On the Effect of the Lambda Parameter on Performance and Stability Indexes in DMC Control

JOSE MANUEL LOPEZ-GUEDE[1], ANA BOYANO[2], IGOR ANSOATEGUI[2], EKAITZ ZULUETA[1]
JOSE ANTONIO RAMOS-HERNANZ[1]
[1]Department of systems Engineering and Automatics
[2]Department of Mechanics
University of the Basque Country (UPV/EHU)
Nieves Cano 12, 01006, Vitoria-Gasteiz
SPAIN
jm.lopez@ehu.es

Abstract: - One of the more outstanding parameters that define the performance and stability of a concrete type of Model Predictive Control (MPC) named Dynamic Matrix Control (DMC) is the λ parameter. The objective of the paper is to analyze its effect considering a set of indexes defined to measure the characteristics of the output of an instable system when it is controlled by a DMC controller. A specific system has been chosen to make such analysis because it has demonstrated to be instable using a discretized PID controller tuned through the Ziegler-Nichols method. A total of 1,200 tests have been performed to assess the effect of the λ parameter, obtaining as conclusion supported by figures that it is a outstanding parameter.

Key-Words: - Model Predictive Control, MPC, Dynamic Matrix Control, DMC, Advanced Control, Stability

1 Introduction

Dynamic Matrix Control (DMC) is a particular type of Model Predictive Control (MPC), which is a family of advanced control schemas. In the literature, this type of advanced controllers have been used and compared with PID controllers [11] showing a good behavior. Based on previous works [1][7], our research group has been working with these controllers obtaining accurate neuronal implementations [4] even for multi-agent systems [5]. However, we have not yet discussed the effect of the λ parameter of the DMC controller because we have always supposed a fixed implementation of the predictive controller, which defines a concrete λ value. The main objective of this paper is to study the effect of the λ parameter in the performance of a model predictive controller. The paper is structured as follows. In the second section, we recall some basic concepts of MPC and DMC to address the importance and the role of the λ parameter in such control scheme. In the third section, we describe the experimental design which we have carried out, detailing the indexes which we have used to describe the performance of the DMC controllers, the system and the working point in which it has been working to assess the effect of changes in the λ parameter and the tested values for that parameter. In the fourth section, we discuss the experimental results that we have obtained; enumerating the particular effect caused in each of the performance indexes. Finally, the last section provides our conclusions.

2 Background

This section reviews some basic concepts about Model Predictive Control (MPC) as a general technique, and about a concrete technique called Dynamic Matrix Control (DMC).

2.1 Model Predictive Control

MPC is an advanced control technique used to deal with systems that are not controllable using classic control schemas. This kind of controllers works like the human brain in the sense that instead of using the past error between the output of the system and the desired value, it controls the system predicting the value of the output in a short time, so the system output is as closer as possible to its desired value for these moments. Predictive Control is not a concrete technique. It is a set of techniques that have several common characteristics: there is a world model which is used to predict the system output from the actual moment until p samples, an objective function that must be minimized and a control law that minimizes the objective function. The predictive controllers follow these steps:

- Each sampling time, through the system model, the controller calculates the system output from now until *p* sampling times (prediction horizon), which depends on the future control signals that the controller will generate.
- A set of *m* control signals is calculated optimizing the objective function to be used along *m* sampling times (control horizon).
- In each sampling time only the first of the set of *m* control signals is used, and in the next sampling time, all the process is repeated again.

The concept of Predictive Control is a set of techniques that share certain characteristics, and the engineer has liberty to choose in each of them. So, there are several types of predictive controllers. These common characteristics are the following:

- There is a plant model, and there can be used a step response model, an impulse step response model, a transfer function, etc.
- There is an objective function that the controller has to optimize.
- There is a control law to minimize the objective function.

To learn more about Predictive Control in general and about diverse predictive control algorithms, see [2-3][6][10][8][9].

2.2 Dynamic Matrix Control

It is a concrete MPC algorithm that fixes each of the three characteristics that we have seen previously as we will see below. To learn more about Dynamic Matrix Control, see [2-3][6][10].

2.2.1 System Model

The plant model used by DMC algorithm is the step response model. This model uses the g_i coefficients that are the output of the lineal system when it is excited using a step. To reduce the number of coefficients we assume that the subsystem is stable and the output does not change after some sampling time *k*. The expression of the output of the system is given through Eq. (1).

$$y(t) = \sum_{i=1}^{k} g_i \Delta u(t-i) \tag{1}$$

2.2.2 Prediction Model

Using the step response model to model the system and maintaining the hypothesis that perturbations over the subsystem are constants, it is possible to calculate a prediction at the instant *t* of the output until the instant $(t+p)$ under the effect of *m* control actions. The prediction is given by the Eq. (2):

$$\hat{y} = G\,\Delta u + f \tag{2}$$

being $\hat{y}$ the prediction of the output, G a matrix which contains the system dynamics and f the free response of the system. In Eq. (3) we show the dimensions of the matrix and vectors involved in Eq. (2).

$$\hat{y} = \begin{bmatrix} \hat{y}(t+1\,|\,t) \\ \hat{y}(t+2\,|\,t) \\ \vdots \\ \hat{y}(t+p\,|\,t) \end{bmatrix}_p \quad G = \begin{bmatrix} g_1 & 0 & \cdots & 0 \\ g_2 & g_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_m & g_{m-1} & \cdots & g_1 \\ \vdots & \vdots & \ddots & \vdots \\ g_p & g_{p-1} & \cdots & g_{p-m+1} \end{bmatrix}_{pxm}$$

$$\Delta u = \begin{bmatrix} \Delta u(t) \\ \Delta u(t+1) \\ \vdots \\ \Delta u(t+m-1) \end{bmatrix}_m \quad f = \begin{bmatrix} f(t,1) \\ f(t,2) \\ \vdots \\ f(t,p) \end{bmatrix}_p \tag{3}$$

In Eq. (4) we describe how the free response of the system $f(t,k)$ is calculated:

$$f(t,k) = y_m(t) + \sum_{i=1}^{N} (g_{k+i} - g_i)\Delta u(t-i) \tag{4}$$

2.2.3 Control Law

The derivation of the control law is based on the existence of an objective function, which uses the future outputs prediction model that we have described before. As objective function we used the described by Eq. (5).

$$J = \sum_{j=1}^{p} \left[\hat{y}(t+j\,|\,t) - w(t+j)\right]^2 + \sum_{j=1}^{m} \lambda \left[\Delta u(t+j-1)\right]^2 \tag{5}$$

We have to minimize the difference between the reference and the output prediction along a prediction horizon *p* with the *m* control actions generated in the control horizon, modulating the roughness in the variation of the manipulated variables using the λ parameter. Minimizing the objective function J described in Eq. (5) we obtain the following expression, which produces *m* control actions, although in *t* only one of them is used:

$$\Delta u = \left\lfloor \left(G^t G + \lambda I \right)^{-1} G^t \left(w - f \right) \right\rfloor_m \quad (6)$$

After this brief introduction, we can understand the role of the λ parameter and intuit which might be the effect on controller performance due to changes in that parameter.

3 Experimental Design

In this section we provide the experimental design of the study which we have carried out to assess the effect of the λ parameter in DMC controllers performance. First, we detail the definition of the performance indexes that we have used. Then, the system whose response will be analyzed is specified in a motivated way. We also discuss the working point of the controlled system. Finally, we specify the set of experiments that we have carried out to obtain significant results.

3.1 Performance Indexes Definition

To assess the effect of the λ parameter of the DMC controller we have defined several indexes. These indexes are focused on the performance reached at the output of the system when it is controlled by a MPC controller when the reference is a unitary step of arbitrary frequency. The indexes which we have used to measure the performance are described in Table 1. On the other hand, Fig. 1 shows a graphical representation of those indexes.

3.2 System and Working Point

We have analyzed the discretized system described by Eq. (7). This is a quite simple but interesting system because it shows an unstable response when the reference is a unitary step and it is controlled by a discretized Proportional-Integral-Derivative (PID) controller in a closed loop configuration, which has been tuned by means of the Ziegler-Nichols method. That unstable response is shown in Fig. 2.

To determine the working point of the system, i.e., the frequency of the unitary step signal which is at the input of the closed loop system as reference, we have used the Bode diagram shown in Fig. 3. Based on that diagram, we have chosen a frequency of 30 sample times because it is a frequency at which the given gain is representative of many other frequencies for that system.

$$G(z) = \frac{1}{z - 0.5} \quad (7)$$

Table 1. Symbol and description of several performance indexes used in the experimental design

Symbol	Description
mse	Mean squared error between the reference and the system output.
M_p	Overshoot of the output over the reference, expressed as a percentage. Usually, it takes place on the first rising of the signal.
t_s	Time elapsed between the output goes from 10% to 90% of the reference value.

Fig.1. Graphical representation of the performance indexes

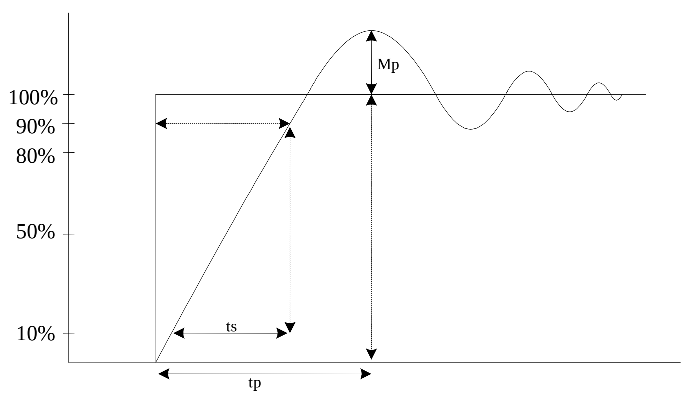
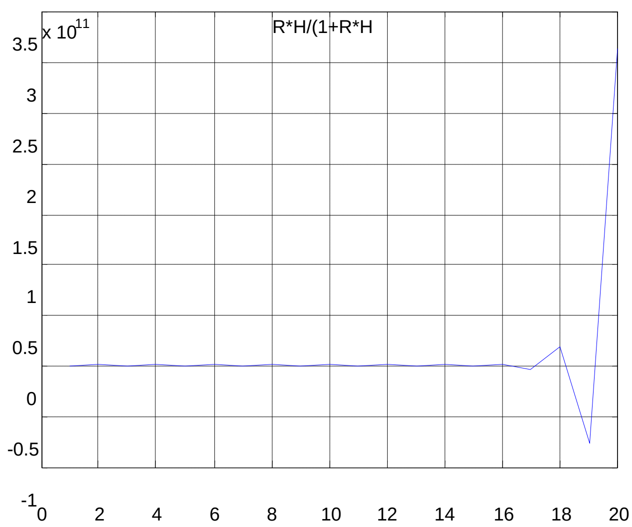
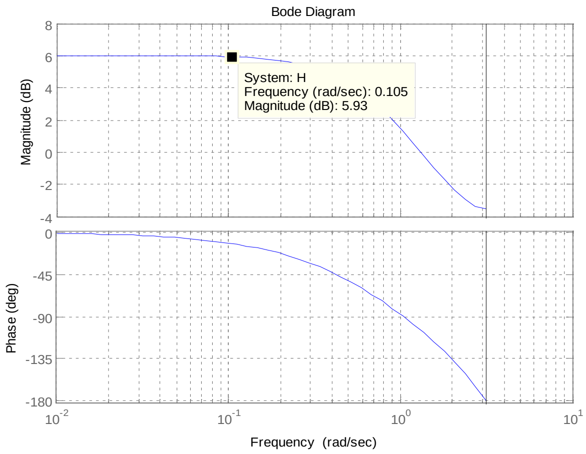


Fig.2. Unstable response of the system when it is controlled by a discretized PID controller.



To determine the working point of the system, i.e., the frequency of the unitary step signal which is at the input of the closed loop system as reference, we have used the Bode diagram shown in Fig. 3. Based on that diagram, we have chosen a frequency of 30 sample times because it is a frequency at which the given gain is representative of many other frequencies for that system.

Fig.3. Bode diagram used to determine the working point of the system



3.3 Sensitivity Analysis
As stated before, the main objective of this paper is to analyze the effect of the λ parameter in DMC controllers. So, we have studied the behavior of the previously specified system at that working point when it is controlled by DMC controllers varying the value of the λ parameter. The values which have been chosen are 0.001, 1 and 100.

4 Experimental Results
In this section we provide several results regarding to the performance indexes which we introduced in the section devoted to the experimental design, when the parameter sensitivity specified in that section is carried out.

4.1 Effect on *mse* value
In this subsection we are going to enumerate the effects on the *mse* performance index values, taking into consideration Figs. 4-6. The following are the main effects:

- In general, we can see that with increasing values of the λ parameter, the value of the *mse* index becomes higher for each combination of (p,m) parameters values.
- The *mse* index value ranges from about 10^{-7} with λ=0.001 to about 10^{-1} with λ=100.

Fig.4. *mse* with λ=0.001

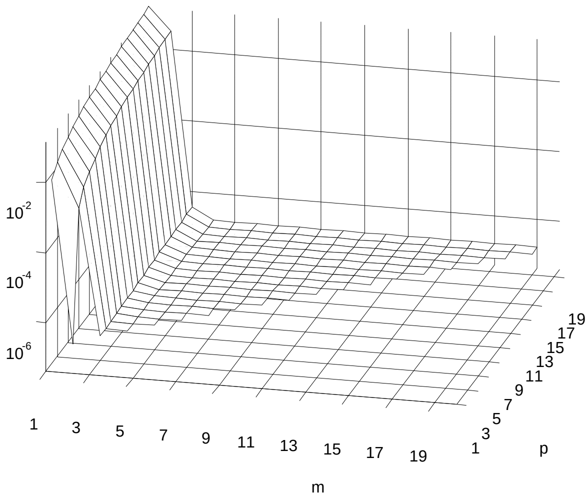


Fig.5. *mse* with λ=1

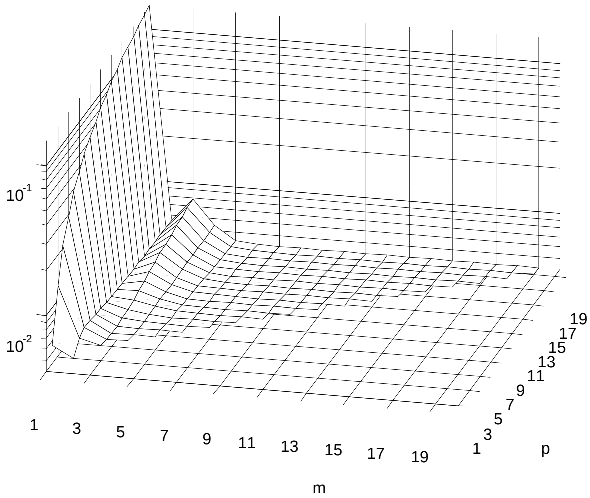
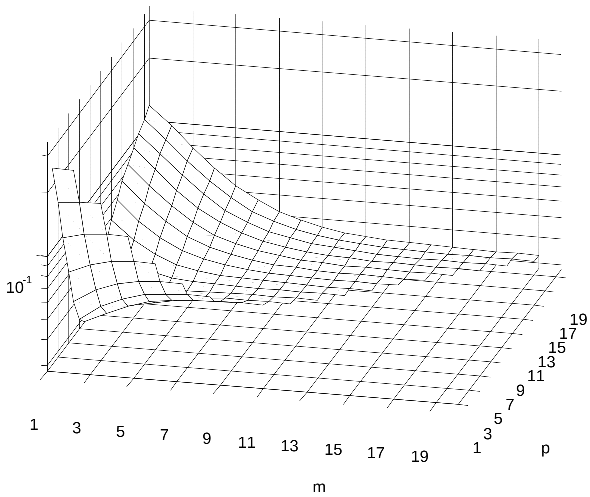


Fig.6. *mse* with λ=100



- For each order of magnitude that the λ parameter increases, the mse index value increases in the same measure.
- As the λ parameter value increases, higher values are needed in the (p,m) parameters combination to maintain the mse index value relatively low, but always within the previously referred increase of order.
- The worst results are produced when m parameter is very low, (independently of p parameter), but if λ parameter increases, the results are relatively better if m is low.

4.2 Effect on M_p values

In this subsection we summarize the effect on the M_p performance index values, taking into consideration Figs. 7-9, being these the main effects:

- Even for the case of very high values of the λ parameter, for most controller structures, quite small overshoot values are obtained, always less than 5%.
- Even with moderate values of the λ parameter, if the controller has very small values of m parameter, the overshoot increases, reaching even 35% if the p parameter value is very high.
- In the case of λ= 100, for virtually all structures, the M_p measure value is less than zero because it is not reached the reference value. Particularly smaller is the amplitude reached by the output with the structure p = 1 y m = 1.
- For moderate values of the λ parameter, it seems that there are m values for which the value of the p parameter is irrelevant: with any of them the peak occurs once that the reference has raised, and for other m values, with any values of p it takes 20 or 30 sample times to occur the peak.

4.3 Effect on t_s value

In this subsection we expose the effect on the t_s performance index values, taking in consideration Figs. 10-12. The following are the main effects:

- As the value of the λ parameter increases, the value of changes in the control signal is slower and this is perceived at the output, so that the ascent speed is slower, increasing the amount of time to move from 10% to 90% of the reference value.

Fig.7. M_p with λ=0.001

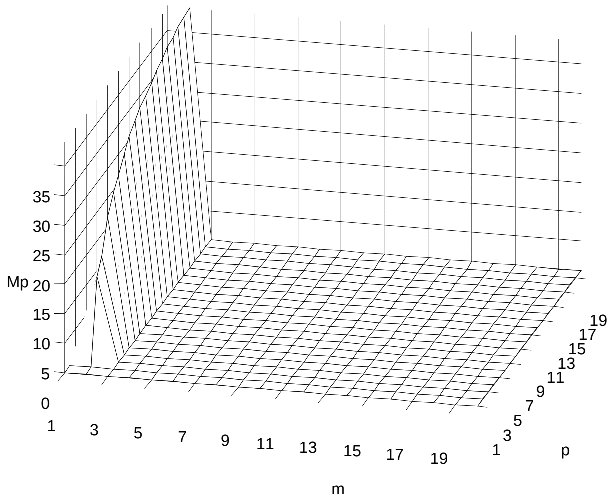


Fig.8. M_p with λ=1

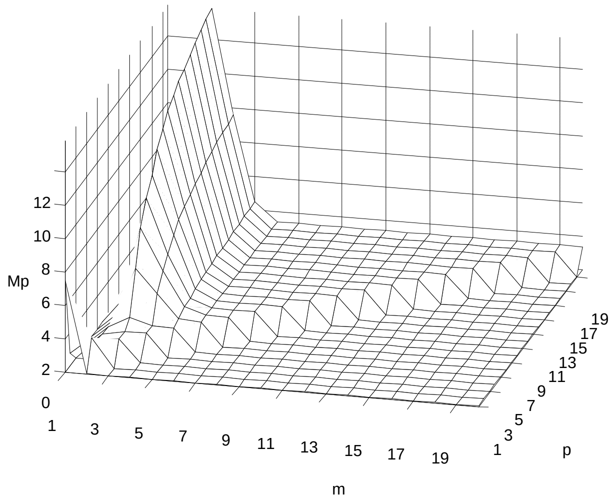
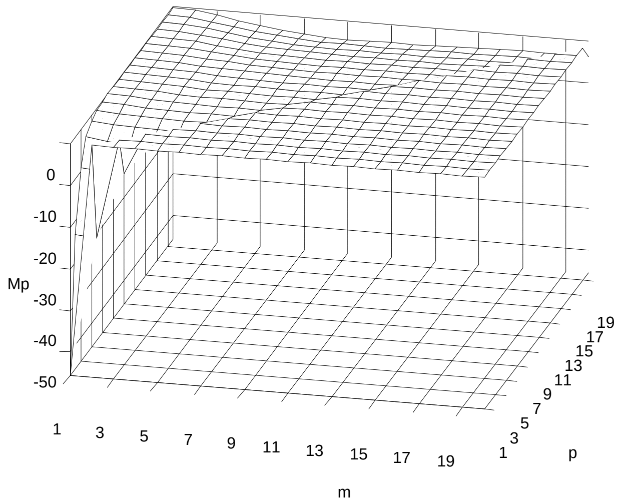


Fig.9. M_p with λ=100



- Values range from 0 sample times (vertical rise) of almost all structures with λ= 0.001 to 20 sample times of the most of the structures with λ= 100.
- Except for very high values of the λ parameter, we can see that in general, the structures with very small values of the *m* parameter are not appropriated, but is particularly bad when the *p* values are high.

5 Conclusion

In this paper we have studied the effect of the λ parameter on DMC controllers performance. We have introduced the MPC and DMC controllers to frame the importance of that parameter. In the experimental design we have specified the performance indexes which we have taken into account and the values used to the λ parameter. Once we have carried out a number of experiments, we have discussed the main results regarding to each of the indexes. We have tested 400 structures of DMC controllers for each value of λ parameter, and as summary, with the specificities that we have shown through figures, we can conclude that a lower value of the λ parameter results in a better performance of the DMC controllers.

References:

[1] Arahal, M.R., Berenguel, M., Camacho, E.F. Neural identification applied to predictive control of a solar plant, *Control Engineering Practice*, Vol. 6, 1998, pp 333-344.

[2] Camacho, E.F., Bordons, C. *Model Predictive Control*, Springer-Verlag, London, 2004.

[3] Camacho, E. F., Bordons, C. *Model Predictive Control in the Process Industry*. Springer-Verlag, London, 1995.

[4] Lopez-Guede, J.M., Graña, M., Zulueta, E., Barambones, O. Economical Implementation of Control Loops for Multi-robot Systems. *15th International Conference, ICONIP 2008*, Aucland, New Zealand.

[5] Lopez-Guede, J.M., Zulueta E., Graña, M., D'Anjou, A. Neuronal Implementation of Predictive Controllers. *Hybrid Artificial Intelligence Systems, HAIS 2010*, Springer Berlin / Heidelberg, Vol. 6077, 2010, pp 312-319.

[6] Maciejowski, J. M. *Predictive Control with Constraints*, Prentice Hall, London, 2002.

Fig.10. t_s with λ=0.001

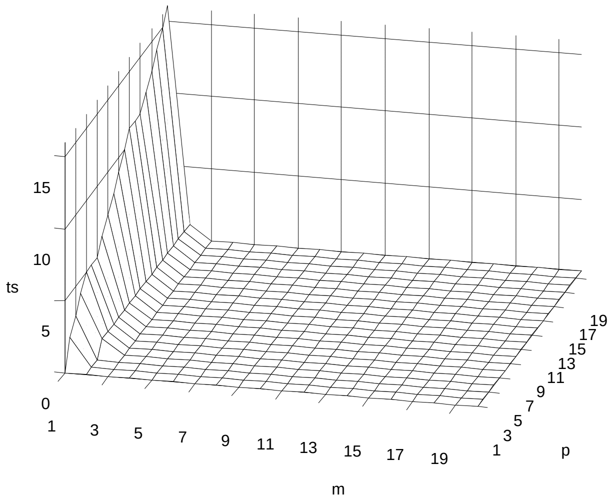


Fig.11. t_s with λ=1

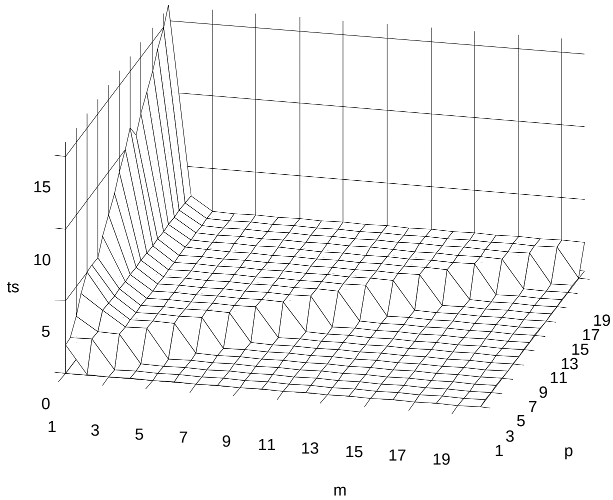
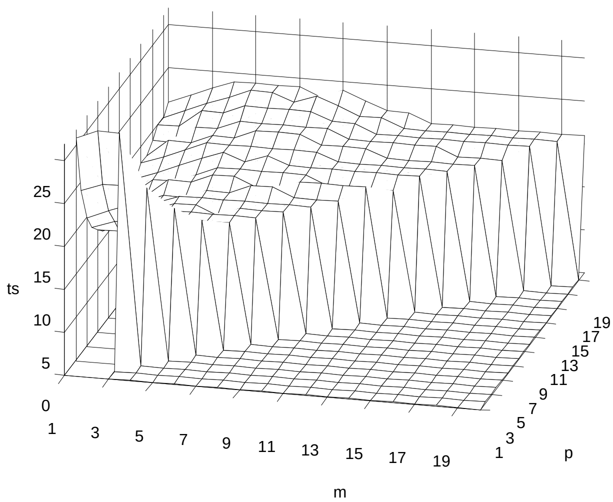


Fig.12. t_s with λ=100



[7] McKinstry, J.L., Edelman, G.M., Krichmar, J.L. A cerebellar model for predictive motor control tested in a brain-based device, *Proceedings of the National Academy of Sciences of The United States of America*, Vol. 103, No 9, 2006, pp 3387-3392.
[8] Rawlings, J.B. Tutorial: Model Predictive Control Technology. *Proceedings of the American Control Conference*, San Diego, California, 1999, pp 662-676.
[9] Soeterboek, R. *Predictive control. A unified approach*, Prentice Hall, 1992.
[10] Sunan, H., Kok T., Tong L. *Applied Predictive Control*, Springer-Verlag, London, 2002.
[11] Voicu, M., Lazär, C., Schönberger, F., Pästravanu, O., Ifrim, S. Predictive Control vs. PID Control of Thermal Treatment Processes. *Control Engineering Solution: a Practical Approach*, 1995, pp 163-174.