two founded models given by (21) and (22), and by using the maximum delay, we have determined the uncertain transfer function of the thermal system which is given by:

$$G(s) = \frac{p_0}{p_3 s^{1.5} + p_2 s + p_1 s^{0.5} + 1} e^{-55s} \quad (24)$$

where:

$$p_0 \in [1.4,\ 1.82];\ p_1 \in [3.08,\ 11.19]$$
$$p_2 \in [90.5,\ 188.42];\ p_3 \in [37.14,\ 216.69]$$

By considering this uncertain transfer function, we proceed the design of the controller by exploiting the new approach exposed in section 3. Thus, the relation (24) can be expressed by equation (4) with the following parameters:

$$\begin{cases} L = 3;\ M = 0; \\ \alpha_{b_0} = 0;\ \alpha_{a_0} = 0;\ \alpha_{a_1} = 0.5;\ \alpha_{a_2} = 1;\ \alpha_{a_3} = 1.5 \\ a_0 = 1;\ b_0 \in [1.4,\ 1.82];\ a_1 \in [3.08,\ 11.19] \\ a_2 \in [90.5,\ 188.42];\ a_3 \in [37.14,\ 216.69] \end{cases}$$

Consequently, the j-step ahead predictor can be computed as described in section 3 with a sample time h that is equal to 0.1. In this case, the optimal control is obtained by the resolution of the min-max problem, exposed previously.

The evolutions of the set point, the thermal flux (control signal) and the temperature measured (output signal), obtained with FPC controller are represented in figure 5. From these practical results, we note that the temperature fluctuates around the set point and the control presented many fluctuations as well as peaks.

The evolutions of the set point, the temperature measured and the control signal, obtained with RFPC controller are shown in figure 6. Based in these results, we notice that the measured temperature meets the desired requirements despite the change of the dynamic of the system due to the change of the sensor position. We remark also that the control signal obtained with the RFPC controller provides a smooth control signal than the one obtained by a FPC with fixed parameters model.

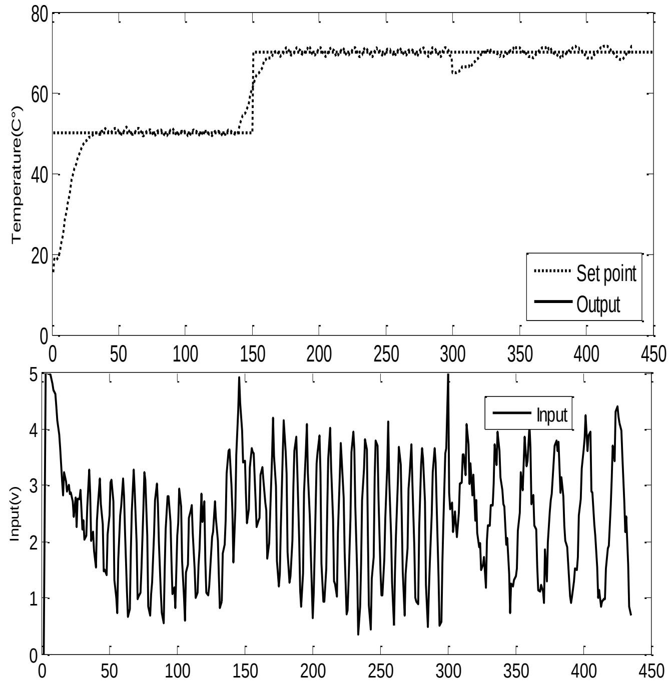

Fig. 5. Closed-loop results obtained with FPC

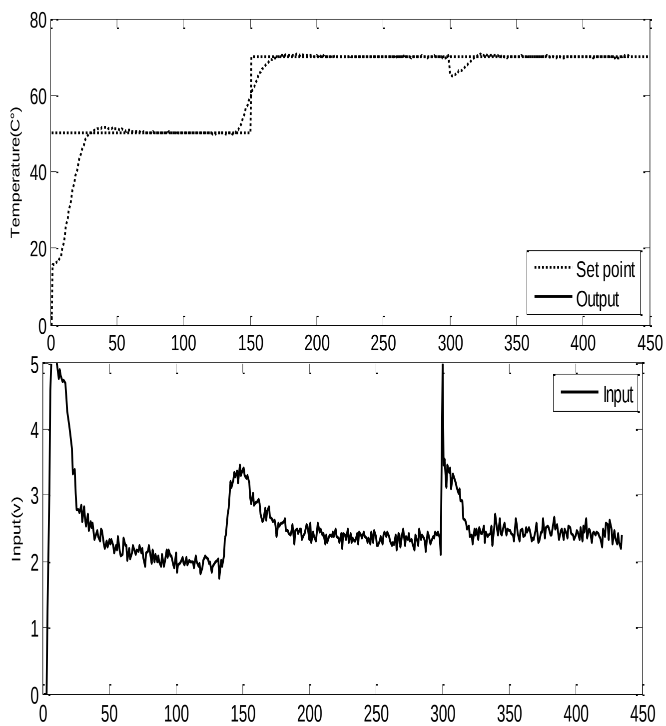

Fig. 6. Closed-loop results obtained with RFPC

5. Conclusion

This paper has presented a Robust Fractional Predictive Control (RFPC) based on an uncertain fractional order model. The proposed controller consists in taking in to account parameters uncertainties during the design of the control law. Therefore, the control law is determined by the resolution of a min–max optimization problem which gives the best solution for the worst case of all possible models. Experimental results on a thermal system show that the RFPC using a fractional order model with parametric uncertainty exhibits good performance compared to the FPC for fixed parameters system.

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