Robust exponential stability of a class of uncertain Lur'e systems of neutral type by Wirtinger-based integral inequality

NUNTAWAN JINATI SAOVALAK VUNPOANG ORATHAI SARADEE WAJAREE WEERA*
University of Phayao
Department of Mathematics
19 Moo 2, Maeka, Muang, Phayao
Thailand
*Corresponding author: wajaree.we@up.ac.th

Abstract: This Paper deals with the problem of exponential stability for a class of uncertain Lur'e systems of neutral with interval time-varying delay and sector-bounded nonlinearily. The interval time-varying delay function is not assumed to be differentiable. We analyze the global exponential stability for uncertain neutral and Lur'e dynamical systems with some sector conditions. By constructing a set of improved Lyapunov-Krasovskii functional and Utilizing Wirtinger-based integral inequality combined with Leibniz-Newton's formula, We establish some new stability criteria in terms of linear matrix inequalities. Numerical examples are given to illustrate the effectiveness of the results. The results in this article generalize and improve the corresponding results of the recent works.

Key–Words: Robust exponential stability, uncertain Lur'e systems, neutral system, Wirtinger-based integral inequality

1 Introduction

In many practical systems, models of system are described by neutral differential equations, in which the models depend on the delays of state and state derivatives. Heat exchanges, distributed networks containing lossless transmission lines, and population ecology are examples of neutral systems. Because of its wider application, therefore, several researchers have studied neutral time delay systems, see [1-4] and references cited therein.

It is known that exponential stability is more favorite property than asymptotic stability since it gives a faster convergence rate to the equilibrium point, the decay rates (i.e., convergent rates) are important indices of practical system, and the exponential stability analysis of time-delay systems has been a popular topic in the past decades; see, for example,[4,6,11] and their references. In [11], delay-dependent exponential stability for uncertain linear systems with interval time-varying delays.

Recently, there are many research study on the asymptotic stability of a class of neutral and Lur'e dynamical systems with time delay,see,for example, [10,13,14,16]. The problems have been dealt with stability analysis for neutral systems with mixed delay and sector-bounded nonlinearity [9], robust absolute stability criteria for uncertain Lur'e systems of neutral type [14], and robust stability criteria for a class of Lur'e systems with interval time-varying delay [10]. However, it is worth pointing out that, even though these results above were elegant, there still exist some points waiting for the improvement. Firstly, most of the work above the time-varying delay are required to be differentiable. In fact, the constraint on the derivative of the time-varying delay is not required which allows the time-delay to be a fast time-varying function. Secondly, in most studies on the asymptotic stability of Lur'e dynamical systems still need to be improved to the exponential stability.

Based on the above discussions, we consider the problem of robust stability for a class of uncertain neutral and Lur'e dynamical systems with sector-bounded nonlinearity The time delay is a continuous function belonging to a given interval, which means that the lower and upper bounded for the time varying delay are available, but the delay function is not necessary to be differentiable. To the best of the authors knowledge, there were no results for uncertain neutral and Lur'e dynamical systems with some sector condition [13,14,16]. Based on the construction of improved Lyapuno-Krasovskii functional and Utilizing seuret and Gouaisbaut, 2013 [5] combined with Leibniz-Newton's formula and the integral terms, new delay dependent sufficient conditions for the uncertain neutral and Lur'e dynamical of system are established of LMIs. The new stability condition is much less conservative and more general than some existing results. Numerical examples are given to illustrate the effectiveness of our theoretical results.

2 Problem statements and preliminaries

The following notation will be used in this paper: $\mathbb{R}^+$ denotes the set of all real nonnegative numbers; R^n denotes the n-dimensional space and the vector norm $\|\cdot\|$; $M^{n\times r}$ denotes the Space of all matrices of $(n\times r)$-dimensions. A^T denotes the transpose of matrix A; A is symmetric if $A=A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{max}(A)=max\{\ Re\lambda\in\lambda(A)\}$. $x_t := \{\ x(t+s) : s\in[\ -h,0]\},\|x_t\| = sup_{s\in[\ -h,0]}\|x(t+s)\|$; $C([\ 0,t],R^n)$ denotes the set of all $\mathbb{R}^n$-valued continuous functions on [0,t]; Matrix A is called semipositive definite $(\ A\geq 0)$ if $x^TAx\geq 0$, for all $x\in\mathbb{R}^n$; a is positive definite $(A>0)$ if $x^TAx>0$ for all $x\neq 0; A>B$ means $A-B>0$; diag$(c_1,c_2,...,c_m)$ denotes block diagonal matrix with diagonal elements $c_i, i=1,2,...,m$. The Symmetric term in a matrix is denoted by*

Consider the following uncertain Lur'e system of neutral type with interval time-varying delays delays and sector-bounded nonlinearity:

$$\begin{aligned}\dot{x}(t)-C\dot{x}(t-\eta(t)) &= (A+\Delta A(t))x(t)\\ &\quad +(A_1+\Delta A_1(t)x(t-h(t))\\ &\quad +(B+\Delta B(t))f(\sigma(t)),\end{aligned}$$

$$\sigma(t)=H^Tx(t)=[\bar{h}_1,\bar{h}_2,...,\bar{h}_m]^Tx(t),\ \ \forall t\geq 0,$$
$$x(t+s)=\phi(t+s),\ \ \dot{x}(t+s)=\varphi(t+s),$$
$$s\in[-m,0],\ \ m=\max\{\ h_2,\eta\} \quad (1)$$

where $x(t)\in\mathbb{R}^n$ is the state vector; $\sigma(t)\in\mathbb{R}^n$ the output vector; $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $C\in\mathbb{R}^{n\times n}$, $A_1\in\mathbb{R}^{n\times n}$, $H\in\mathbb{R}^{n\times m}$, are constant known matrices; $f(\sigma(t))\in\mathbb{R}^m$ is the nonlinear function in the feedback path, which is denoted as f for simplicity in the sequel. Its form is formulated as

$$\begin{aligned}f(\sigma(t)) &= [f_1(\sigma_1(t))\ f_2(\sigma_2(t)\ ...\ f_m(\sigma_m(t))]^T,\\ \sigma(t) &= [\sigma_1(t)\ \sigma_2(t)\ ...\ \sigma(t)]^T\\ &= [h_1^Tx(t)\ h_2^Tx(t)\ ...\ h_m^Tx(t)]\end{aligned}$$

where $f_i(\sigma(t)), i=1,2,...,m$ satisfies any one of the following sector condition:

$$f_i(\sigma_i(t))\in K_{[0,k_i]}=\Big\{f_i(\sigma_i(t))\mid f_i(0)=0,$$
$$0<\sigma_i(t)f_i(\sigma_i(t))\leq k_i\sigma_i(t)^2,\sigma_i(t)\neq 0\Big\} \quad (2)$$

or

$$f_i(\sigma_i(t))\in K_{[0,\infty]}=\Big\{f_i(\sigma_i(t))\mid f_i(0)=0, \quad (3)$$
$$0<\sigma_i(t)f_i(\sigma_i(t))>0,\sigma_i(t)\neq 0\Big\}.$$

$\Delta A(t),\Delta B(t)$, and $\Delta A_1(t)$ are time-varying uncertainties of appropriate dimensions, which are assumed to be of the following form:

$$[\Delta A(t)\ \Delta B(t)\ \Delta A_1(t)]=DF(t)[E_1\ E_2\ E_3], \quad (4)$$

where D,E_1,E_2, and E_3 are known matrices of appropriate dimension, and the time-varying matrix $F(t)$ satisfies

$$F^T(t)F(t)\leq I,\ \forall t\geq 0. \quad (5)$$

The delays $h(t)$ and $\tau(t)$ are time-varying continuous function that satisfy

$$0\leq h_1\leq h(t)\leq h_2,\ 0\leq\eta(t)\leq\eta,\ \dot{\eta}(t)\leq\eta_d<1. \quad (6)$$

We introduce the following technical well-known propositions and definition, which will be used in the proof of our results.

Definition 1. *If there exist $\gamma>0$ and $\psi(\gamma)>0$ such that*

$$\|x(t)\|\leq\psi(\gamma)e^{-\gamma t},\ \forall t>0.$$

system (1) is said to be exponentially stable at the equilibrium point, where γ is called the degree of exponential stability.

Lemma 2. *(Cauchy inequality,[2]). For any symmetric positive definite matrix $N\in M^{n\times n}$ and $x,y\in\mathbb{R}^n$ we have*

$$\pm 2x^Ty\leq x^TNx+y^TN^{-1}y.$$

Lemma 3. *(Schur complement lemma, [2]). Given constant symmetric matrices X,Y,Z with appropriate dimensions satisfying $X=X^T$,$Y=Y^T>0$. Then $X+Z^TY^{-1}Z<0$ if and only if*

$$\begin{pmatrix} X & Z^T\\ Z & -Y\end{pmatrix}<0\ or\ \begin{pmatrix} -Y & Z\\ Z^T & X\end{pmatrix}<0. \quad (7)$$

Lemma 4. *(Seuret - Gouaisbaut, 2013). For a given matrix $R>0$, the following inequality hold for any continuously differentiable function $\omega:[a,b]\to\mathbb{R}^n$*

$$\int_a^b\dot{\omega}(u)du\geq\frac{1}{b-a}(\Gamma_1^TR\Gamma_1+3\Gamma_2^TR\Gamma_2)$$

where
$\Gamma_1:=\omega(b)-\omega(a)$,
$\Gamma_2:=\omega(b)+\omega(a)-\frac{2}{b-a}\int_a^b\omega(u)du.$

3 Main results

Now we present a new delay-dependent condition for the uncertain system (1) satisfying the sector conditions (2).

Assumption 5. *All the eigenvalues of matrix C are inside the unit circle.*

Theorem 6. *Under Assumption 5, given $\alpha > 0$. The system (1) satisfying the sector condition (2) is $\alpha-$exponentially stabilizable if there exist symmetric positive definite matrices P, Q, R, U, F, L symmetric positive semidefinite matrices $Z = diag(z_1, z_2, ..., z_m)$ and $J = diag(j_1, j_2, ..., j_m)$ scalars $\epsilon > 0$ matrices $N_i; i = 1, 2, 3$ of appropriate dimensions such that the following LMIs hold:*

$$\mathcal{M}_1 = \mathcal{M} + \mathcal{M}_1^* < 0, \tag{8}$$

$$\mathcal{M}_2 = \mathcal{M} + \mathcal{M}_2^* < 0, \tag{9}$$

$$\mathcal{M} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \phi_{18} & \phi_{19} & \phi_{110} & 0 & 0 \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \phi_{28} & 0 & 0 & \phi_{211} & \phi_{212} \\ * & * & \phi_{33} & 0 & 0 & 0 & 0 & 0 & \phi_{39} & 0 & 0 & \phi_{312} \\ * & * & * & \phi_{44} & 0 & 0 & 0 & 0 & 0 & \phi_{410} & \phi_{411} & 0 \\ * & * & * & * & \phi_{55} & \phi_{56} & \phi_{57} & \phi_{58} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \phi_{66} & \phi_{67} & \phi_{68} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \phi_{77} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \phi_{88} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \phi_{99} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \phi_{110} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \phi_{111} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \phi_{112} \end{bmatrix},$$

where
$\phi_{11} = 2\alpha P + 2PA + 2Q - e^{-2\alpha h_1} - e^{-2\alpha h_2}4R$
$+\epsilon E_1 E_1, \phi_{13} = -e^{-2\alpha h_1}2R, \quad \phi_{14} = -e^{-2\alpha h_2}2R,$
$\phi_{12} = PA_1 + \frac{1}{2}N_1^T A + \frac{1}{2}N_2^T A + \frac{1}{2}N_3^T A + \epsilon E_1 E_3,$
$\phi_{15} = PB + \frac{1}{2}HKJ + ZH^T A + \frac{1}{2}N_1^T A + \frac{1}{2}N_2^T A$
$+\frac{1}{2}N_3^T A + \epsilon E_1 E_2, \quad \phi_{17} = PC, \quad \phi_{18} = PD,$
$\phi_{16} = LA + \frac{1}{2}HKJ + ZH^T A + \frac{1}{2}N_1^T A + \frac{1}{2}N_2^T A$
$+\frac{1}{2}N_3^T A, \ \phi_{19} = -e^{-2\alpha h_1}6R, \ \phi_{110} = -e^{-2\alpha h_2}2R,$
$\phi_{22} = -e^{-2\alpha h_2}4U - e^{-2\alpha h_2}4U + \frac{1}{2}HKJ + ZH^T A$
$+\frac{1}{2}N_1^T A + \frac{1}{2}N_2^T A + \frac{1}{2}N_3^T A + \epsilon E_3 E_3,$
$\phi_{23} = -e^{-2\alpha h_1}2U, \quad \phi_{24} = -e^{-2\alpha h_1}2U,$
$\phi_{25} = ZH^T A_1 + \frac{1}{2}HKJ + ZH^T A + \frac{1}{2}N_1^T A_1$
$+\frac{1}{2}N_2^T A_1 + \frac{1}{2}N_3^T A_1 + \frac{1}{2}HKJ + ZH^T B + \frac{1}{2}N_1^T B$
$+\frac{1}{2}N_2^T B + \frac{1}{2}N_3^T B + \epsilon_1 E_3 E_2, \ \phi_{211} = -e^{2\alpha h_2}6U,$
$\phi_{26} = LA_1 - \frac{1}{2}N_1^T - \frac{1}{2}N_2^T - \frac{1}{2}N_3^T + \frac{1}{2}N_1^T A_1$
$+\frac{1}{2}N_2^T A_1 + \frac{1}{2}N_3^T A_1, \quad \phi_{212} = -e^{2\alpha h_2}6U,$
$\phi_{27} = \frac{1}{2}N_1^T C + \frac{1}{2}N_2^T C + \frac{1}{2}N_3^T C, \quad \phi_{39} = e^{2\alpha h_1}6R,$
$\phi_{28} = \frac{1}{2}N_1^T D + \frac{1}{2}N_2^T D + \frac{1}{2}N_3^T D, \phi_{312} = e^{2\alpha h_2}6U,$
$\phi_{33} = -e^{-2\alpha h_1}Q - e^{2\alpha h_1}4R - e^{2\alpha h_2}4U,$
$\phi_{44} = -e^{-2\alpha h_2}Q - 4R - 4U - (1-\beta)U,$
$\phi_{410} = e^{2\alpha h_2}6R, \phi_{411} = e^{2\alpha h_2}6U,$
$\phi_{55} = 2ZH^T B - J + \frac{1}{2}N_1^T B + \frac{1}{2}N_2^T B + \frac{1}{2}N_3^T B$
$+\epsilon E_2 E_2, \quad \phi_{56} = LB - \frac{1}{2}N_1^T - \frac{1}{2}N_2^T - \frac{1}{2}N_3^T$
$+\frac{1}{2}N_1^T B + \frac{1}{2}N_2^T B + \frac{1}{2}N_3^T B, \quad \phi_{68} = LD,$
$\phi_{57} = 2ZH^T C + \frac{1}{2}N_1^T C + \frac{1}{2}N_2^T C + \frac{1}{2}N_3^T C,$
$\phi_{58} = 2ZH^T D + \frac{1}{2}N_1^T D + \frac{1}{2}N_2^T D + \frac{1}{2}N_3^T D,$
$\phi_{66} = h_1^2 R + h_2^2 R + (h_2 - h_1)^2 U + F - 2L - N_1^T - N_2^T$
$-N_3^T, \quad \phi_{67} = LC + \frac{1}{2}N_1^T + \frac{1}{2}N_2^T + \frac{1}{2}N_3^T,$
$\phi_{77} = -e^{-2\alpha\eta}(1-\eta_d)F, \quad \phi_{88} = -\epsilon I,$
$\phi_{99} = -e^{2\alpha h_1}12R, \quad \phi_{1010} = -e^{2\alpha h_2}12R,$
$\phi_{1111} = -e^{2\alpha h_2}12U, \quad \phi_{1212} = -e^{2\alpha h_2}12U,$

$$\mathcal{M}_1^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & \bar{\phi}_{22} & \bar{\phi}_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{\phi}_{212} \\ * & * & \bar{\phi}_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{\phi}_{312} \\ * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \bar{\phi}_{1212} \end{bmatrix},$$

where $\bar{\phi}_{22} = -e^{2\alpha h_2}4U, \ \bar{\phi}_{23} = -e^{2\alpha h_2}2U, \ \bar{\phi}_{212} = -e^{2\alpha h_2}6U, \ \bar{\phi}_{33} = -e^{2\alpha h_2}2U, \ \bar{\phi}_{312} = -e^{2\alpha h_2}6U, \ \bar{\phi}_{1212} = -e^{2\alpha h_2}12U,$

$$\mathcal{M}_2^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & \hat{\phi}_{22} & 0 & \hat{\phi}_{24} & 0 & 0 & 0 & 0 & 0 & 0 & \hat{\phi}_{211} & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\phi}_{44} & 0 & 0 & 0 & 0 & 0 & 0 & \hat{\phi}_{411} & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \hat{\phi}_{1111} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & 0 \end{bmatrix},$$

where
$\hat{\phi}_{22} = -e^{2\alpha h_2}4U, \quad \hat{\phi}_{24} = -e^{2\alpha h_2}2U,$
$\hat{\phi}_{211} = -e^{2\alpha h_2}6U, \quad \hat{\phi}_{44} = -e^{2\alpha h_2}4U,$
$\hat{\phi}_{411} = -e^{2\alpha h_2}6U, \quad \hat{\phi}_{1111} = -e^{2\alpha h_2}12U.$
The solution $x(t)$ of the system satisfies,

$$\|x(t)\| \leq \sqrt{\frac{a\|\phi\|^2 + b\|M_1\|^2 + c\|M_2\|^2}{\lambda_m(P)}} \tag{10}$$

where $a = \lambda_M(P) + 2h_2\lambda_m(R) + h_2\lambda_M(U) + 2\lambda_M(HZKH^T), b = 2\lambda_M(Q)(1 - e^{-2\alpha h_2})/2\alpha +$

$2h_2\lambda_M((1 - e^{-2\alpha}h_2)/2\alpha) + h_2\lambda_M(U)((1 - e^{-2\alpha h_2}/2\alpha)$, *and* $c = \lambda_M(F)((1 - e^{-2\alpha\eta})/2\alpha), \|M_1\| = \sup_{-m\le s\le 0}\|x(s)\|, \|M_2\| = \sup_{-m\le s\le 0}\|\dot{x}(s)\|$.

Proof. Using (2), the uncertain system (1) can be represented as

$$\begin{aligned}\dot{x}(t) &= Ax(t) + A_1x(t-h(t)) + Bf(\sigma(t)) \\ &\quad +C\dot{x}(t-\eta(t)) + Dp(t), \\ p(t) &= F(t)(E_1x(t) + E_2f(\sigma(t)) + E_3x(t-h(t))), \\ \sigma(t) &= H^Tx(t) = [\bar{h}_1\ \bar{h}_2\ \cdots\ \bar{h}_m]^Tx(t), \forall t \ge 0, \\ x(s) &= \phi(s),\ s \in [-\max(h_2,\eta), 0]. \end{aligned} \tag{11}$$

We consider the following Lyapunov-Krasovskii functional

$$V(x(t)) = \sum_{i=1}^{8} V_i \tag{12}$$

where

$$\begin{aligned}V_1(x(t)) &= e^{2\alpha t}x^T(t)Px(t), \\ V_2(x(t)) &= \int_{t-h_1}^{t} e^{2\alpha s}x^T(s)Qx(s)ds, \\ V_3(x(t)) &= \int_{t-h_2}^{t} e^{2\alpha s}x^T(s)Qx(s)ds, \\ V_4(x(t)) &= h_1\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha s}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau ds, \\ V_5(x(t)) &= h_2\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha s}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau ds, \\ V_6(x(t)) &= (h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha s}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau ds, \\ V_7(x(t)) &= \int_{t-\eta_t}^{t} e^{2\alpha s}\dot{x}^T(s)F\dot{x}(s)ds, \\ V_8(x(t)) &= 2\sum_{i=1}^{m}\lambda_i e^{2\alpha t}\int_0^{\sigma_i} f_i(\sigma)d\sigma_i. \end{aligned}$$

Taking the derivative of $V(x_t)$ along the solution of system (11), we have

$$\begin{aligned}\dot{V}_1(x(t)) &= 2\alpha e^{2\alpha t}x^T(t)Px(t) + 2e^{2\alpha t}x^T(t)P\dot{x}(t) \\ &= e^{2\alpha t}[2\alpha x^T(t)Px(t) + 2x^T(t)P(Ax(t) \\ &\quad +A_1x(t-h(t)) + Bf(\sigma(t)) \\ &\quad +C\dot{x}(t-\eta(t)) + Dp(t))], \\ \dot{V}_2(x(t)) &= e^{2\alpha t}[x^T(t)Qx(t) - e^{-2\alpha h_1}x^T(t-h_1) \\ &\quad \times Qx(t-h_1)], \\ \dot{V}_3(x(t)) &= e^{2\alpha t}[x^T(t)Qx(t) - e^{-2\alpha h_2}x^T(t-h_1) \\ &\quad \times Qx(t-h_2)], \\ \dot{V}_4(x(t)) &= e^{2\alpha t}[h_1^2\dot{x}(t)R\dot{x}(t) - h_1e^{-2\alpha h_1} \\ &\quad \times\int_{t-h_1}^{t}\dot{x}^T(s)R\dot{x}(s)ds], \\ \dot{V}_5(x(t)) &= e^{2\alpha t}[h_2^2\dot{x}(t)R\dot{x}(t) - h_2e^{-2\alpha h_2} \\ &\quad \times\int_{t-h_2}^{t}\dot{x}^T(s)R\dot{x}(s)ds], \\ \dot{V}_6(x(t)) &= e^{2\alpha t}[(h_2-h_1)^2\dot{x}(t)U\dot{x}(t) \\ &\quad -(h_2-h_1)e^{-2\alpha h_2}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds], \\ \dot{V}_7(x(t)) &= e^{2\alpha t}[\dot{x}^T(t)F\dot{x}(t) - e^{-2\alpha\eta}(1-\eta_d) \\ &\quad \times\dot{x}^T(t-\eta(t))F\dot{x}(t-\eta(t))], \\ \dot{V}_8(x(t)) &= e^{2\alpha t}[2\sum_{i=1}^{m}\lambda_i e^{2\alpha t}(2\alpha\int_0^{\sigma_i(t)} f_i(\sigma_i)d\sigma_i \\ &\quad +f_i(\sigma_i(t))\dot{\sigma}_i(t))], \\ &\le e^{2\alpha t}[4\alpha f^T(\sigma(t))Z\sigma(t) + 2f^T(\sigma(t))Z\dot{\sigma}(t)], \\ &\le e^{2\alpha t}[4\alpha f^T(\sigma(t))ZHx(t) + 2f^T(\sigma(t)) \\ &\quad \times ZH^T\dot{x}(t)], \\ &= e^{2\alpha t}[4\alpha f^T(\alpha(t))ZHx(t) \\ &\quad +2f^T](\sigma(t))ZH^T\times(Ax(t) \\ &\quad +A_1x(t-h(t)) + Bf(\sigma(t)) \\ &\quad +C\dot{x}(t-\eta(t)) + Dp(t))]. \end{aligned} \tag{13}$$

Applying the Seuret and Gouaisbaut, 2013, we have

$$\begin{aligned} &-h_1\int_{t-h_1}^{t}\dot{x}^T(s)R\dot{x}(s)ds \\ \le\ &-(x(t)-x(t-h_1))^TR(x(t)-x(t-h_1)) \\ &+3[(x(t)+x(t-h_1)-\frac{2}{t-(t-h_1)}\int_{t-h_1}^{t}x(s)ds)^T \\ &\times R(x(t)+x(t-h_1)-\frac{2}{t-(t-h_1)}\int_{t-h_1}^{t}x(s)ds)] \\ \le\ &-4x^T(t)Rx(t) - 2x^T(t)Rx(t-h_1) - 2x^T(t-h_1) \\ &\times Rx(t) - 4x^T(t-h_1)Rx(t-h_1) + x^T(t) \\ &\times\frac{6R}{h_1}\int_{t-h_1}^{t}x(s)ds + x^T(t-h_1)\frac{6R}{h_1}\int_{t-h_1}^{t}x(s)ds \\ &+\frac{6}{h_1}\int_{t-h_1}^{t}x^T(s)dsRx(t) \\ &+\frac{6}{h_1}\int_{t-h_1}^{t}x^T(s)dsRx(t-h_1) \\ &-\frac{6}{h_1}\int_{t-h_1}^{t}x^T(s)ds\frac{2R}{h_1}\int_{t-h_1}^{t}x(s)ds, \end{aligned} \tag{14}$$

$$\begin{aligned} &-h_2\int_{t-h_2}^{t}\dot{x}^T(s)R\dot{x}(s)ds \\ \le\ &-[(x(t)-x(t-h_2))^TR(x(t)-x(t-h_2)) \\ &+3((x(t)+x(t-h_2)-\frac{2}{t-(t-h_2)}\int_{t-h_2}^{t}x(s)ds)^T \end{aligned}$$

$$\begin{aligned}
&\times R(x(t)+x(t-h_2)-\frac{2}{t-(t-h_2)}\int_{t-h_2}^{t}x(s)ds)]\\
\leq\;& -4x^T(t)Rx(t)-2x^T(t)Rx(t-h_2)-2x^T(t-h_2)\\
&\times Rx(t)-4x^T(t-h_2)Rx(t-h_2)+x^T(t)\\
&\times\frac{6R}{h_2}\int_{t-h_2}^{t}x(s)ds+x^T(t-h_2)\frac{6R}{h_2}\int_{t-h_2}^{t}x(s)ds\\
&+\frac{6}{h_2}\int_{t-h_2}^{t}x^T(s)dsRx(t)\\
&+\frac{6}{h_2}\int_{t-h_2}^{t}x^T(s)dsRx(t-h_2)\\
&-\frac{6}{h_2}\int_{t-h_2}^{t}x^T(s)ds\frac{2R}{h_2}\int_{t-h_2}^{t}x(s)ds. \qquad (15)
\end{aligned}$$

Note that

$$\begin{aligned}
&-(h_2-h_1)\int_{t-h_2}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds\\
=\;& -(h_2-h_1)\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)U\dot{x}(s)ds\\
&-(h_2-h_1)\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds\\
=\;& -(h_2-h(t))\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)U\dot{x}(s)ds\\
&-(h(t)-h_1)\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)U\dot{x}(s)ds\\
&-(h(t)-h_1)\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds\\
&-(h_2-h(t))\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds.
\end{aligned}$$

Using the Seuret - Gouaisbaut, 2013 gives

$$\begin{aligned}
&-(h_2-h(t))\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)U\dot{x}(s)ds\\
\leq\;& [(x(t-h(t))-x(t-h_2))^TU(x(t-h(t))\\
&-x(t-h_2))+3((x(t-h(t))+x(t-h_2)\\
&-\frac{2}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)^TU(x(t-h(t))\\
&-x(t-h_2)-\frac{2}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)]\\
\leq\;& -4x^T(t-h(t))Ux(t-h(t))-2x^T(t-h(t))\\
&\times Ux(t-h_2)-2x^T(t-h_2)Ux(t-h(t))\\
&-4x^T(t-h_2)Ux(t-h_2))\\
&+x^T(t-h(t))\frac{6U}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)\\
&+x^T(t-h_2)\frac{6U}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)\\
&+\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)ds)Ux(t-h(t))\\
&+\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)ds)Ux(t-h_2)\\
&-\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)ds)\\
&\times\frac{2U}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds), \qquad (16)
\end{aligned}$$

$$\begin{aligned}
&-(h(t)-h_1)\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds\\
\leq\;& [(x(t-h_1)-x(t-h(t)))^TU(x(t-h_1)\\
&-x(t-h(t)))+3((x(t-h_1)+x(t-h(t))\\
&-\frac{2}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)^TU(x(t-h_1)\\
&-x(t-h(t))-\frac{2}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)]\\
\leq\;& -4x^T(t-h_1)Ux(t-h_1)-2x^T(t-h_1)\\
&\times Ux(t-h(t))-2x^T(t-h(t))Ux(t-h_1)\\
&-4x^T(t-h(t))Ux(t-h(t)))\\
&+x^T(t-h_1)\frac{6U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)\\
&+x^T(t-h(t))\frac{6U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)\\
&+\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)ds)Ux(t-h_1)\\
&+\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)ds)Ux(t-h(t))\\
&-\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)ds)\\
&\frac{2U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds). \qquad (17)
\end{aligned}$$

Let $\beta=(h_2-h(t))/(h_2-h_1)\leq 1$, we get

$$\begin{aligned}
&-(h_2-h(t))\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds\\
=\;& -\beta\int_{t-h(t)}^{t-h_1}(h_2-h_1)\dot{x}^T(s)U\dot{x}(s)ds\\
\leq\;& -\beta\int_{t-h(t)}^{t-h_1}(h(t)-h_1)\dot{x}^T(s)U\dot{x}(s)ds\\
\leq\;& -\beta[(x(t-h_1)-x(t-h(t)))^TU(x(t-h_1)\\
&-x(t-h(t)))+3((x(t-h_1)+x(t-h(t))\\
&-\frac{2}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)^TU(x(t-h_1)\\
&+x(t-h(t))-\frac{2}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)]\\
\leq\;& \beta[-4x^T(t-h_1)Ux(t-h_1)-2x^T(t-h_1)\\
&Ux(t-h(t))-2x^T(t-h(t))Ux(t-h_1)
\end{aligned}$$

$$
\begin{aligned}
& -4x^T(t-h(t))Ux(t-h(t)) \\
& +x^T(t-h_1)\frac{6U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds \\
& +x^T(t-h(t))\frac{6U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds \\
& +\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)dsUx(t-h_1) \\
& +\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)dsUx(t-h(t)) \\
& -(\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)ds) \\
& \times(\frac{2U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds], \qquad (18) \\
& -(h(t)-h_1)\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)U\dot{x}(s)ds \\
= & -(1-\beta)\int_{t-h_2}^{t-h(t)}(h_2-h_1)\dot{x}^T(s)U\dot{x}(s)ds \\
\leq & -(1-\beta)\int_{t-h_2}^{t-h(t)}(h_2-h(t))\dot{x}^T(s)U\dot{x}(s)ds \\
\leq & -(1-\beta)(\frac{h_2-h(t)}{h_2-h(t)})[(x(t-h(t))-x(t-h_2))^T \\
& \times U(x(t-h(t))-x(t-h_2))+3((x(t-h(t)) \\
& +x(t-h_2)-\frac{2}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)^T \\
& \times U(x(t-h(t))+x(t-h_2) \\
& -\frac{2}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)] \\
\leq & (1-\beta)[-4x^T(t-h(t))Ux(t-h(t)) \\
& -2x^T(t-h(t))Ux(t-h_2)-2x^T(t-h_2) \\
& Ux(t-h(t))-4x^T(t-h_2)Ux(t-h_2) \\
& +x^T(t-h(t))\frac{6U}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds \\
& +x^T(t-h_2)\frac{6U}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds \\
& +\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)dsUx(t-h(t)) \\
& +\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)dsUx(t-h_2) \\
& -(\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)ds) \\
& \times(\frac{2U}{h(h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds]. \qquad (19)
\end{aligned}
$$

Therefore from (16)-(19), we obtain

$$
\begin{aligned}
& -(h_2-h_1)\int_{t-h_2}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds \\
\leq & -4x^T(t-h(t))Ux(t-h(t))-2x^T(t-h(t)) \\
& \times Ux(t-h_2)-2x^T(t-h_2)Ux(t-h(t)) \\
& -4x^T(t-h_2))Ux(t-h_2) \\
& +x^T(t-h(t))\frac{6U}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds) \\
& +x^T(t-h_2)\frac{6U}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds) \\
& +\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)ds)Ux(t-h(t)) \\
& +\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)ds)Ux(t-h_2) \\
& -\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)ds) \\
& \times\frac{2U}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds) \\
& -4x^T(t-h_1)Ux(t-h_1) \\
& -2x^T(t-h_1)Ux(t-h(t)) \\
& -2x^T(t-h(t))Ux(t-h_1) \\
& -4x^T(t-h(t))Ux(t-h(t))) \\
& +x^T(t-h_1)\frac{6U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds) \\
& +x^T(t-h(t))\frac{6U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds) \\
& +\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)ds)Ux(t-h_1) \\
& +\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)ds)Ux(t-h(t)) \\
& -\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)ds) \\
& \times\frac{2U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds) \\
& +\beta[-4x^T(t-h_1)Ux(t-h_1)-2x^T(t-h_1) \\
& Ux(t-h(t))-2x^T(t-h(t))Ux(t-h_1) \\
& -4x^T(t-h(t))Ux(t-h(t)) \\
& +x^T(t-h_1)\frac{6U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds \\
& +x^T(t-h(t))\frac{6U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds \\
& +\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)dsUx(t-h_1) \\
& +\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)dsUx(t-h(t)) \\
& -(\frac{6}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)ds \\
& \times\frac{2U}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s))ds]
\end{aligned}
$$

$$
\begin{aligned}
&+(1-\beta)[-4x^T(t-h(t))Ux(t-h(t))\\
&-2x^T(t-h(t))Ux(t-h_2)-2x^T(t-h_2)\\
&Ux(t-h(t))-4x^T(t-h_2)Ux(t-h_2)\\
&+x^T(t-h(t))\frac{6U}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds\\
&+x^T(t-h_2)\frac{6U}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds\\
&+\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)dsUx(t-h(t))\\
&+\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)dsUx(t-h_2)\\
&-(\frac{6}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)ds)\\
&\times(\frac{2U}{(h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds]. \qquad (20)
\end{aligned}
$$

We add the following zero equation:

$$
\begin{aligned}
&2\xi^T(t)\overline{N}[Ax(t)+A_1x(t-h(t))+Bf(\sigma(t))\\
&+C\dot{x}(t-\eta(t))+Dp(t)-\dot{x}^T(t)]=0, \qquad (21)
\end{aligned}
$$

where $\overline{N} = [N_1^T \ N_2^T \ N_3^T]^T$, $\xi(t) = [x^T \ (t-h(t)) \ f^T(\sigma(t))\dot{x}(t)]^T$ and by using the identity relation

$$
\begin{aligned}
&-\dot{x}(t)+Ax(t)+A_1(t-h(t))+Bf(\sigma(t))\\
&+C\dot{x}(t-\eta(t))+Dp(t)=0,
\end{aligned}
$$

we have

$$
\begin{aligned}
&-2\dot{x}^T(t)L\dot{x}(t)+2\dot{x}^T(t)LAx(t) \qquad (22)\\
&+2\dot{x}^T(t)LA_1x(t-h(t))+2\dot{x}^T(t)LBf(\sigma(t))\\
&+2\dot{x}^T(t)LC\dot{x}(t-\eta(t))+2\dot{x}^T(t)LDp(t)=0.
\end{aligned}
$$

For system (3) with nonlinearity located in the sectors $[0,k_j](j = 1,2,...,m)$, if there exists $J = diag(j_1, j_{2,...,j_m})$, then we have

$$j_if_i(\sigma_i)[k_ih_i^Tx(t)-f_i(\sigma_i)]\geq 0, \quad i=1,2,...,m$$

which is equivalent to

$$x^T(t)HKJf(\sigma(t))-f^T(\sigma(t))Jf(\sigma(t))\geq 0. \qquad (23)$$

Similarly, for any $\epsilon > 0$, we have

$$
\begin{aligned}
&-\epsilon p^T(t)p(t)+\epsilon(E_1x(t)+E_2f(\sigma(t))\\
&+E_3x(t-h(t)))^T(E_1x(t)+E_2f(\sigma(t)) \qquad (24)\\
&+E_3x(t-h(t)))\geq 0.
\end{aligned}
$$

Hence, according to (13)-(20) and by adding the zero term (21)-(22) and (23)-(24), we get

$$\dot{V}(x(t))\leq e^{2\alpha t}\{\xi^T(t)[(1-\beta)\mathcal{M}_1+\beta\mathcal{M}_2]\xi(t)\}$$

where M_1 and M_2 defined as in (8) and (9), respectively, and $\xi(t) = [x(t) \quad x(t-h(t)) \quad x(t-h_1) \quad x(t-h_2) \quad f(\sigma(t)) \quad \dot{x}(t) \quad \dot{x}(t-\eta(t)) \quad p(t) \quad \frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds \quad \frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds \quad \frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds \quad \frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds]$. by $(1-\beta)M_1+\beta M_2 < 0$ hold if and only if $M_1 < 0$ and $M_2 < 0$. For showing the convergence rate, we have $\dot{V}(x(t)) \leq 0$, and then $V(x(t)) \leq V(x(0))$. However, $V(x(t)) \geq e^{2\alpha t}x^T(t)Px(t) \geq e^{2\alpha t}\lambda_m(t)P\|x(t)\|^2$. Then, form Definition 1 we conclude that the equilibrium point is globally exponentially stable. This completes the proof. □

Remark 7. *The following stability criteria are presented for finite and infinite sector conditions, for systems without uncertainties.*

Corollary 8. *Under Assumption 5, given $\alpha > 0$. The system (1) without uncertainties satisfying the sector condition (3) is α-exponentially stabilizable if there exist symmetric positive definite matrices P, Q, R, U, F, L symmetric positive semidefinite matrices $Z = diag(z_1, z_2, ..., z_m)$ and $J = diag((j_1, j_2, ..., j_m)$; matrices $N_i; i = 1,2,3$ of appropriate dimension such that the following $LMIs$ holds:*

$$\mathcal{M}_1=\mathcal{M}+\mathcal{M}_1^*>0, \qquad (25)$$

$$\mathcal{M}_2=\mathcal{M}+\mathcal{M}_2^*>0, \qquad (26)$$

$$
\mathcal{M}=\begin{bmatrix}
\Phi_{11} & \Phi_{12} & \phi_{13} & \phi_{14} & \Phi_{15} & \phi_{16} & \phi_{17} & \Phi_{18} & \Phi_{19} & 0 & 0\\
* & \Phi_{22} & \phi_{23} & \phi_{24} & \Phi_{25} & \phi_{26} & \phi_{27} & 0 & 0 & \Phi_{210} & \Phi_{211}\\
* & * & \Phi_{33} & 0 & 0 & 0 & 0 & \Phi_{38} & 0 & 0 & \Phi_{311}\\
* & * & * & \Phi_{44} & 0 & 0 & 0 & 0 & \Phi_{49} & \Phi_{410} & 0\\
* & * & * & * & \Phi_{55} & \phi_{56} & \phi_{57} & 0 & 0 & 0 & 0\\
* & * & * & * & * & \phi_{66} & \phi_{67} & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & \phi_{77} & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & \Phi_{88} & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & \Phi_{99} & 0 & 0\\
* & * & * & * & * & * & * & * & * & \Phi_{110} & 0\\
* & * & * & * & * & * & * & * & * & * & \Phi_{111}
\end{bmatrix},
$$

where

$\Phi_{11} = 2\alpha P + 2PA + 2Q - e^{-2\alpha h_1} - e^{-2\alpha h_2}4R,$

$\Phi_{12} = PA_1 + \frac{1}{2}N_1^TA + \frac{1}{2}N_2^TA + \frac{1}{2}N_3^TA$,
$\Phi_{15} = PB + \frac{1}{2}HKJ + ZH^TA + \frac{1}{2}N_1^TA + \frac{1}{2}N_2^TA + \frac{1}{2}N_3^TA$, $\Phi_{18} = -e^{-2\alpha h_1}6R$, $\Phi_{19} = -e^{-2\alpha h_1}6R$,
$\Phi_{22} = -e^{-2\alpha h_2}4U - e^{-2\alpha h_2}4U + \frac{1}{2}HKJ + ZH^TA + \frac{1}{2}N_1^TA + \frac{1}{2}N_2^TA + \frac{1}{2}N_3^TA$, $\Phi_{25} = ZH^TA_1 + \frac{1}{2}HKJ + ZH^TA + \frac{1}{2}N_1^TA_1 + \frac{1}{2}N_2^TA_1 + \frac{1}{2}N_3^TA_1 + \frac{1}{2}HKJ + ZH^TB + \frac{1}{2}N_1^TB + \frac{1}{2}N_2^TB + \frac{1}{2}N_3^TB$, $\Phi_{210} = -e^{2\alpha h_2}6U$,
$\Phi_{211} = -e^{2\alpha h_2}6U$, $\Phi_{38} = e^{2\alpha h_2}6R$, $\Phi_{311} = e^{2\alpha h_2}6U$,
$\Phi_{49} = e^{2\alpha h_2}6R$, $\Phi_{410} = e^{2\alpha h_2}6U$,
$\Phi_{55} = 2ZH^TB - J + \frac{1}{2}N_1^TB + \frac{1}{2}N_2^TB + \frac{1}{2}N_3^TB$,
$\Phi_{88} = -e^{2\alpha h_1}12R$, $\Phi_{99} = -e^{2\alpha h_2}12R$,
$\Phi_{1010} = -e^{2\alpha h_2}12U$, $\Phi_{1111} = -e^{2\alpha h_2}12U$.

Corollary 9. *Under Assumption 5, the system (1) without uncertainties satisfying the sector condition (3) is asymptotically stable if there exist symmetric positive definite matrices P, Q, R, U, F, L symmetric positive semidefinite matrices $Z = diag(z_1, z_2, ..., z_m)$ and $J = diag((j_1, j_2, ..., j_m)$; matrices $N_i; i = 1, 2, 3$ of appropriate dimension such that the following $LMIs$ holds:*

$$\mathcal{M}_1 = \mathcal{M} + \mathcal{M}_1^* > 0, \qquad (27)$$

$$\mathcal{M}_2 = \mathcal{M} + \mathcal{M}_2^* > 0, \qquad (28)$$

$$\mathcal{M} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \phi_{13} & \phi_{14} & \omega_{15} & \phi_{16} & \phi_{17} & \Phi_{18} & \Phi_{19} & 0 & 0 \\ * & \Phi_{22} & \phi_{23} & \phi_{24} & \Phi_{25} & \phi_{26} & \phi_{27} & 0 & 0 & \Phi_{210} & \Phi_{211} \\ * & * & \Phi_{33} & 0 & 0 & 0 & 0 & \Phi_{38} & 0 & 0 & \Phi_{311} \\ * & * & * & \Phi_{44} & 0 & 0 & 0 & 0 & \Phi_{49} & \Phi_{410} & 0 \\ * & * & * & * & \omega_{55} & \phi_{56} & \phi_{57} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \phi_{66} & \phi_{67} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \phi_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Phi_{88} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Phi_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Phi_{110} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Phi_{111} \end{bmatrix},$$

where
$\omega_{15} = PB + \frac{1}{2}HJ + ZH^TA + \frac{1}{2}N_1^TA + \frac{1}{2}N_2^TA + \frac{1}{2}N_3^TA$,
$\omega_{55} = 2ZH^TB + \frac{1}{2}N_1^TB + \frac{1}{2}N_2^TB + \frac{1}{2}N_3^TB$.

4 Numerical Example

In this section, we provide numerical examples to show the the effectiveness of our theoretical results.

Example 4.1. Consider the following nominal Lur'e system with time-varying delays which is studied in :

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t - \eta(t)) &= Ax(t) + A_1x(t - h(t) \\ &\quad + Bf(\sigma(t)) \\ \sigma(t) &= H^Tx(t) = [h_1, h_2]^Tx(t), \\ &\quad \forall t \geq 0, \end{aligned}$$

where

$$A = \begin{bmatrix} -2 & 0.5 \\ 0 & -1 \end{bmatrix}, \ A_1 = \begin{bmatrix} 1 & 0.4 \\ 0.4 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.5 \\ -0.75 \end{bmatrix}, \ C = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \ H = \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}$$

$Solution.$ From the conditions (8) and (9) of Theorem 6, we let $\alpha = 0.01$, $h_1 = 0.1, h_2 = 0.4$, $t = 0.8$, $\eta = 0.1$, and $K = 0.5$. By using the $LMIs$ Tool-box in $MATLAB$, we obtain $e = 90.4781$,

$$P = \begin{bmatrix} 19.788 & -1.338 \\ -1.339 & 3.417 \end{bmatrix}, Q = \begin{bmatrix} 14.444 & -2.146 \\ -2.146 & 3.859 \end{bmatrix},$$

$$R = \begin{bmatrix} 9.006 & 0.343 \\ 0.343 & 11.791 \end{bmatrix}, U = \begin{bmatrix} 9.330 & -1.725 \\ -1725 & 9.010 \end{bmatrix},$$

$$F = \begin{bmatrix} 1.183 & 0.511 \\ 0.511 & 0.965 \end{bmatrix}, L = \begin{bmatrix} 1.062 & 0.339 \\ 0.339 & 0.640 \end{bmatrix},$$

$$J = \begin{bmatrix} 0.186 & 0 \\ 0 & 139.706 \end{bmatrix}, Z = \begin{bmatrix} 34.472 & 0 \\ 0 & 77.241 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} -2.195 & 0.153 \\ -0.001 & 8.241 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 199.141 & 14.002 \\ -15.095 & -50.721 \end{bmatrix},$$

$$N_3 = \begin{bmatrix} -172.333 & -19.683 \\ 12.910 & 68.330 \end{bmatrix}.$$

Give comparison of maximum allowable value of h_2 for obtained in Corollary 8 with nonlinearity satisfying (2), where $k = 100$ and Corollary 9 with nonlinearity satisfying (2), respectively. We see that, when

$h_1 = 0$ the maximum allowable bounds for h_2 obtained from Corollaries 8 and 9 are much better than those obtained in [8, 9, 17]. The results obtained in [8, 9] may not be used for the case when $h_1 \neq 0$. Moreover, the differentiability of the time delay $h(t)$ is not required in Corollaries 8 and 9.

Example 4.2. Consider the following uncertain Lur'e system with interval time-varying delays with the following parameters :

$$\dot{x}(t) - C\dot{x}(t-\eta(t)) = (A + \Delta A(t))x(t)$$
$$+(A_1 + \Delta A_1(t)x(t-h(t)) + (B + \Delta B(t))f(\sigma(t))$$

where

$$A = \begin{bmatrix} -2 & 0.5 \\ 0 & -0.9 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}, C = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, H = \begin{bmatrix} 0.4 \\ 0.8 \end{bmatrix},$$

$$D = E_1 = E_3 \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$$

$Solution.$ From the conditions (8) and (9) of Theorem 6, we let $\alpha = 0.01,\ h_1 = 0.1, h_2 = 0.4,\ t = 0.8,\ \ \eta_d = 0.2,\ \eta = 0.1$, and $K = 0.5$. By using the $LMIs$ Tool-box in $MATLAB$, we obtain $e = 19.7170$,

$$P = \begin{bmatrix} 2.949 & -1.087 \\ -1.087 & 1.762 \end{bmatrix}, Q = \begin{bmatrix} 0.724 & 0.170 \\ -0.170 & 4.449 \end{bmatrix},$$

$$R = \begin{bmatrix} 10.830 & -0.927 \\ -0.927 & 6.855 \end{bmatrix}, U = \begin{bmatrix} 9.158 & -1.710 \\ 1.710 & 7.854 \end{bmatrix},$$

$$F = \begin{bmatrix} 0.108 & -0059 \\ -0.063 & 0.170 \end{bmatrix}, L = \begin{bmatrix} 0.100 & -0.063 \\ -0.063 & 0.170 \end{bmatrix},$$

$$J = \begin{bmatrix} 0.018 & 0 \\ 0 & 68.320 \end{bmatrix}, Z = \begin{bmatrix} 6.210 & 0 \\ 0 & 1.435 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 3.376 & -0.054 \\ 1.054 & 1.788 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 349.530 & 223.699 \\ 194.921 & 108.959 \end{bmatrix},$$

$$N_3 = \begin{bmatrix} -343.448 & -223.977 \\ -193.654 & -107.171 \end{bmatrix}$$

Thus, the system (1), is 0.4-exponentially stabilizable. Given $\alpha > 0$, we will give the values of the maximum allowable upper bounds of the uncertain Lur'e system with interval time-varying delay (6) for difference η_d of the delay for different decay rates $0.1 \leq \alpha \leq 0.4$. From Theorem 6, we obtain the maximum allowable upper bound of the time-varying h_2, as shown in Table 3.

Table 3. Maximum allowable upper bounds h_2 of the uncertain Lur'e system with interval time-varying delay (6) for different values of the η_d and decay

	$\eta_d = 0.2$	$\eta_d = 0.4$	$\eta_d = 0.6$	$\eta_d = 0.8$
$\alpha = 0.1$	0.6400	0.6360	0.6300	0.6150
$\alpha = 0.2$	0.6200	0.6170	0.6120	0.6100
$\alpha = 0.3$	0.5950	0.5930	0.5900	0.5800
$\alpha = 0.4$	0.5710	0.5700	0.5670	0.5600

5 Conclusion

In this paper, we have investigated robust exponential stability criteria for uncertain neutral and Lure dynamical systems with sector-bounded nonlinearity. Based on Lyapunov-krasovskii theory, new delay-dependent sufficient conditions for robust exponential stability have been derived in terms of LMIs. The interval time-varying delay function is not required to be differentiable which allows time-delay function to be a fast time-varying function. The global exponential stability for uncertain neutral and Lure dynamical systems with some conditions are investigated. Numerical examples are given to illustrate the effectiveness of the theoretic results which show that our results are much less conservative than some existing results in the literature.

6 Acknowledgements

This work is supported by University of Pha Yao, Pha Yao, Thailand.

References:

[1] J. H. Park, Novel robust stability criterion for a class of neutral systems with mixed delays and nonlinear perturbations, *Appl. Math. Comput.*, 161, 2005, pp. 413421.

[2] F. Qiu, B. Cui, and Y. Ji, Further results on robust stability of neutral systemwith mixed time-varying delays and nonlinear perturbations, *Nonlinear Anal. Real World Appl.*, 11, 2010, pp. 895–906.

[3] P.-L. Liu, Improved results on delay-interval-dependent robust stability criteria for uncertain neutral-type systems with time-varying delays, *ISA Transactions*, 60, 2016, pp. 53–66.

[4] W. Weera and P. Niamsup, Novel delay-dependent exponential stability criteria for

neutral-type neural networks with non-differentiable time-varying discrete and neutral delays, *Neurocomputing*, 173, 2016, pp. 886–898.

[5] A. Seuret and F. Gouaisbaut, Wirtinger-based integral inequality: application to time-delay systems, *Automatica*, 49, 2013, pp. 2860–2866.

[6] Y. Ma and L. Zhu, New exponential stability criteria for neutral system with time-varying delays and nonlinear perturbations, *Adv. Difference Equ.*, 44, 2014, pp. 1–12.

[7] Y. Chen, W. Qian and S. Fei, Improved robust stability conditions for uncertain neutral systems with discrete and distributed delays, *J. Franklin Inst.* , 352, 2015, pp. 2634–2645.

[8] B. Li, A further note on stability criteria for uncertain neutral systems with mixed delays, *Chaos Solitons Fractals*, 77, 2015, pp. 72–83.

[9] J. Gao, H. Su, X. Ji, and J. Chu, Stability analysis for a class of neutral systems with mixed delays and sector-bounded nonlinearity, *Nonlinear Anal. Real World Appl.*, 9, 2008, pp. 2350–2360.

[10] K. Ramakrishnan and G. Ray, Improved delay-range-dependent robust stability criteria for a class of Lur'e systems with sector-bounded nonlinearity, *J. Comput. Appl. Math.*, 235, 2011, 2147–2156.

[11] T. Botmart, P. Niamsup, and V. N. Phat, Delay-dependent exponential stabilization for uncertain linear systems with interval non-differentiable time-varying delays, *Appl. Math. Comput.*, 217, 2011, pp. 8236–8247.

[12] K.-W. Yu and C.-H. Lien, Stability criteria for uncertain neutral systems with interval time-varying delays, *Chaos Solitons Fractals*, 38, 2008, pp. 650–657.

[13] Q.-L. Han, A. Xue, S. Liu, and X. Yu, Robust absolute stability criteria for uncertain Lur'e systems of neutral type, *Int. J. Robust Nonlinear Control*, 18, 2008, pp. 278–295.

[14] C. Yin, S.-m. Zhong, and W.-F. Chen, On delay-dependent robust stability of a class of uncertain mixed neutral and Lur'e dynamical systems with interval time-varying delays, *J. Franklin Inst.* , 347, 2010, pp. 1623–1642.

[15] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay System*, Birkh aauser, Boston, Mass, USA, 2003.

[16] W. Duan, B.Du, Z.Liu, Y. Zou, Improved stability criteria for uncertain neutral-type Lur'e systems with time-varying delays, *J. Franklin Inst.* , 351, 2014, pp. 4538–4554.