Delay-dependent H_∞ Performance for Neutral System with Interval Time-varying Delays

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Abstract: This paper is concerned with the problem of H_∞ performance for the neutral systems with interval time-varying delays, which the time delay is not necessary to be differentiable. Based on the Lyapunov-Krasovskii functional, Leibiz-Newton formula, Cauchy inequality and modified version of Jensen's inequality. The delay-dependent criteria for the H_∞ performance with asymptotically stable are represented in term of linear matrix inequality.

Key–Words: H_∞ performance, Neutral system, Lyapunov-Krasovskii functional, Interval time-varying delay, Linear matrix inequality.

1 Introduction

The one of the interesting topics in the field control society is stability analysis of dynamic systems with delay since time-delay occur in many practical systems such as chemical engineering systems, biological system, chaos system, transportation systems, economics, neural networks, and so on [13]. The problem of various stability and stabilization for dynamical systems with or without state delays and nonlinear perturbations have been intensively studied in the past years by many researchers mathematics and control communities [28, 29]. Stability criteria for dynamical systems with time delay are generally divided into two classes: delay-independent one and delay-dependent one. Delay-independent stability criteria tend to be more conservative, especially for small size delay, such criteria do not give any information on the size of the delay. On the other hand, delay-dependent stability criteria are concerned with the size of the delay and usually provide a maximal delay size.

The H_∞ method has been presented in control theory to integrate controllers succeeding stabilization with guaranteed performance. H_∞ technique has been used to minimize the effects of the external disturbances. It is the objective of H_∞ control to design the controllers such that the closed-loop system is internally stable and its $H_\infty-$ norm of the transfer function between the controlled output and the disturbances will not exceed a given level γ. Moreover, the studies H_∞ control systems with interval time-varying delays have been developed so the improvement of the theory of H_∞ control have extend the region to study.

The problems which concerned about delay-dependent robust H_∞ for linear system with interval time-varying delay and restricted the derivative of the interval time-varying delay, that mean a fast interval time-varying delay is allowed [32], [17]. For [19] paid attention on the H_∞ performance of linear system with parameter uncertainties. In other hand, [31] showed the time derivative of the Lyapunov Krasovskii functional produced not only the strictly proper rational functions but also the non-strictly proper rational functions of the time-varying delays with first-order denominators, which was fully handled using reciprocally convex approach.

From the many above researcher, our works concern in two sections, there are investigating about stability analysis and considering H_∞ performance is continuous modified. We investigate the robust stability analysis and the problem of H_∞ performance for neutral systems with interval time-varying delay. The parameter uncertainties are assumed to be norm-bounded and nonlinear perturbation are bounded in magnitude as some inequality. Base on Lyapunov-Krasovskii theory which construct in term quadruple integral of Lyapunov-Krasovskii functional, Leibniz-Newton formula, Cauchy inequality, modified version of Jensen's inequality and linear matrix inequality technique, then reduce conservatism stability criteria and improve the H_∞ performance criteria for neutral system with interval time-varying delay will be obtain in term LMIs. Finally, numerical examples will be given to show the effectiveness of the obtained results.

Notations. We introduce some notations that will be used throughout the paper. Lebesgue space $\mathcal{L}_{2+} = \mathcal{L}_2[0,\infty]$ consists of square-integral functions on $[0,\infty]$. R^+ denotes the set of all real non-negative numbers; R^n denotes the n-dimensional space with the vector norm $\|\cdot\|$; $\|x\|$ denotes the Euclidean vector norm of $x \in R^n$; $R^{n\times r}$ denotes the set of $n \times r$ real matrices; A^T denotes the transpose of the matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\max}(A) = \max\{\lambda_{\max}(A_i) : i = 1,2,...,N\}$; $\lambda_{\min}(A) = \min\{\lambda_{\min}(A_i) : i = 1,2,...,N\}$; $C([-b,0],R^n)$ denotes the space of all continuous vector functions mapping $[-b,0]$ into R^n, where $b = \max\{h,r\}$, $h_2, r \in R^+$; $*$ represents the elements below the main diagonal of a symmetric matrix.

2 Problem statement and preliminaries

Consider the system described by the following state equations of the form

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h(t)) + C\dot{x}(t-r(t)) \\ \qquad +E_\omega \omega(t), \\ z(t) = A_1x(t) + B_1x(t-h(t)) + E_{1\omega}\omega(t), \\ x(t+t_0) = \phi(t),\ \dot{x}(t+t_0) = \psi(t),\ t \in [-b,0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state variable, $\omega(t) \in R^m$ denotes the disturbance input such that $\omega(t) \in \mathcal{L}_{2+}$, $z(t) \in R^q$ is the performance output, $\phi(t), \psi(t)$ are continuously real-valued initial functions on $[-b,0]$. $A, B, C, E_\omega,\ A_1, B_1,\ E_{1\omega}$ are known real constant matrices with appropriate dimensions.The delay $h(t)$ and neutral delay $r(t)$ are time-varying continuous function that satisfies

$$0 \le h_1 \le h(t) \le h_2, \tag{2}$$
$$0 \le r(t) \le r, \quad \dot{r}(t) \le r_d \tag{3}$$

where h_1, h_2, r, and r_d are given real constants. Consider the initial functions $\phi(t), \psi(t) \in C([-b,0],R^n)$ with the norm $\|\phi\| = \sup_{t\in[-b,0]} \|\phi(t)\|$ and $\|\psi\| = \sup_{t\in[-b,0]} \|\psi(t)\|$.

Definition 1 *The system* (1) *is robustly exponentially stable, if there exist positive real constants k and M such that for each $\phi(t), \psi(t) \in C([-b,0],R^n)$, the solution $x(t,\phi,\psi)$ of the system ** satisfies*

$$\|x(t,\phi,\psi)\| \le M \max\{\|\phi\|, \|\psi\|\} e^{-kt}, \quad \forall t \in R^+.$$

Definition 2 *Given a scalar $\gamma > 0$, system* (1) *is said to be asymptotically stable with the H_∞ performance level γ, if it is asymptotically stable and satisfies the $H_\infty-$ norm constraint*

$$\|z(t)\|_2 < \gamma \|\omega(t)\|_2,$$

for all nonzero $\omega(t) \in \mathcal{L}_2[0,\infty]$ under zero initial condition.

Definition 3 *[5] A system governed by* (1) *is said to be robustly asymptotically stable with an H_∞ norm bound γ if the following conditions hold:*

1) For the system with $\omega(t) = 0$, the trivial solution (equilibrium point) is globally asymptotically stable if $\lim_{t\to\infty} x(t) = 0$; and

2) Under the assumption of zero initial condition, the controlled output $z(t)$ satisfies

$$\|z(t)\|_2 \le \gamma \|\omega(t)\|_2 \tag{4}$$

for any nonzero $\omega(t) \in \mathcal{L}_2[0,\infty)$.

Lemma 4 *[Cauchy inequality] For any constant symmetric positive definite matrix $P \in R^{n\times n}$ and $a, b \in R^n$, we have*

$$\pm 2a^Tb \le a^TPa + b^TP^{-1}b. \tag{5}$$

Lemma 5 *[38] The following inequality holds for any $a \in R^n$, $b \in R^m$, $N, Y \in R^{n\times m}$, $X \in R^{n\times n}$, and $Z \in R^{m\times m}$:*

$$-2a^TNb \le \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ * & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}, \tag{6}$$

where $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \ge 0.$

Lemma 6 *[24] For any constant symmetric matrix $Q \in R^{n\times n}$, Q is semi-positive definite and $h(t)$ is discrete time-varying delays with* (2)*, vector function $\omega : [-h,0] \to R^n$ such that the integrations concerned are well defined, then*

$$h\int_{-h}^{0} \omega^T(s)Q\omega(s)ds \\ \ge \int_{-h(t)}^{0} \omega^T(s)dsQ \int_{-h(t)}^{0} \omega(s)ds.$$

Lemma 7 *[K. Mukdasai] For any constant matrices $Q_{11}, Q_{22}, Q_{12} \in R^{n\times n}$, $Q_{11} \geq 0$ $Q_{22} \geq 0$, $\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \geq 0$, $h(t)$ is time-varying delays with 2 and vector function $\dot{x} : [-h_2, 0] \rightarrow R^n$ such that the following integration is well defined, then*

$$
\begin{aligned}
&-[h_2 - h_1] \int_{t-h_2}^{t-h_1} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\
&\leq \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t-h_1} x(s)ds \\ \int_{t-h_2}^{t-h(t)} x(s)ds \end{bmatrix}^T \\
&\times \begin{bmatrix} -Q_{22} & Q_{22} & 0 & -Q_{12}^T & 0 \\ * & -Q_{22} - Q_{22} & Q_{22} & Q_{12}^T & -Q_{12}^T \\ * & * & -Q_{22} & 0 & Q_{12}^T \\ * & * & * & -Q_{11} & 0 \\ * & * & * & * & -Q_{11} \end{bmatrix} \\
&\times \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t-h_1} x(s)ds \\ \int_{t-h_2}^{t-h(t)} x(s)ds \end{bmatrix}. \qquad (7)
\end{aligned}
$$

Lemma 8 *[K. Mukdasai] For any constant matrices $Q_{11}, Q_{22}, Q_{12} \in R^{n\times n}$, $Q_{11} \geq 0$ $Q_{22} \geq 0$, $\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \geq 0$, $h(t)$ is time-varying delays with 2 and vector function $\dot{x} : [-h_2, 0] \rightarrow R^n$ such that the following integration is well defined, then*

$$
\begin{aligned}
&-h_2 \int_{t-h_2}^{t} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\
&\leq \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t} x(s)ds \\ \int_{t-h_2}^{t-h(t)} x(s)ds \end{bmatrix}^T \\
&\times \begin{bmatrix} -Q_{22} & Q_{22} & 0 & -Q_{12}^T & 0 \\ * & -Q_{22} - Q_{22} & Q_{22} & Q_{12}^T & -Q_{12}^T \\ * & * & -Q_{22} & 0 & Q_{12}^T \\ * & * & * & -Q_{11} & 0 \\ * & * & * & * & -Q_{11} \end{bmatrix} \\
&\times \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t} x(s)ds \\ \int_{t-h_2}^{t-h(t)} x(s)ds \end{bmatrix}.
\end{aligned}
$$

Corollary 9 *[13] For matrices A, B, C, the inequality*

$$
M = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix} > 0 \qquad (8)
$$

is equivalent to the following two inequalities

$$
\begin{aligned}
A &> 0, && (9) \\
C - B^T A^{-1} B &> 0. && (10)
\end{aligned}
$$

Lemma 10 *[9] For any constant matrix $Z = Z^T > 0$ and scalars h, $\bar{h}$, $0 < h < \bar{h}$ such that the following integrations are well defined, then*

$$
\begin{aligned}
- &\int_{t-h}^{t} x^T(s) Z x(s) ds \\
&\leq -\frac{1}{h} \left(\int_{t-h}^{t} x(s)ds \right)^T Z \left(\int_{t-h}^{t} x(s)ds \right) \\
- &\int_{-\bar{h}}^{-h} \int_{t+s}^{t} x^T(\tau) Z x(\tau) d\tau ds \\
&\leq -\frac{2}{\bar{h}^2 - h^2} \left(\int_{-\bar{h}}^{-h} \int_{t+s}^{t} x(\tau) d\tau ds \right)^T \\
&\times Z \left(\int_{-\bar{h}}^{-h} \int_{t+s}^{t} x(\tau) d\tau ds \right).
\end{aligned}
$$

Lemma 11 *[33] For any constant matrix $X \in R^{n\times n}$, $X = X^T > 0$, a scalar function $h : h(t) > 0$, and a vector-valued function $\dot{x}(t) : [-h, 0] \rightarrow R^n$ such that the following integrations are well-defined, then*

$$
\begin{aligned}
&-h \int_{-h}^{0} \dot{x}^T(t+s) Z \dot{x}(t+s) ds \\
&\quad \leq \xi_1^T(t) \begin{bmatrix} -X & X \\ X & -X \end{bmatrix} \xi_1(t), \\
&-\frac{h^2}{2} \int_{-h}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Z \dot{x}(s) ds d\theta \\
&\quad \leq \xi_2^T(t) \begin{bmatrix} -X & X \\ X & -X \end{bmatrix} \xi_2(t),
\end{aligned}
$$

where $\xi_1(t) = \begin{bmatrix} x^T(t) & x^T(t-h) \end{bmatrix}$, *and* $\xi_2(t) = \begin{bmatrix} hx^T(t) & \int_{t-h}^{t} x^T(s)ds \end{bmatrix}$.

Proposition 12 *(Schur complement lemma, S. Boyd et al.)[22] Given constant matrices X, Y, Z, where $Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$
\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0.
$$

3 Main results

We consider in asymptotically stable and H_∞ performance of neutral system with interval time-varying delays. Concerning the systems about the result of system (1), the notations of several matrix variables are defined:

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & 0 & 0 & -Q_2^T & 0 & 0 & \Sigma_{18} & \Sigma_{19} & \Sigma_{112} & \Sigma_{113} & 0 \\ * & \Sigma_{2,2} & Q_6^T & \Sigma_{2,4} & Q_2^T & Q_5^T & \Sigma_{27} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Sigma_{3,3} & 0 & 0 & -Q_5^T & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Sigma_{4,4} & 0 & 0 & \Sigma_{4,7} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -Q_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Sigma_{77} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -S_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -S_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Sigma_{12,12} & \Sigma_{12,13} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Sigma_{13,13} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -R_2 \end{bmatrix},$$

where

$$\begin{aligned}
\Sigma_{11} &= PA + A^TP + N_1^TA + A^TN_1 + M_1 + M_2 \\
&\quad + h_2^2Q_1 - Q_3 - R_2 + (h_2-h_1)^2Q_4 - h_2^2S_1 \\
&\quad - (h_2-h_1)^2S_2 \\
\Sigma_{12} &= PB + N_1^TB + Q_3, \quad \Sigma_{18} = h_2S_1, \\
\Sigma_{19} &= (h_2-h_1)S_2, \quad \Sigma_{113} = PC + N_1^TC, \\
\Sigma_{112} &= h_2^2Q_2 + (h_2-h_1)^2Q_5 + A^TP - N_1^T \\
&\quad + A^TN_2, \\
\Sigma_{22} &= -Q_3 - Q_3 - Q_6 - Q_5, \quad \Sigma_{24} = Q_3 + Q_6, \\
\Sigma_{27} &= -Q_2^T - Q_5^T, \quad \Sigma_{33} = -M_1 - Q_6, \\
\Sigma_{44} &= -M_2 - Q_3 - Q_6, \quad \Sigma_{47} = Q_2^T + Q_5^T, \\
\Sigma_{77} &= -Q_1 - Q_4, \\
\Sigma_{1212} &= h_2^2Q_3 + (h_2-h_1)^2Q_6 + \frac{1}{4}h_2^4S_1 \\
&\quad + \frac{1}{4}(h_2-h_1)^2S_2 - P - P - N_2 - N_2^T, \\
\Sigma_{1213} &= PC + N_2^TC, \quad \Sigma_{1313} = -(1-r_d)R_1,
\end{aligned}$$

and

$$\begin{aligned}
\xi_1(t) &= \big[x(t), x(t-h(t)), x(t-h_1), x(t-h_2), \\
&\quad \int_{t-h(t)}^{t} x(s)ds, \int_{t-h(t)}^{t-h_1} x(s)ds, \int_{t-h_2}^{t-h(t)} x(s)ds, \\
&\quad \int_{t-h_2}^{t} x(s)ds, \int_{t-h_2}^{t-h_1} x(s)ds, \dot{x}(t-r(t)), \\
&\quad \int_{t-r(t)}^{t} x(s)ds\big]. \qquad (11)
\end{aligned}$$

Theorem 13 *For $\|C\| < 1$ and given positive scalars h_1, h_2, r, r_d and and a prescribed $\gamma > 0$, if there exist positive symmetric matrices $P, M_i, S_i, R_i, (i = 1,2)$ and $\begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \geq 0$, $\begin{bmatrix} Q_4 & Q_5 \\ * & Q_6 \end{bmatrix} \geq 0$, and any matrices $N_j, (j = 1,2,3)$ with appropriate dimensions that the following LMIs hold*

$$\Omega = \begin{bmatrix} \Sigma & \Lambda_1 & \Lambda_2 \\ * & -\gamma^2 I & E_{1\omega} \\ * & * & -I \end{bmatrix} < 0, \qquad (12)$$

where

$$\begin{aligned}
\Lambda_1^T &= [E_\omega^TP + E_\omega^TN_1, 0,0,0,0,0,0,0,0,0, \\
&\qquad E_\omega^TN_2 + E_\omega^TP, 0, 0], \\
\Lambda_2^T &= [A_1^T, B_1^T, 0,0,0,0,0,0,0,0, C_1^T, 0],
\end{aligned}$$

then the system (1) for any time-delays (2), (3) is asymptotically stable and satisfies $\|z\|_2 < \gamma\|\omega\|_2$ for all nonzero $\omega \in \mathcal{L}_2[0,\infty)$.

Proof: Construct a Lyapunov-Krasovskii functional as

$$V(t) = \sum_{i=1}^{5} V_i(t),$$

where

$$\begin{aligned}
V_1(t) &= \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \int_{t-h_2}^{t}\int_{s}^{t}\dot{x}(\theta)d\theta ds \end{bmatrix}^T \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
&\quad \times \begin{bmatrix} P & 0 & 0 \\ 0 & 0 & 0 \\ N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \int_{t-h_2}^{t}\int_{s}^{t}\dot{x}(\theta)d\theta ds \end{bmatrix}, \\
V_2(t) &= \int_{t-h_1}^{t} x^T(s)M_1x(s)ds \\
&\quad + \int_{t-h_2}^{t} x^T(s)M_2x(s)ds, \\
V_3(t) &= h_2\int_{-h_2}^{0}\int_{t+s}^{t} \begin{bmatrix} x(\theta) \\ \dot{x}(\theta) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} x(\theta) \\ \dot{x}(\theta) \end{bmatrix} d\theta ds \\
&\quad + (h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+s}^{t} \begin{bmatrix} x(\theta) \\ \dot{x}(\theta) \end{bmatrix}^T \\
&\qquad \times \begin{bmatrix} Q_4 & Q_5 \\ * & Q_6 \end{bmatrix} \begin{bmatrix} x(\theta) \\ \dot{x}(\theta) \end{bmatrix} d\theta ds, \\
V_4(t) &= \frac{h_2^2}{2}\int_{-h_2}^{0}\int_{s}^{0}\int_{t+\omega}^{t} \dot{x}^T(\theta)S_1\dot{x}(\theta)d\theta d\omega ds \\
&\quad + \frac{h_2^2-h_1^2}{2}\int_{-h_2}^{-h_1}\int_{s}^{0}\int_{t+\omega}^{t} \dot{x}^T(\theta)S_2\dot{x}(\theta)d\theta d\omega ds, \\
V_5(t) &= \int_{t-r(t)}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds \\
&\quad + r\int_{-r}^{0}\int_{t+s}^{t} \dot{x}^T(\theta)R_2\dot{x}(\theta)d\theta ds.
\end{aligned}$$

Calculating the time derivatives of $V_i(t)$, $i = 1, 2, 3, ..., 6$, along the trajectory of (1) yields

$$
\begin{aligned}
\dot{V}_1(t) &= 2\begin{bmatrix} x(t) \\ \dot{x}(t) \\ \int_{t-h_2}^{t}\int_{s}^{t}\dot{x}(\theta)d\theta ds \end{bmatrix}^T \begin{bmatrix} P & 0 & N_1^T \\ 0 & 0 & N_2^T \\ 0 & 0 & N_3^T \end{bmatrix} \\
&\times \begin{bmatrix} \dot{x}(t) \\ 0 \\ 0 \end{bmatrix}, \\
&= 2x^T(t)P[Ax(t) + Bx(t-h(t) + C\dot{x}(t-r(t)) \\
&\quad +E_\omega\omega(t)] + 2x^T(t)N_1^T[Ax(t) + Bx(t-h(t) \\
&\quad +C\dot{x}(t-r(t)) + E_\omega\omega(t) - \dot{x}(t)] \\
&\quad +2\dot{x}^T(t)N_2^T[Ax(t) + Bx(t-h(t) \\
&\quad +C\dot{x}(t-r(t)) + E_\omega\omega(t) - \dot{x}(t)] \\
&\quad +2\int_{t-h_2}^{t}\int_{s}^{t}\dot{x}^T(\theta)d\theta ds N_3^T[Ax(t) \\
&\quad +Bx(t-h(t) + C\dot{x}(t-r(t)) + E_\omega\omega(t) - \dot{x}(t)]. \\
\dot{V}_2(t) &= x^T(t)M_1x(t) - x^T(t-h_1)M_1x(t-h_1) \\
&\quad +x^T(t)M_2x(t) - x^T(t-h_2)M_2x(t-h_2) \\
&\quad -2kV_2(t).
\end{aligned}
$$

The time derivative of $V_3(t)$ is

$$
\begin{aligned}
\dot{V}_3(t) &= h_2^2\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\
&\quad -h_2\int_{t-h_2}^{t}\begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds, \\
&\quad +(h_2-h_1)^2\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_4 & Q_5 \\ * & Q_6 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\
&\quad -(h_2-h_1)\int_{t-h_2}^{t-h_1}\begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q_4 & Q_5 \\ * & Q_6 \end{bmatrix} \\
&\quad \times \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds,
\end{aligned}
$$

using Lemma 7 and Lemma 8 to estimate the integral inequality of $\dot{V}_3(t)$, then we obtain

$$
\begin{aligned}
\dot{V}_3(t) &\leq h_2^2\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\
&\quad +(h_2-h_1)^2\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_4 & Q_5 \\ * & Q_6 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\
&\quad +\begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t} \\ \int_{t-h_2}^{t-h(t)} \end{bmatrix}^T \\
&\quad \times \begin{bmatrix} -Q_3 & Q_3 & 0 & -Q_2^T & 0 \\ * & -Q_3-Q_3 & Q_3 & Q_2^T & -Q_2^T \\ * & * & -Q_3 & 0 & Q_2^T \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t} \\ \int_{t-h_2}^{t-h(t)} \end{bmatrix} + \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t-h_1} \\ \int_{t-h_2}^{t-h(t)} \end{bmatrix}^T \\
&\quad \times \begin{bmatrix} -Q_6 & Q_6 & 0 & -Q_5^T & 0 \\ * & -Q_6-Q_6 & Q_6 & Q_5^T & -Q_5^T \\ * & * & -Q_6 & 0 & Q_5^T \\ * & * & * & -Q_4 & 0 \\ * & * & * & * & -Q_4 \end{bmatrix} \\
&\quad \times \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t-h_1} \\ \int_{t-h_2}^{t-h(t)} \end{bmatrix}.
\end{aligned}
$$

For derivative of $V_4(t)$, then we get

$$
\begin{aligned}
\dot{V}_4(t) &= \left(\frac{h_2^2}{2}\right)^2\dot{x}^T(t)S_1\dot{x}(t) \\
&\quad -\frac{h_2^2}{2}\int_{-h_2}^{0}\int_{t+s}^{t}\dot{x}^T(\omega)S_1\dot{x}(\omega)d\omega ds \\
&\quad +\left(\frac{h_2^2-h_1^2}{2}\right)^2\dot{x}^T(t)S_2\dot{x}(t) \\
&\quad -\left(\frac{h_2^2-h_1^2}{2}\right)\int_{-h_2}^{-h_1}\int_{t+s}^{t}\dot{x}^T(\omega)S_2\dot{x}(\omega)d\omega ds \\
&\quad -2kV_4(t),
\end{aligned}
$$

and using Lemma (11) to estimate integral inequality, then we obtain

$$
\begin{aligned}
\dot{V}_4(t) &\leq \left(\frac{h_2^2}{2}\right)^2\dot{x}^T(t)S_1\dot{x}(t) \\
&\quad +\left(\frac{h_2^2-h_1^2}{2}\right)^2\dot{x}^T(t)S_2\dot{x}(t) \\
&\quad +\begin{bmatrix} h_2x(t) \\ \int_{t-h_2}^{t}x^T(s)ds \end{bmatrix}^T \begin{bmatrix} -S_1 & S_1 \\ S_1 & -S_1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} h_2x(t) \\ \int_{t-h_2}^{t}x^T(s)ds \end{bmatrix} \\
&\quad +\begin{bmatrix} (h_2-h_1)x(t) \\ \int_{t-h_2}^{t-h_1}x^T(s)ds \end{bmatrix}^T \begin{bmatrix} -S_2 & S_2 \\ S_2 & -S_2 \end{bmatrix}
\end{aligned}
$$

$$\times \begin{bmatrix} (h_2 - h_1)x(t) \\ \int_{t-h_2}^{t-h_1} x^T(s)ds \end{bmatrix} - 2kV_4(t).$$

$$\begin{aligned} \dot{V}_5(t) \;\; &\leq \;\; \dot{x}^T(t)R_1\dot{x}(t) + r^2\dot{x}^T(t)R_2\dot{x}(t) \\ &\quad -(1-r_d)\dot{x}^T(t-r(t))R_1\dot{x}(t-r(t)) \\ &\quad -r\int_{t-r(t)}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds - 2kV_6(t), \end{aligned}$$

using Jensen's inequality to estimate integral inequality, then

$$\begin{aligned} - \;\; & r\int_{t-r(t)}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds \\ & \leq -\left(\int_{t-r(t)}^{t} \dot{x}^T(s)ds\right) R_2 \left(\int_{t-r(t)}^{t} \dot{x}(s)ds\right). \end{aligned}$$

Hence,

$$\begin{aligned} \dot{V}_5(t) \;\; &\leq \;\; \dot{x}^T(t)R_1\dot{x}(t) + r^2\dot{x}^T(t)R_2\dot{x}(t) \\ &\quad -(1-r_d)\dot{x}^T(t-r(t))R_1\dot{x}(t-r(t)) \\ &\quad -\left(\int_{t-r(t)}^{t} \dot{x}^T(s)ds\right) R_2 \left(\int_{t-r(t)}^{t} \dot{x}(s)ds\right) \\ &\quad -2V_5(t). \end{aligned}$$

From the following zero equation is for positive symmetric matrices P with:

$$\begin{aligned} &2\dot{x}^T(t)P[Ax(t) + Bx(t-h(t)) + C\dot{x}(t-r(t)) \\ &\quad +E_\omega\omega(t) - \dot{x}(t)] = 0. \end{aligned} \tag{13}$$

Using zero equation and the whole time derivative of $V(t)$ for all $0 \leq h_1 \leq h(t) \leq h_2$, we obtain

$$\begin{aligned} \dot{V}(t,x_t) \;\; &\leq \sum_{i=1}^{6} \dot{V}(t) + 2\dot{x}^T(t)P \\ &\quad \times[Ax(t) + Bx(t-h(t)) + C\dot{x}(t-r(t)) \\ &\quad\quad +E_\omega\omega(t) - \dot{x}(t)] \\ &\quad +[A_1x(t) + B_1x(t-h(t)) + E_{1\omega}\omega(t)]^T \\ &\quad\quad \times[A_1x(t) + B_1x(t-h(t)) + E_{1\omega}\omega(t)] \\ &\quad -\gamma^2\omega^T(t)\omega(t) - z^T(t)z(t) + \gamma^2\omega^T(t)\omega(t) \end{aligned} \tag{14}$$

Therefore, we yield

$$\begin{aligned} \dot{V}(t) \;\; &\leq \;\; \begin{bmatrix} \xi_1(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} \Sigma & \Lambda_1 \\ * & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \omega(t) \end{bmatrix} \\ &\quad +[A_1x(t) + B_1x(t-h(t)) + E_{1\omega}\omega(t)]^T \\ &\quad \times[A_1x(t) + B_1x(t-h(t)) + E_{1\omega}\omega(t)] \\ &\quad -z^T(t)z(t) + \gamma^2\omega^T(t)\omega(t), \\ &= \;\; \begin{bmatrix} \xi_1(t) \\ \omega(t) \end{bmatrix}^T \left(\begin{bmatrix} \Sigma & \Lambda_1 \\ * & -\gamma^2 I \end{bmatrix} + \mathfrak{H}^T\mathfrak{H} \right) \begin{bmatrix} \xi_1(t) \\ \omega(t) \end{bmatrix} \\ &\quad -z^T(t)z(t) + \gamma^2\omega^T(t)\omega(t), \end{aligned} \tag{15}$$

where

$$\mathfrak{H}^T = \begin{bmatrix} A_1 & B_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_{1\omega} \end{bmatrix},$$

By using Schur complement Lemma [22], therefore (15) can define as (12).

Then, combining (12) and (15), we can show that

$$\dot{V}(t) \leq -z^T(t)z(t) + \gamma^2\omega^T(t)\omega(t). \tag{16}$$

Integrate both sides of (16) from t_0 to t, yield

$$\begin{aligned} V(t) - V(t_0) \;\; &\leq \;\; -\int_{t_0}^{t} z^T(s)z(s)ds \\ &\quad + \int_{t_0}^{t} \gamma^2\omega^T(s)\omega(s)ds \end{aligned} \tag{17}$$

Then, letting $t \to \infty$ and under zero initial condition, we have $V(t_0) = V(0) = 0$ and $V(\infty) = 0$, that leads to

$$\int_{t_0}^{t} z^T(s)z(s)ds \leq \int_{t_0}^{t} \gamma^2\omega^T(s)\omega(s)ds, \tag{18}$$

therefore $\|z(t)\|_2 \leq \gamma\|\omega(t)\|_2$ is satisfied for any non-zero $\omega(t) \in \mathcal{L}_2[0,\infty)$.

Next, we can prove the asymptotically stable for system (1). When $\omega(t) = 0$, we yield the result as

$$\dot{V}(t) \leq \xi(t)\Sigma\xi(t) - z^T(t)z(t) < 0, \tag{19}$$

combining (12) and using Schur complement Lemma, we obtain

$$\begin{bmatrix} \Sigma & \Lambda_2 \\ * & -I \end{bmatrix} < 0, \tag{20}$$

which guarantees $\dot{V} < 0$. Therefore, the system (1) is asymptotic stability for any delay satisfying (2) and (3). Thus, by Definition 3 the result is shown. This completes the proof. □

4 Conclusion

The problem of robust H_∞ performance of neutral systems has presented. Based on Lyapunov-Krasovskii functional, combination of Leibniz-Newton formula, free weighting matrices, linear matrix inequality, Cauchy inequality and modified version of Jensen's inequality. The delay-dependent stability and H_∞ performance criteria are formulated in terms of LMIs.

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